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No. 51

PRINCIPLES AND PRACTICE OF ASSEMBLING MACHINE TOOLS

PART II

By ALFRED SPANGENBERG

CONTENTS

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CHAPTER I

ASSEMBLING AN ENGINE LATHE*

While the problems encountered in assembling engine lathes are not as difficult of solution as those met with in assembling machine tools of a more complicated nature, thorough and careful consideration of the methods employed is essential in order to minimize the cost. The most important operations involved are the scraping of the bed and carriage and the lining up of the head- and tail-stock. To a large extent the cost of these operations is dependent on the accuracy of the machine work.

It is the object of the present chapter to discuss the methods employed in machining, and to illustrate and describe the erecting process on the bed. The principles involved in assembling the units have been fully outlined in Part I of this treatise. (MACHINERY'S Reference Series No. 50.) For the purpose of giving a concrete example, a 24-inch engine lathe with quick-change gear device is selected, the general features of which are shown in Fig. 1.

Planing the Bed and Carriage

The practice of some makers to rough out the surfaces to be planed on the bed and carriage, and allow them to season before taking the finishing cuts is indicative of the modern tendency toward accurate machine work as a means to avoid unnecessary scraping. The seasoning process simply consists in letting the casting stand in some convenient place in such a way that it will not be subjected to any outside forces, and allowing the stresses in the casting itself to become equalized.

Next in importance to providing accurate planers on which to machine the bed and carriage, is the necessity for gages that will enable interchangeable work to be produced. The gages shown in Fig. 2 are particularly well adapted to this class of work, and are far superior to the common type having a bearing on both sides of each V. Feelers or thickness gages are used in connection with the gages illustrated, in order to measure the amount of error. The advantages this form possesses over the common type are that the V's on the casting being fitted will both be of the same width, and no difficulty is experienced in keeping the gage level, since it always lies in a horizontal plane by resting on the top of each V. At *A* and *B* are shown gages for the bed and carriage respectively; these gages are made of steel ¼ inch thick.

Assuming, for example, that the V-surface on the bed at *C* is tight to the gage, and a 0.002 inch feeler will just pass between the bed and the gage at *D*, then 0.002 is the exact amount that must be planed off the surface *C*. When these two surfaces fit the gage so that no error

*MACHINERY, November, 1909.

Fig. 1. Twenty-four-inch Engine Lathe with Quick-change Gear Device

can be detected with a 0.001 feeler, the gage is turned end for end, and the surfaces *E* and *F* are tested in the same manner. It is, of course, necessary to set the gage square with the bed and this is accomplished by trying the feelers on both sides of the gage. The same remarks apply to the carriage gage *B*. At *G* and *H* in the same engraving are shown gages in the form of cast-iron blocks about 6 inches long which are used for testing the ways on bed and carriage as indicated; *I* is a sheet steel strip fastened to one end of gage *H* and this is used to test the apron seat *J*. All other measurements are tested with ordinary height blocks or caliper gages, as the case may require.

Referring now to the carriage, the sequence of operations in finish planing it is to plane the bearing for the shoe, and square up the ends, and then turn it over in the position shown in the engraving and plane the V's and other surfaces on this side. It is good practice to plane lathe carriages of this size in lots of six at a time by placing them in

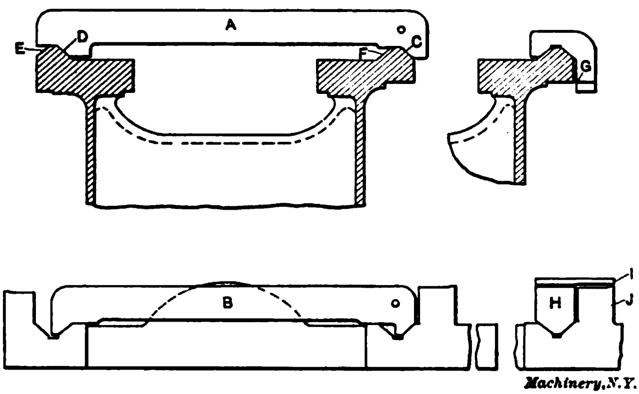

Fig. 2. Gages for Testing the Planing of Lathe Bed and Carriage

a "string" on the planer table. When completing the final operation, that of planing the V's, it is evident, however, that the carriages farthest away from the angle plate by which they are squared up, are particularly liable to error. With careful setting up, this error should not exceed 0.003 inch, which is easily and quickly scraped off by the assemblers.

Machining the Head- and Tail-stock

The methods of machining head- and tail-stocks vary greatly in different shops, and also with different sizes. Some makers first finish all the planing and then perform the boring operations and scrape in the head-stock spindle, while others leave a finishing cut to be taken on the bottom of the head- and tail-stock after the boring and scraping are completed, by setting up the castings on arbors held in V-blocks on the planer table. Both systems, or modifications of both, are frequently used in the same shop. While the first-mentioned system is the most economical in that it saves setting up the work on the planer twice, it is essential that the machining be such that very little scraping is required on the head-stock boxes; otherwise the head-stock will be thrown out of line in fitting the spindle, which necessitates replaning the head-stock and possibly the tail-stock.

In any event, it will always pay to leave 1/16 inch stock to be bored out of the bearing boxes, so that after the boxes are fitted in the head-stock it can be replaced in the boring jig and a finishing cut taken in the boxes, allowing 0.005 inch to be reamed out. Adjustable shell reamers are mounted on the boring bar for this purpose. When this precaution is taken, and the spindle is accurately ground to size, very little scraping should be necessary to make a good bearing. The object of leaving the 1/16 inch to be bored out with a cutter before reaming, is to insure that the reamer has an even cut and that sufficient stock is left for reaming.

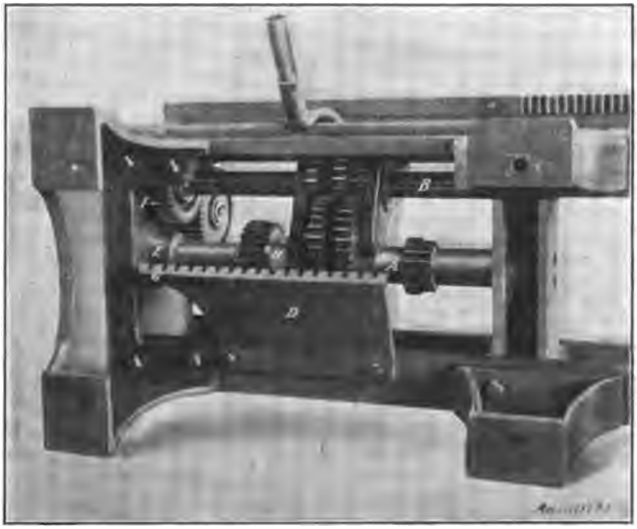

Fig. 3. Quick-change Gear Mechanism of Lathe shown in Fig. 1

Boring and Drilling the Bed

When all the planing is finished on the bed, it is next sent to the drilling department where the first operation consists in boring the shaft holes for the tumbler mechanism shown in Fig. 3. This operation is performed on a horizontal drilling and boring machine with the aid of a jig which also locates the sweep clamping bolt hole. The shafts *A* and *B* run in long cast-iron bushings that are a light drive fit in the bed, and to provide for standardization and enable the bushings to be finished in large quantities, the holes in the bed are reamed to gage, adjustable shell reamers being mounted in the boring bar for this purpose. The hole for stud *C* is drilled and then finished with a rose reamer, after which it is hand reamed to gage. This hole and the sweep clamping bolt hole are the only ones that require facing, the method of taking the measurements being apparent from Fig. 4; *A* is a sheet-iron templet having an outline indicated by the heavy line; the opening at *B* locates the surface to which the stud boss is faced, while at *C* is shown whether there is sufficient clearance cored out for a feed gear. The templet is located sidewise by the spline *D* which is riveted to the templet and fits the keyway in the bed; and endwise by being brought flush with the end of the bed. When in this position lines are scribed at *B* and *C*, after which the templet is removed and the hole counterbored to the line; *C* is chipped out later on if necessary.

To locate the surface *E* for the sweep clamping bolt hole, the gage *F* is used, its construction being clearly shown in Fig. 4. All lateral measurements on the bed are taken from the surface *B*. Facing surface *E* completes the operations for this setting, and the bed is then moved around for drilling and tapping the lead-screw box bolt holes.

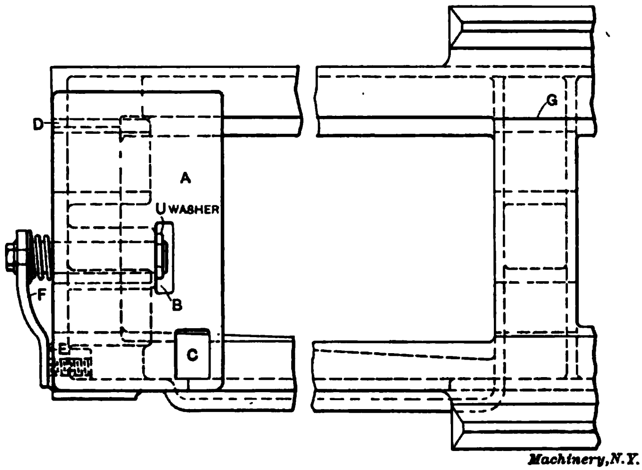

Fig. 4. Templet and Gage used when Boring the Bed, shown in Position

In Fig. 12, *A* and *B* are jigs for drilling the front and rear lead-screw box seats, respectively. As will be seen, both jigs are located from the flat surface on top of the bed, the rear jig clearing the front V. Jig *A* is located endwise by seating against the hub *C*; the wooden pole *D* is used to set jig *B*. This pole has lines cut on one side, giving the settings for various lengths of beds up to 16 feet, so that the distance between the boxes, when bolted on, will be correct for the lead-screw. For longer beds, especially those in two sections, jig *B* is set from the rear end of the bed, and after the boxes are bolted on, the measurement is taken for the length to cut the lead-screw; it is not practicable to set the rear box or cut the lead-screw in the same way in this case as in the previous one, due to the error that is likely to occur in the length of the bed. This completes the work at the horizontal drill, and the bed is now moved to a radial drill, where the holes for the rack are next drilled. Special eccentric clamps are used for holding the rack in place, one of which is seen resting on top of the bed in Fig. 5. The advantages of this clamp over the common C-form are that it does not mar the work, no copper packing is needed, the clamping and loosening is quickly accomplished, and there is no danger of shifting the rack when clamping it.

When the holes for the rack are all drilled, including the pin holes, the drill-hand taps and enters one screw in each rack section, so as to keep the rack in place after the clamps are removed, and until the bed reaches the assemblers. This avoids tying up the radial drill while the rack is being fastened on the bed, which is the practice in some shops.

On top of the bed in Fig. 5 are shown two fixtures *A* and *B*, respectively, which are used to locate the tumbler locking bar *D*, Fig. 3. Fixture *A* is also used to set the bushings *E* and *F*, Fig. 3, by inserting the stud in the fixture in place of the stud *C*, and bringing the bushings (which are straight) up against the hubs shown on the fixture. Now, with fixture *B* in position in the tumbler shaft hole, the locking bar is set so that its first notch *G*, Fig. 3, fits over the projection *C* of fixture *A*, Fig. 5. The plates *D* and *E* on fixture *B* are then moved so as to enter slots near the ends of the locking bar, after which the locking bar is set so that its slots bottom in the fixture plates. In this position the bar is marked off, and after drilling the two bolt holes, which have 1/16-inch clearance, it is again placed in position and clamped by its bolts. It is then shifted so as to bring it in the correct relation to the fixtures, and then the pin holes are drilled. With the bushings *E* and *F* (Fig. 3) located as previously mentioned, their set-screw holes are now drilled.

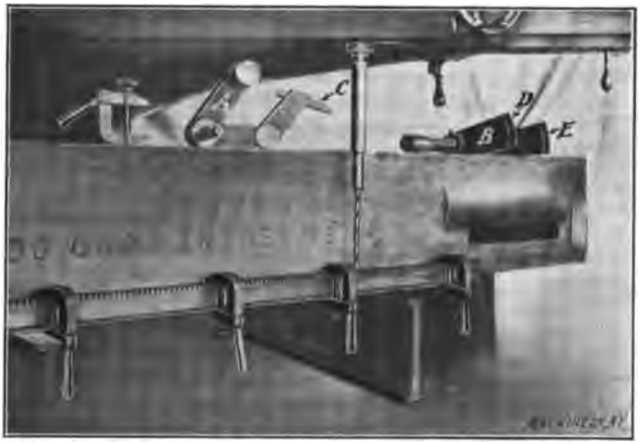

Fig. 5. Special Clamps for Holding Rack in Position when Drilling, and Fixtures used for Locating Tumbler Locking Plate

The legs are set and marked off on the bed with the aid of a wooden pole having lines scribed on it representing the center line of each inner leg. After these holes are drilled and tapped and the legs bolted on, the bed is next turned right side up for drilling the head-stock clamping bolt holes. The jig for drilling these holes is located on the bed by the keyway *D*, surface *G*, and hub *B*, Fig. 4, so that when the head-stock, which also was drilled by this jig, is bolted on, the feed gear on the spindle will line up with the intermediate feed gear in the bed, and the head-stock casting will match the end of the bed. This completes the drilling, and the bed is next sent to the assemblers, where the actual work of assembling proper begins.

Scraping the Carriage and Assembling the Tumbler Mechanism

It is the general practice to keep the head-stocks, tail-stocks, carriages, tumbler members, etc., in stock, completely assembled, these units being identical for any length of bed. This method will be considered here, it being assumed that the units already have been brought to the assemblers. Two men usually are employed in assembling a lathe of this size, since a larger number cannot advantageously be used. Two men should be able to assemble such a lathe in 40 hours, total time. The operations of scraping on the carriage and assembling the tumbler mechanism are, of course, carried on simultaneously, one man working on each job; but for the purpose of description, each operation will be considered separately.

The preliminary operations on the bed consist of rough scraping the V's and inside bearing for the tail-stock, fastening on the rack, and polishing the sides and top. When fastening on the rack, and polishing the sides, the bed is turned over for the convenience of the workmen. The rack, which was temporarily fastened on by the drill-hand, is now removed and all the holes tapped, after which it is screwed fast, the pin holes reamed, and the pins driven in. A carpenter's brace is used for the taps and reamer. As the rack already has been polished on a disk grinder, it is only necessary to rub it with emery cloth to obtain a good finish. The bed is now turned right side up and carefully leveled, using iron wedges. During these preliminary operations the painting and any necessary chipping is done.

For scraping the carriage to fit the bed a special lifting device is used to facilitate turning the carriage over. This device is described by the author in MACHINERY'S Reference Series No. 50. When the carriage is being pulled along the bed for the purpose of finding the bearing, the lifter bolts are slackened off so as to prevent any danger of springing the carriage.

Fig. 6 clearly indicates the method of squaring the carriage with the bed. As will be seen, the sweep bar *A* is held in firm contact with the angle and bottom bearing of the carriage by means of two flat steel springs *B*, bent to the shape shown. Brass shoes *C*, riveted to these springs, prevent the latter from cutting or scratching the bar. The function of coiled spring *D* is to hold the sweep bar collar *E* in close contact with the end of the carriage. With this device, the sweep is easily and quickly applied or removed, more precise measurements are obtained than by having an operator hold it, as is the usual practice; and besides, only one man is required to perform the operation of testing. In operation, micrometer point F is set to one position as shown, using a piece of cigarette paper as a "feeler"; then the sweep is rotated to the opposite side and a measurement taken as before. The carriage is scraped so as to turn the face-plate about 0.001 inch concave.

Very little scraping is necessary on the bed, merely enough to smooth the V's and break up the bearing, the tool marks being visible after the scraping is completed. When this work is accomplished, the carriage gibs are fitted, the apron is bolted in place, and the cross-feed screw, shoe, etc., are placed in position. Next, the tail-stock cricket is placed on the bed and its packing fitted, after which it is pulled the entire length of the bed to determine if the latter is straight. After any high spots on the bed are scraped off, the tail-stock traverse bracket

Fig. 6. Top View of Carriage, showing Method of Squaring the Carriage with the Bed. Springs B and D hold the Sweep in Position, enabling One Man to make the Test

is bolted onto the cricket, and then the tail-stock and its shoes are placed in position.

Referring again to Fig. 3, the tumbler mechanism is assembled in the bed as shown in the illustration, the operation being so simple that no explanation is necessary. Holes in the bed for the oil pipes (not shown) are drilled by means of a pneumatic drill, this being done, of course, preceding the assembling operation. The slot in the interlocking plate H is now marked off, the plate removed, and the slot cut. The position of this slot is determined by having the tumbler gears in a central position between the largest change gears. With the interlocking plate again screwed onto the tumbler, the thirty-four holes in the bed for the tumbler locking pin are now ready to be drilled.

Drilling these holes is the most interesting operation on this part of the lathe, the manner of accomplishing it being immediately apparent from Fig. 7, which shows the tools used, some of them being seen on top of the bed. The method of holding the tumbler in engagement with the various change gears while drilling the holes in the bed is clearly shown in Fig. 8. Paper, to the thickness of about 0.005 inch, is placed between the teeth of the engaging gears before they are brought into mesh, and when the jack screw *A* is tightened sufficiently to hold the tumbler rigidly in place, the gears are in proper mesh. The holes are first drilled with a special drill that fits the locking pin hole in the tumbler and has flutes milled only a short distance up from the point, so as to avoid cutting the hole in the tumbler. Fig. 7 shows this drill in position in the air drill. For finishing the holes, a special rose reamer, shown at *B*, is used in the same manner.

The tools are fed into the work by means of the bar *C*, which is pointed on one end so that it can be driven into the floor to prevent

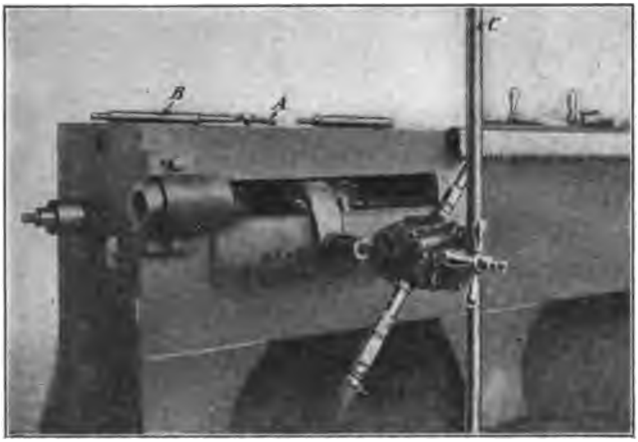

Fig. 7. Method of Drilling and Reaming the Holes in the Lathe Bed for the Tumbler Locking Pins

slipping. The operator presses his shoulder against the upper end of the bar, holds the throttle of the air drill with the left hand and pulls on the bed with the right hand. Each hole is drilled in succession, alternating between the top and bottom rows. The record for drilling and reaming these thirty-four holes, including the time required to set the tumbler, is 50 minutes. In setting the tumbler for drilling, its lateral movement for the various positions is controlled by the interlocking plate engaging the respective slots in the locking bar. (See Fig. 8.)

To mark off the groove which is seen between the two rows of holes in Fig. 1, a special scriber is used that fits the tumbler locking-pin hole, the tumbler being held in a neutral position by the locking bar. Two circles are scribed, one at each end of the groove to be cut, and then a straightedge is used in scribing lines connecting the two, the lines acting as a guide when chipping the groove with an air hammer.

Inserting the handle, chipping the groove, and fastening on the number and index plates completes the operation on the tumbler mechanism.

Lining Up the Head-stock and Tail-stock

The bed is now ready to receive the head-stock and while this is being fitted on by one of the assemblers, the other is working on the lead-screw, lead-screw boxes, and change gear sweep. As the head- and tail-stock were tested for alignment before being sent to the store-room, it is now only necessary to line up the head-stock on the bed and fit the taper dowel pins.

The method of testing these parts is interesting, inasmuch as the test arbor used is somewhat out of the ordinary. A jig is used that represents the head-end section of a 24-inch lathe bed. The jig and whole outfit of fixtures used, together with a head- and tail-stock in

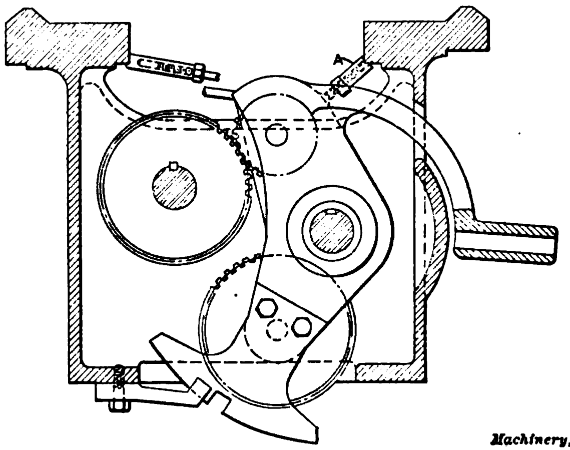

Fig. 8. Method of Holding Tumbler when Drilling and Reaming the Locking Pin Holes by the Method shown in Fig. 7

position for testing, are shown in Fig. 9. As will be seen, the test bar is square at *A*, near each end, the object being to use the indicator on a flat surface when testing the spindle for parallelism with the V's, and on a cylindrical surface when testing the spindle taper hole for concentricity. The two squares are integral with a sleeve that can be turned independently of the bar, and in this way one plane surface can be set at a mean between the "high" and "low" point on the bar. This adjustment is obviously necessary, since the bar is particularly liable to run out at its free end, due to a number of conflicting elements, the error in any one of which may be infinitesimal. When using the plane surfaces, one of these is always trued up with the square *B*, as shown in the engraving.

The reason for providing a plane surface on which to indicate is this: Suppose, for example, that we are testing the spindle for alignment sideways, and further that the axis of the spindle actually is parallel sideways with the bed, but that the axis does not lie in a horizontal plane, it being high on the front end, say 0.005 inch in the length of the test bar. Then in indicating on the older type of bar with cylindrical collars, the line of motion of the indicator point in traveling from one collar to the other is not parallel with the axis of the test bar and therefore the readings are false. Now, with the form of bar having plane surfaces, assuming the conditions to be the same as regards the alignment of the spindle, the reading will show that the alignment is perfect sideways, because the indicator point is moving on a plane surface.

Referring again to Fig. 9, the fixture *C* is guided on the V tracks of the bed and is constructed so that it is adjustable for holding a Starret indicator *D* either on the side or top of the bar as the case may require Having explained the use of the tools and fixtures, the method

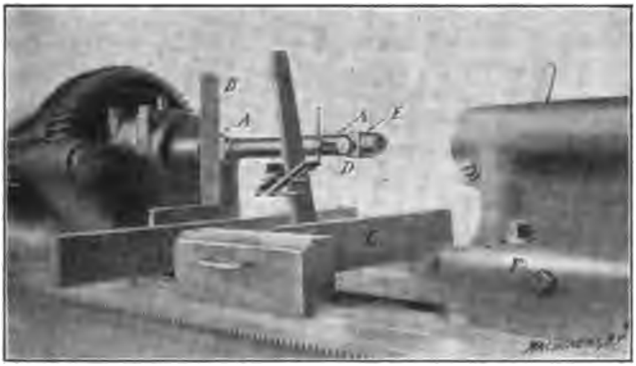

Fig. 9. Special Form of Test Bar for Aligning Head- and Tail-stock. Plane Surfaces are used for Testing Parallelism, and Cylindrical Surfaces for Testing Concentricity

of "lining up" the head-stock and tail-stock, both with reference to the V's and to each other, is to place them on the bed and approximately set the head-stock true sideways by inserting keys in the keyways planed in the head-stock and bed for this purpose.

With the clamping bolts tightened lightly (the bolts have 1/16-inch clearance), the head-stock spindle taper hole is first tested for concentricity by indicating on the cylindrical surface *E* while slowly rotating the spindle. Then with the squares *A* set as previously explained, the head-stock is moved around by knocking it with a babbitt hammer until the reading on both surfaces is the same. The bar is then moved over into the tail-stock spindle hole, and after the set-over screw *F* is adjusted so as to bring this spindle in line with that in the head-stock (the reading being the same as before) the tail-stock spindle is tested for alignment sideways. Now the indicator is set on top of the bar, with the latter turned so as to use the same surface, and then the tail-stock spindle is tested for alignment in a horizontal plane, after which the bar is again set into the head-stock spindle hole and that spindle tested both with reference to the V's and to the tail-stock spindle. It is understood, of course, that if any errors are discovered which exceed the allowable limits of variation, the part at fault is either filed and scraped to bring it true, or machined, if the circumstances warrant.

Turning again to the lathe bed, we will assume that the head-stock is lined up as just described, and the clamping bolts are tightened down hard. The taper dowel pin holes are then drilled, reamed, and the pins driven in, care being exercised to see that the pins are a good fit. Next the faceplate is screwed on, ready to be turned off.

Lining Up the Lead-screw Boxes

As was previously stated, work has been progressing on the lead-screw boxes and other minor details. For the purpose of lining up the boxes and also to test the alignment of the lead-screw bearings in the apron, a short arbor is used that represents the lead-screw. First the apron is tested, and then the lead-screw boxes are bolted onto the bed and lined up with the apron. Sometimes it is necessary to file the apron seats or adjust the boxes to bring them into proper alignment; but with careful planing and thorough inspection of the parts, this should not be required. Two special gages are used to facilitate the aligning; one gage reaches down from the flat track on top of the bed for horizontal measurements, while another is held against the side of the bed just below the V to test sideways. Both gages are provided with micrometer points to enable the accurate measurement of error.

Referring to Fig. 1, the head-end lead-screw box is set longitudinally when the change gear sweep is in place, so as to line the box with reference to the sweep bearing on the end of the bed. When properly set, the boxes are drilled and reamed for the taper dowel pins; then the lead-screw is put in place, its checknuts screwed on, and the gears on the sweep brought into proper mesh.

The Finishing Operations and Inspection

With the cone belted up to a countershaft or other source of power, the bearings are thoroughly oiled, and the lathe is run idle for a while preparatory to turning off the face-plate. The V's on the bed and bearings on top of the carriage are now spotted with a scraper, while all other finished surfaces receive their final polishing. The centers and their respective holes in the spindles were fitted to male and female gages during the machining process, so that now it is only necessary to place the centers in position. Clamping on the center-rest is the final assembling operation.

All machines are more or less defective, as it is practically impossible to make anything absolutely perfect. Knowing this, the builder establishes a limit within which the error will not materially affect the working of the machine, and furnishes the inspector with a list of allowable limits.

The inspection is carried on as the work proceeds, so that no part is neglected, and no defective material or faulty workmanship is allowed to pass. Gearing of all kinds is inspected and tested for alignment and smoothness of operation. The fits of all wearing surfaces are tested, as well as the fit of the various screws and binding and clamping fixtures. All information obtained from the inspection is entered on a printed form. Each machine is given a serial number, and the reports are filed in the office, so that in case of any trouble arising or any repairs being required for a given lathe, an exact record of its condition when it left the shop is available.

After truing up the face-plate, it is tested by means of a straight-edge and cigarette paper, this kind of paper being the best for the purpose. The spindle is tested for end motion when running, by application of the Starrett indicator to the face of the spindle nose. To prove the alignment of the head spindle with the shears of the lathe and the alignment of the taper hole with the spindle under actual working conditions, a steel test bar is provided which fits the taper hole and projects 18 inches from the spindle. This bar carries three cast-iron collars, placed one at each end and one in the middle, from which all measurements are taken. A light cut is taken across these collars with a keen diamond-point tool, and the collars are then measured with a micrometer. As it is desirable when boring a hole, to have the taper, if any, large toward the front end, the front end of the head-stock was purposely set slightly toward the rear of the bed so that the outer collar should be found 0.0005 inch larger than the collar next the spindle. The alignment of the spindle in the vertical plane is again tested by attaching the indicator to the tool-post and traversing the carriage along its ways with the contact point of the indicator pressing against the top surface of the collars.

In testing the alignment of the taper hole with the spindle itself, after the collars have been turned off, the test bar is removed, turned half way round and replaced in the spindle; the indicator is then put in the tool-post in place of the turning tool, and with the contact point on the center line, the indicator is traversed past the collars, the variation in readings showing twice the error in the alignment of the taper. The alignment of the tail-stock spindle is now tested, the method being the same as in the previous instance.

CHAPTER II

ASSEMBLING A MOTOR-DRIVEN PLANER*

In planer erection, the principal points to be observed are that the housings must be parallel with each other and square with the bed; accuracy is essential in the fit of all sliding members and in the truth of all plane bearing surfaces; the gears should mesh properly and run smoothly; and the system must be such as to permit the various parts to be easily and quickly assembled, and avoid the necessity of fitting the members together for the laying-out operations. This, of course, presupposes the employment of jigs and gages, but, owing to the fact that planers of the 48-inch size and upwards are seldom built in large numbers at a time, and further, that there are many different types of drive, it is impracticable to indulge very freely in the use of elaborate jigs for duplicating the larger parts in the larger sizes. However, many of the members used in the construction of planers are common to several different sizes and different types of drive, so that with a few very simple jigs and gages, the standard members can be made interchangeable, and in this way much expensive handling in laying-out, and the consequent lost time, is avoided.

This chapter will deal principally with the erecting process on the bed, since the methods and processes employed in assembling the smaller units do not differ greatly from the practice used in assembling those of other machine tools. All the principles involved in the erection of a small planer are encountered in the case of a large machine, and many other complicating factors are added; hence, the erection of a planer of the latter class will be described in detail. For this purpose a 48-inch motor-driven planer is selected, the general features of which are apparent from a study of the half-tone Fig. 10.

As the machining processes are so intimately correlated with those of assembling, and as the methods employed in the latter are controlled to a great extent by the former, a brief description of the points to be observed in machining, together with the gages used for testing the larger members, will be illustrated and described. Referring to Fig. 11, *A* indicates a gage for testing the V-surfaces on both the bed and table, it being shown in position on the latter. The surfaces *B*, which support the gage, are finished first, and in this way the gage is always kept in a horizontal plane and both tracks are the same width, so that when the table is placed in position on the bed, the top of the table will be square with the housings. Another advantage of this gage over the usual form having a bearing on both sides of the V, is that only two parallel surfaces are finished and tested at a time, which often saves changing the planing tools and resetting the tool-heads. The gage is squared by trying a 0.001-inch feeler on both sides of the gage at C and C_1. To determine the width of the

* MACHINERY, December, 1909, and January, 1910.

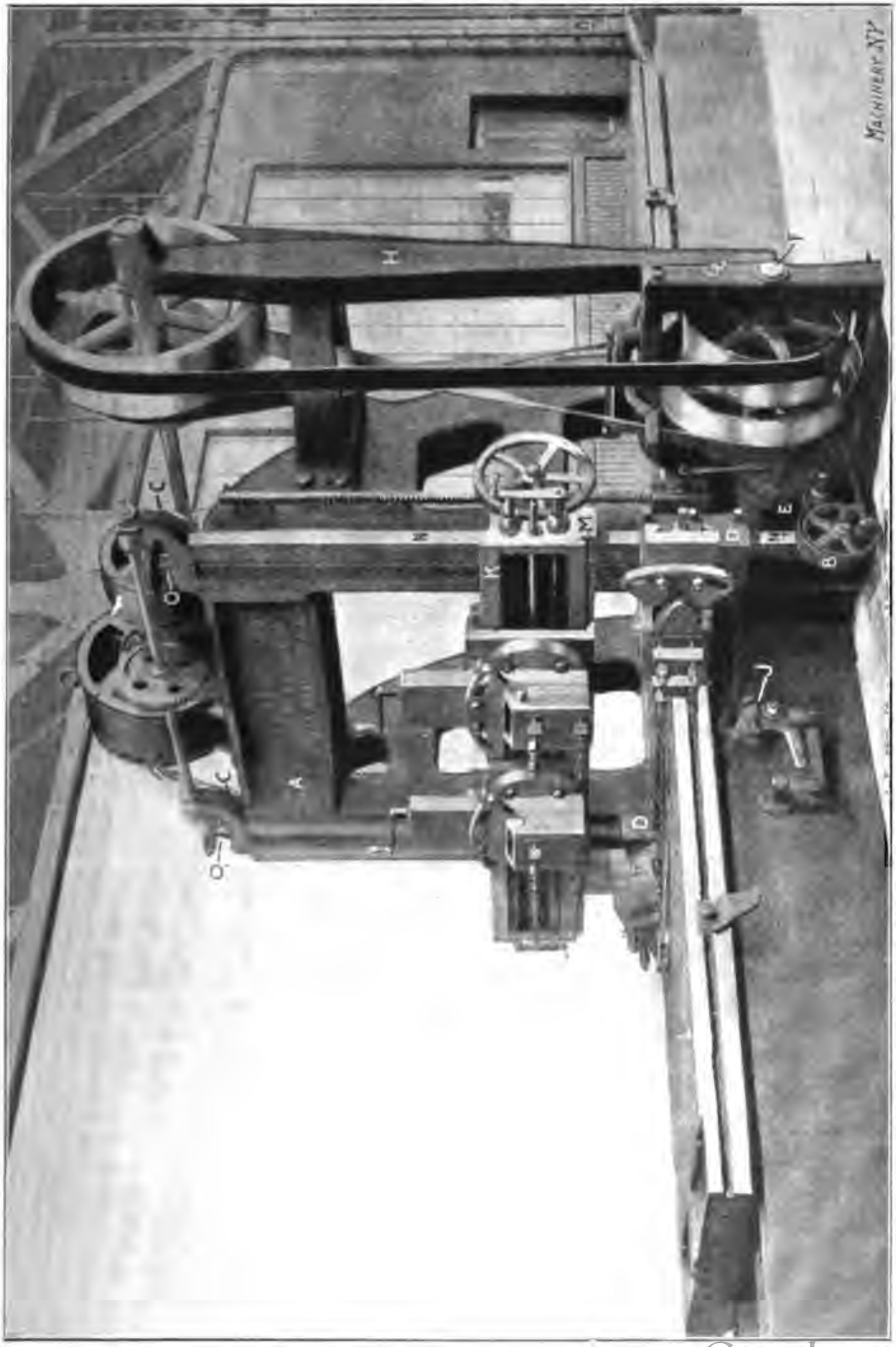

Fig. 10. Forty-eight-inch Motor-driven Planer, used as Example for Illustrating Principles involved in Assembling Planers

V-tracks on the bed and table, measurements are taken at *D* and D_1, respectively.

The gage just described is not adapted for measuring the rack seat, however, and, therefore, another gage is provided which fits both sides of the V's, and is represented by the dotted lines. This gage carries a slide *E* which measures the rack seat. At *F* is shown a sheet-iron support which fits either gage and prevents it from tipping over.

In the same illustration, at *G*, is shown the method of testing the table on surfaces *H* and H_1, which have a clearance of 0.005 inch between corresponding surfaces on the bed. As will be seen, the cast-iron gage block *I* fits the V on the table and is provided with two surfaces, one, *J*, for setting the planer tool, and another, *K*, for testing surface *H* after the finishing cut is taken. To the right, at *L*, is shown another cast-iron gage, this being used for setting the tools and testing the surfaces *M* and *N*; at L_1 the gage is shown in position in the V-track of the bed. As will be pointed out later, the object of finishing the surface *N* last is to provide a locating surface for the jig for boring the bed. In this way, the rack gear shaft hole is bored the correct distance from the V-tracks, so that when the table is in position, the table rack will mesh properly with its gears. Length gages *O*, *P*, *Q*, and *R* are for testing the measurements indicated, the latter also being used for taking the length of the arch *A*, Fig. 10.

It is essential, of course, that the housing cheeks on the bed be perfectly square with the V-tracks and parallel with each other. To accomplish this, the sweep *S*, carrying the Starrett indicator *T*, is used in connection with the straight-edge *U*, which reaches across the bed and extends a sufficient amount beyond one side to accommodate the swing of the indicator. By this means very accurate results are obtained. The operator holds the bar *V* in contact with the bed, and the flanged bearing *W*, being of ample diameter and ground true with the bar, keeps the bar in a vertical plane.

Boring and Drilling the Bed

The bed, having passed inspection with regard to the accuracy of the planing operations, is now sent to the horizontal boring and drilling machine where all the boring, drilling, and tapping operations are completed; one setting only is required, as the machine is provided with two separate columns carrying spindle heads, both working on the sides of the bed simultaneously. In the line-engraving Fig. 13 the bed is seen resting on parallels *A* with the jig *B* in position ready for the operations just mentioned. As will be observed from the top view, the jig consists of three main castings *C*, *D*, and *E*, respectively, which are bolted to the three cast-iron tie bars *F*; this construction permits adjustment of members *D* and *E* to compensate for beds having different widths over the housing cheeks. The jig rests on the top of the bed, and is located endwise with reference to the jig members *D* and *E* matching the bed casting, so that when the housings, which have been drilled by a separate jig, are bolted onto the bed, the castings will match properly. Set-screws *G* square the jig with the bed by holding

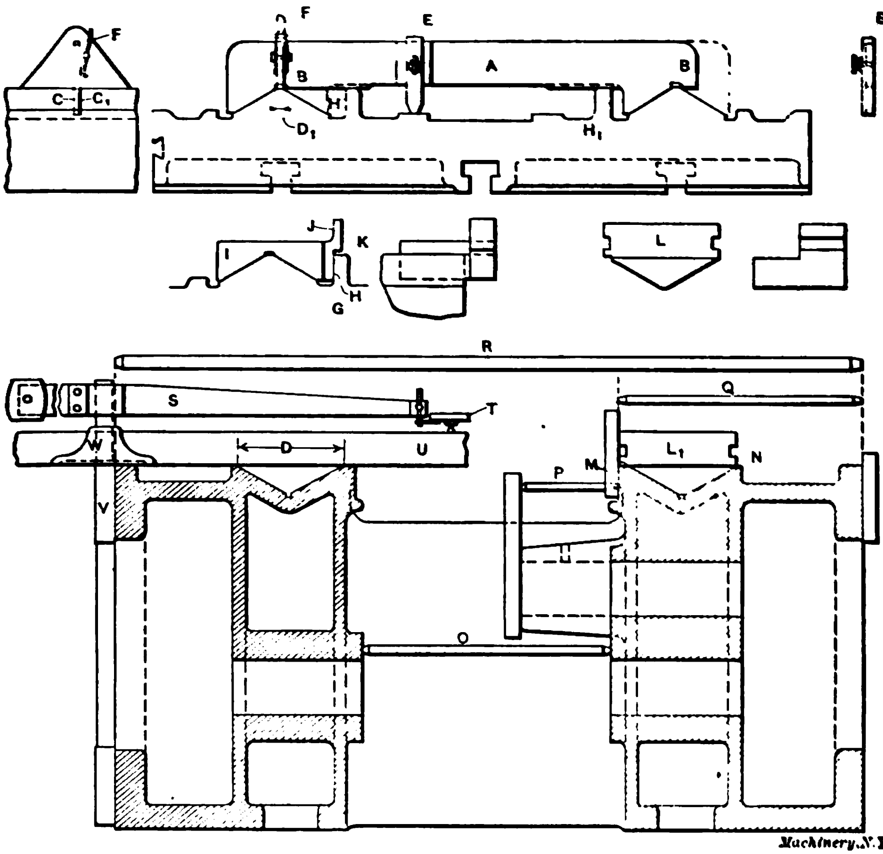

Fig. 11. Gages used for Testing Planing of Bed and Table

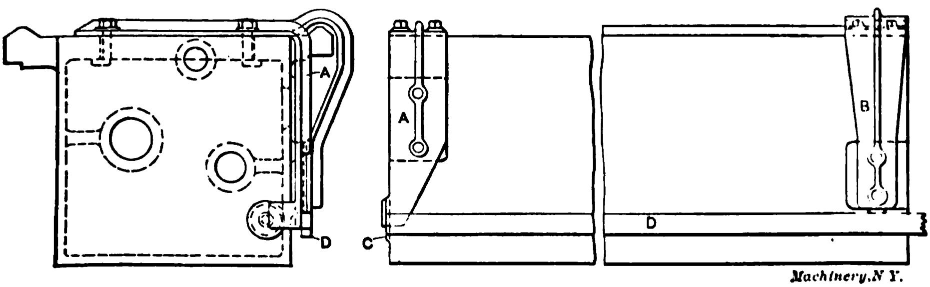

Fig. 12. Jigs used for Drilling Lead-screw Box Bolt Holes in the Bed

the jig against surface *H*. Suitable stops, straps and bolts secure the jig and bed to the base-plate during these operations.

For boring and reaming the shaft holes *I*, *J*, *K*, *L*, and *M*, two boring bars having suitable cutters and reamers are used.* The jig is provided with removable hardened steel drill bushings for the housing bolt holes *N*, the tapping being accomplished at the same setting of the spindle. Drill and reamer bushings are used at *O* and *P*, while a fixed drill bushing *Q* permits a small hole to be drilled for the taper dowel pin, ample stock being left for reaming after the housings are bolted on and properly located. After the boring and drilling operations are completed, the bed is moved over to the erecting foundation where the erecting process proper begins.

Drilling the Housings

In Fig 14, the front housing is shown at *A*, with jigs *B* and *C* in position for drilling the cheek bolt holes and the arch seat tap holes, respectively; the same jigs are used for the back housing, and jig *C* is also used for drilling the arch casting. The locating points and method of clamping the jigs are indicated in the engraving; as a matter of precaution, after the first hole is drilled in each case, a pin is inserted through the jig bushing into the drilled hole.

All drilling and tapping on this side of the housing being completed, jig *C* is removed, while *B* is secured by four bolts *D* having round heads fitting in place of the drill bushing collars, after which the housing is turned over in the position shown in Fig. 16. The drilling operations are performed on a cast-iron base plate provided with a portable motor-driven radial drill, this base also serving the purpose of a surface plate for laying-out the work. It is important that the driving shaft bracket hole *A* and feed box shaft hole *C* line up nicely with corresponding holes in the bed, so that the shafts will run perfectly free when assembled. In order to accomplish this without having to assemble the members and housing on the bed, jig *B* is provided with flanged bearings, as at *D*, which support arbors located in the exact center position of the respective driving and feed shafts. The location of the bearings in this jig, and also of the bolt holes *E* is found by clamping the jig to the bed jig member *D*, Fig. 13, and boring the former in this position, so that the two jigs are identical with respect to the locating points and center distances of the holes.

Referring again to Fig. 16, driving shaft bracket *F* is first centered by the bushing *G* being pushed down into the hole; then the outboard bearing *H* and its member *I* are set approximately correct by means of shaft *J* and jig *K*, and held in this position by C-clamps, after which the truth of bearing *H* with respect to its being square is tested by means of sweep *L*, indicator *M* and the test block *N*, as shown in the engraving. When it is determined that bearing *H* is square and properly set, so that bushing *O* enters the hole in jig *K* without springing the shaft, all the clamping bolt holes are marked off,

* These boring bars are similar to those shown in Figs. 137 and 142, page 21, of MACHINERY's Reference Series No. 43, Jigs and Fixtures, except that in the present case a middle support enables each bar to carry two cutters and two reamers.

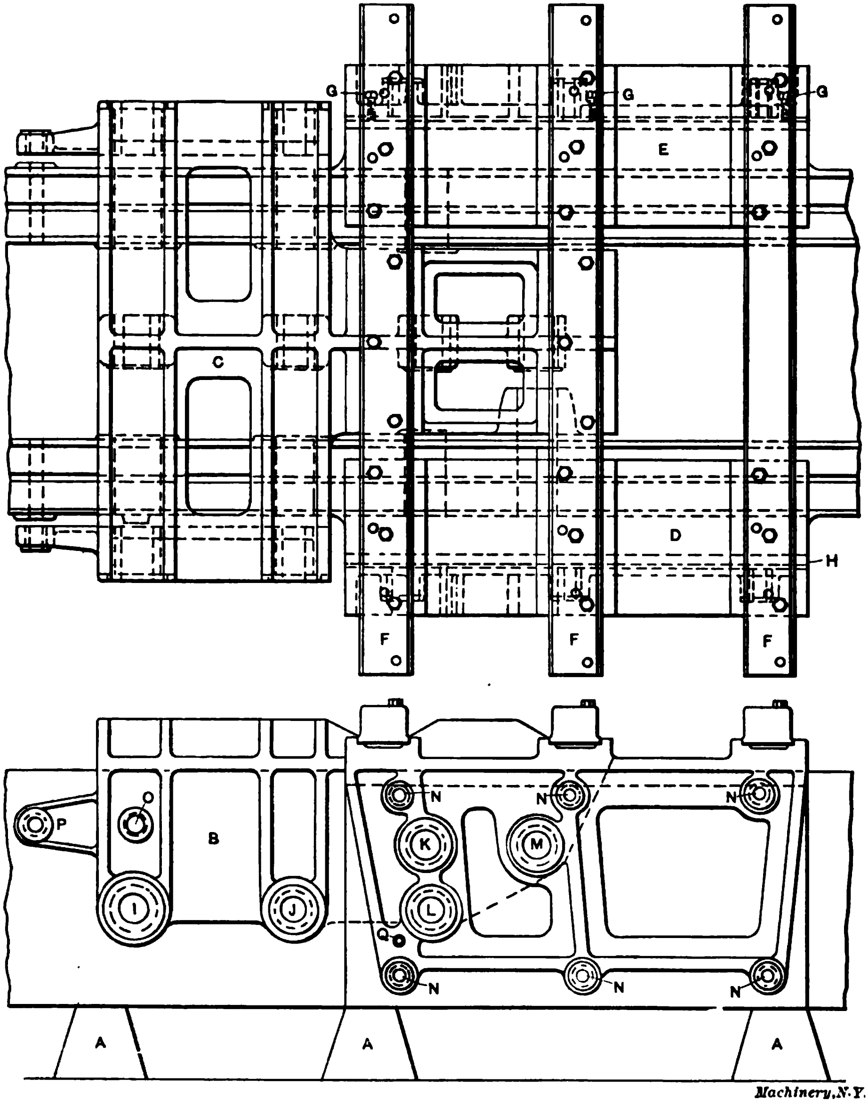

Fig. 13. Combination Boring and Drilling Jig in Position on the Bed

the brackets removed, and the holes drilled and tapped; then the brackets are bolted on, reset in the same manner, and the dowel pins fitted. In setting and testing these bearing brackets, particular care is exercised to insure the accuracy of the work, thereby saving much time when assembling the parts. As was stated at the outset, the fact that these driving works gener-

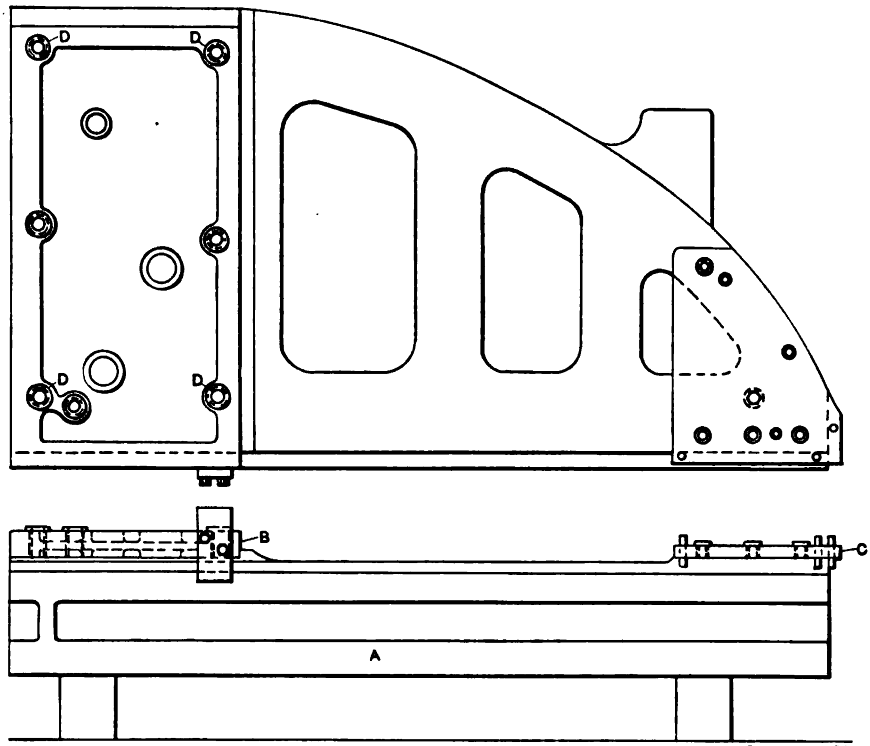

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Fig. 14. Front Housing with Jigs in Position for Drilling Clamping Bolt Holes. Same Jigs are used for Similar Operations on the Back Housing

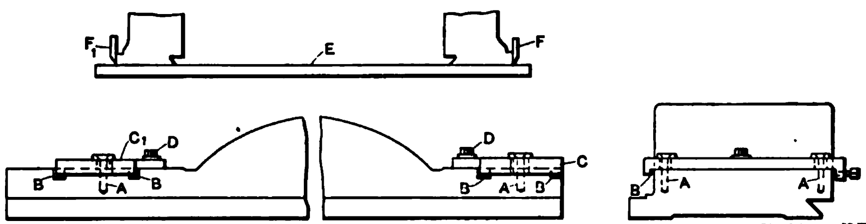

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Fig. 15. Jigs for Drilling Stud Holes in Back of Cross-rail. Setting of Jig C_1 endwise is accomplished by transferring Measurement from Housings by Means of Wooden Straightedge E

ally are of a special nature, is the reason why jigs are not provided for each individual member.

The cam operating lever bracket *P* is marked off after being set lengthwise to the correct dimension *Q*, and sidewise so that the center line of its shaft will coincide with a line laid off on the housing the right distance

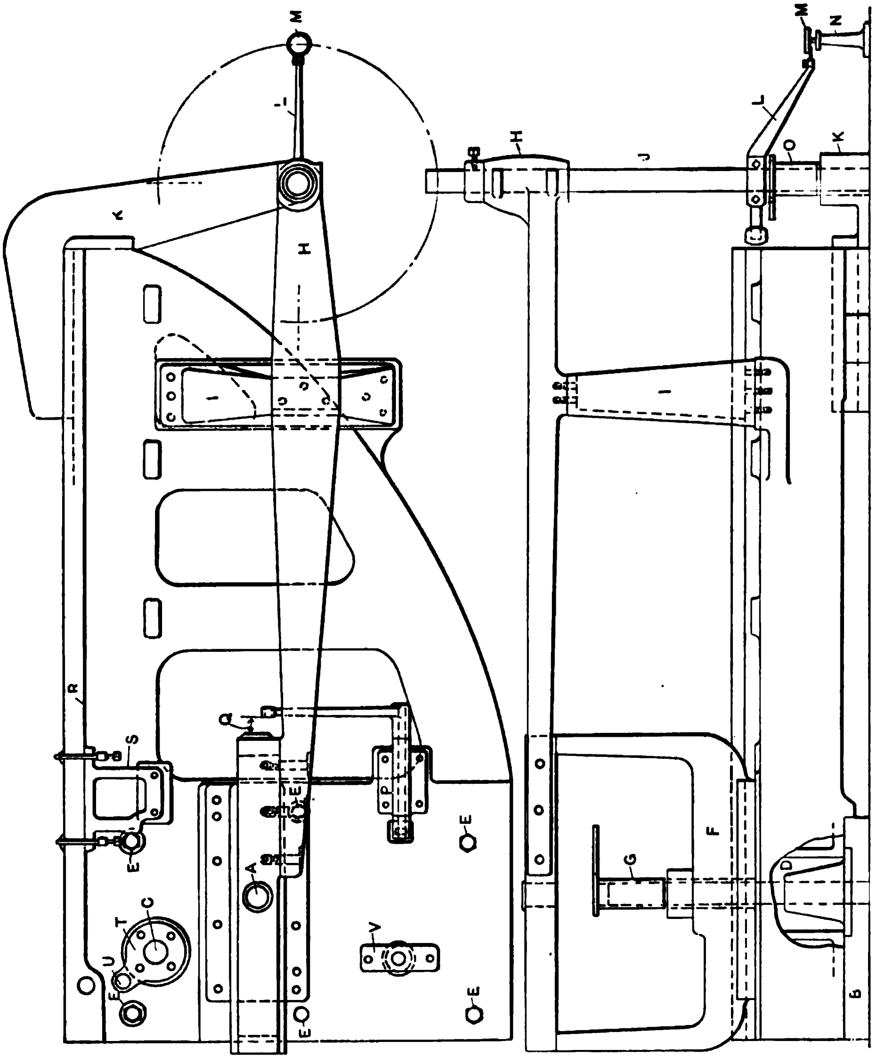

Fig. 16. Method of Setting and Laying out Holes for the Driving and Feed Members on the Front Housing

from surface *R*. A simple jig for drilling the feed rack casing holes is shown at *S*, the method of locating and clamping it being immediately apparent. Jig *T* is for drilling the feed-box clamping bolts holes; the jig consists of a flat plate centered by means of an arbor the same as at *D*, and located by a pin fitting into housing dowel pin hole *U*. A jig of similar construction and centered in the same manner, is illustrated at *V*; this jig drills the clamping bolt holes for a bracket carrying the side-head feed shaft. Simple jigs, not shown, are provided for drilling for the feed bracket *B* and elevating screw brackets *C*, Fig. 10, these two operations being performed on a horizontal drill. This completes the drilling on the front housing, and after the necessary drilling is performed on the back housing, using the same jigs as previously explained, the housings are tested to determine the accuracy of the planing.

Testing the Housings

One of the essential requirements of a first-class planer is that it must produce accurate work when using the side-heads, and this means that the ways on the housings must be true and parallel. When making this test, as shown in Fig. 17, the housings occupy the same position as when assembled on their bed, and it is at once apparent that whether or not the front faces stand perfectly plumb, is a matter of little consequence, so long as the faces lie in the same plane. With respect to the side faces and angles, however, the conditions are different; these must be square with the bed. Casting *A* which corresponds to the cheeks on the planer bed is bolted to a suitable concrete foundation and carries two V-blocks *B*, forming bearings for the sweep bar *C* which in turn supports sweep *D* and indicator *E*. The bar is held in the V-blocks by straps *F* and wooden blocks *G*, while collar *H* and its thrust bearing *I* take up all lateral motion. The construction of sweep proper, *D*, is such that clamp *J*, carrying the indicator, may be secured in any position of its travel between the two seamless steel tubes *K*, which enables readings to be taken at various points.

After being bolted onto the jig, the housings are located against strap *L* by means of screws as at *M*. It is desirable that the front faces show about 0.001 inch low at their outer edges, as at *N*, so that when the cross-rail is in position it will surely have a bearing across the entire face of each housing. Measurements are also taken across *O* at various heights, and between the arch seats *P*, to determine the parallelism of these surfaces. It should be explained that, in the side view, the front housing only is shown for the purpose of more clearly illustrating the sweep bearings. The housings having passed inspection in this test, are next turned face uppermost in a suitable pit for convenience in scraping on the side-head shoes *D*, Fig. 10, after which the housings are ready to be placed in position on the bed.

As has already been stated, the principal points to be observed in planer erection are: housings parallel with each other and square with the bed; accurate fit of all sliding members and truth of all plane bearing surfaces; proper mesh and smooth working of gears, and a system that permits the various parts to be easily and quickly assembled, and avoids the necessity of fitting the members together for the laying-out operations. We have now completed the boring and drilling of the bed, the drilling of the housings, and the testing of the housings. We will now take up the laying out of the arch bars, the assembling operations on the bed, the setting of the housings, the assembling of

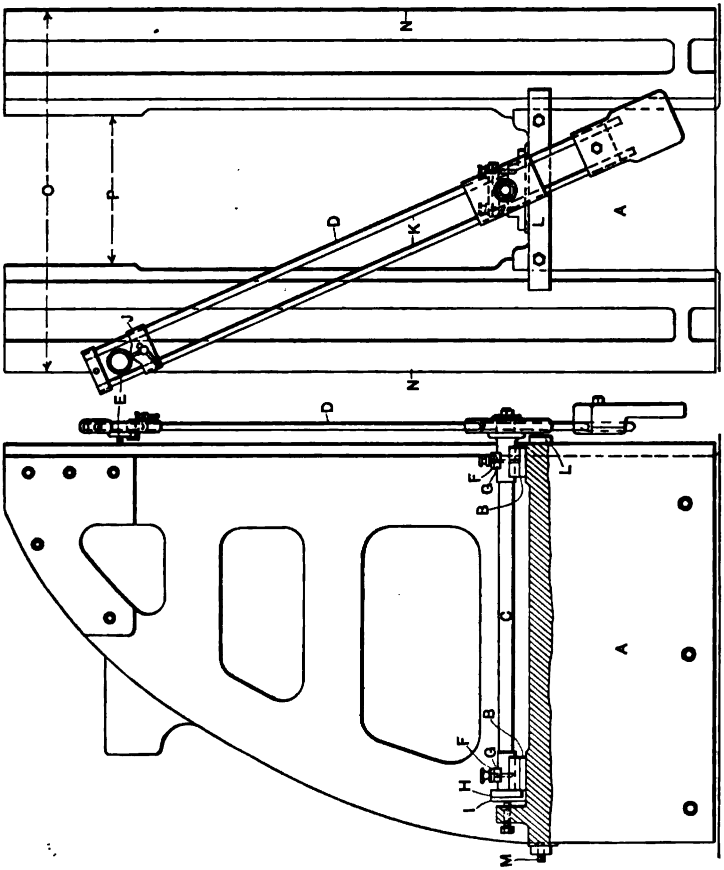

Fig. 17. Special Fixture used for Testing Alignment of Housing Faces, provided with Sweep for Carrying Starrett Indicator

the driving, feed and arch members, the laying out and setting of the cross-rail, and the final testing and inspection.

Laying Out the Arch Members

The first drilling operation on the arch is for the housing bolts, the jig for this being shown at *C*, Fig. 14; then the arch is set under the drill in its normal position and the brackets and motors are located for marking off the bolt holes, as illustrated in Fig. 18. When these holes have been drilled and tapped, the arch members are bolted in place for drilling the dowel pin holes, this time more care being exercised in the setting. This is another instance where the varying character of the parts precludes the use of drill jigs, and a method of laying out the work must be resorted to. For convenience in obtaining accurate measurements, and to facilitate the work, a pair of adjustable angle-plates are used, as shown at *A*, the idea being to use an arbor in the bracket hole *B* and to provide a positive locating surface at each end, against which the arbor just touches.

The location endwise not being as particular, a line is scribed on the arch the correct distance from end *C*, and the bracket, with a scale held against the end, is set so that the edge of the scale coincides with the line just mentioned. As will be seen from the detail view, the angle-plate is adjustable on its base *D*, and may be turned end for end and clamped in any position as the case may require, fine adjustment being made by the screw *E*. Fastened to base plate *D* is a scale, while attached to angle-plate *F* is a vernier, this combination enabling very accurate settings; all readings are taken from the lip *G* to surface *H*, and it is necessary, of course, to add half the diameter of the shaft to

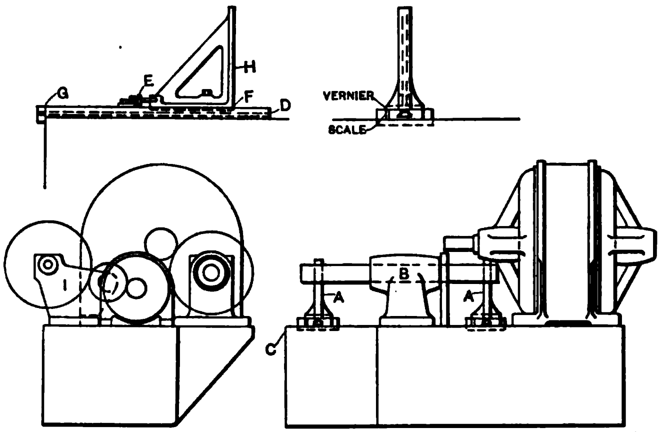

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Fig. 18. Arch with Bracket Members and Motor in Position for Laying Out

obtain the center distance. Bracket *I*, for the top elevating shaft, is set in the same manner, except that the angle-plates are reversed on account of the bracket being so close to the edge of the arch. With both brackets bolted fast and pinned, and their gears in place supported on short arbors, the motors are set so that their pinions mesh properly with the respective gears; then the motor clamping bolt holes are marked off, drilled and tapped, and the motors reset for pinning. In case the design is such that the entire top surface of the arch casting is not planed, *i. e.*, where finished seats are provided for the brackets and motors, additional spots are required for the angle plates. These spots are conveniently located on the casting and are finish planed with the other seats.

Assembling Operations on the Bed

The first assembling operations proper on the bed consist of drilling the various set-screw and oil pipe holes; drilling and fitting the track oiling device, a drill jig for which is shown in Fig. 19, and assembling the rack gear, its shaft, and the two intermediate compound gears and shafts. These operations, together with placing the housings on the bed, are done before the leveling operation, as otherwise the consequent hammering and additional weight of the housings might throw the bed out of level.

During erection, the bed is supported on cast-iron parallel blocks placed about six feet apart along the whole length of the bed and also under the housings. Planed cast-iron wedges, having screw adjustment, are placed between the parallel blocks and the bed, thus enabling very accurate leveling to be accomplished. The arrangement of all the blocking is such that none of it will interfere with the driving and feed mechanism during erection, and as these details vary in different machines, the blocking must be arranged to suit each machine.

Several methods may be followed in leveling a planer bed, any one of which will give good results if the work is carefully done. The

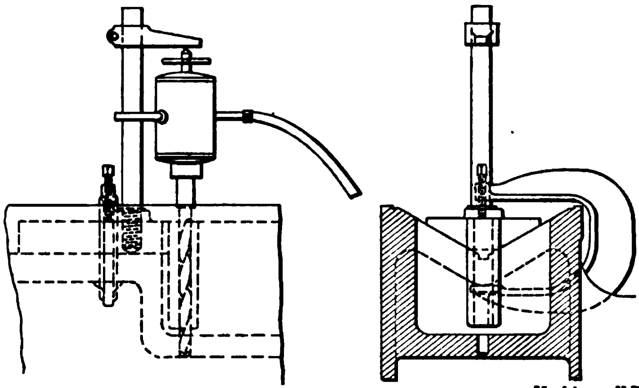

Fig 19. Combination Drill Jig and "Old Man" for Drilling Hole in Oil Pockets for Oiling Device. Jig insures Hole being drilled in Center of Way

prime requisites are, of course, a first-class, sensitive, spirit-level—one at least 18 inches long—and an accurate parallel that will reach across both tracks. It is obvious that since the tracks in a new bed are not worn, just as good results are obtained in leveling by using the top surface on the tracks as by using either V-shaped parallels or cylindrical pieces in the ways. The leveling is done as follows: The level is used on the top surface of one track, and that side of the bed is carefully leveled by moving the instrument short distances at a time, over the entire length. Then, by placing the level on the parallel, the bed is leveled crosswise; the operation of first leveling one side and then cross-leveling to the other is repeated several times, or at least until no further errors can be detected.

Setting the Housings

Fig. 20 is a top view of the planer, and shows the general method of setting the housings; the operation involves the alignment of the driving shaft bearing in bracket *A* with that in the bed at *B*, and also includes bringing the faces of the housings into the same plane. With the housings bolted to the bed only sufficiently tight to hold them in place, and with driving shaft bracket *A* bolted and pinned fast to the front housing *C*, an arbor *D* is used as indicated. This arbor is ground true and is a wringing fit in the bed at *B*, and, being of smaller diameter where it passes through bracket *A*, permits it to be easily introduced into the bed bearing even though the bracket hole is out of alignment. This condition is possible, of course, since the clamping bolts *E* have 1/8 inch clearance in the housings. Now, with bushing *F* in position to enter hole *G*, the front housing is driven with a babbitt hammer, either forward or backward as the case may require, until the bushing enters the hole freely without springing the arbor.

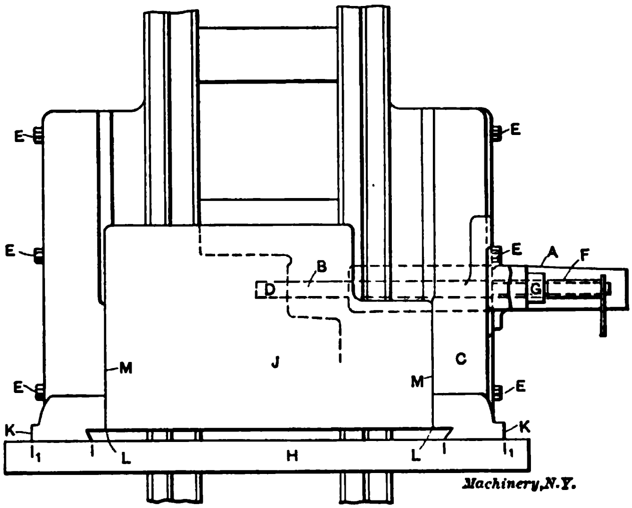

Fig. 20. Method of Setting the Housings. First the Front Housing is set for Alignment of Driving Shaft Bracket A; then the Back Housing is brought into the Same Plane by means of Straightedge H

To set the back housing, a straightedge is laid across the bed as shown at *H*, and narrow strips of tissue paper are introduced at *I* and I_1; then, by moving the back housing until papers *I* are tight and I_1 can just be moved, it is determined that both housings are in proper alignment. The fact that the outside papers are slightly loose is due to conditions already stated. After reaming the housing dowel pin holes by means of an air drill, and driving in the pins, measurement is taken for arch casting *J*, and the housings are calipered over surfaces *K* just above the bed, and at the top, to test their parallelism.

While an attempt is made at interchangeability with respect to the length of the arch member, it sometimes happens that certain elements make it necessary to slightly deviate from the standard measurement. For instance, the above test may show the measurement over *K* to be from 0.002 to 0.003 inch wide or narrow at the top, in which case the housings are made parallel by means of a jack-screw, or tie-rod, as the case may require, and then the arch is machined to suit. The arch is now bolted in place, and set to match the housings at *L* and *M*, after which it is pinned.

Assembling the Driving, Feed and Arch Members

The cross-rail, side-saddles, and the various driving and feed units are assembled in a department separate from the erecting department, and these parts usually are duplicated in quantities and come from the store-room to the erectors completely finished. In all cases where possible in assembling these units, standardization is provided for, and in this way much time is saved by the erectors avoiding unnecessary adjustments.

Referring back to Fig. 10, the feed box *E* is next bolted in place, and after assembling the driving shaft *F* with its members, and bolting on the bracket member *G*, the outboard bearing *H* is bolted in position. The arch bearing for the fly-wheel shaft *I* already being secured in place, this shaft is red-leaded and tried in its bearings to test their alignment. When proper care is exercised in the aligning operations, very little scraping is necessary on these bearings.

While work is proceeding in assembling the top works, elevating screws, and motors, other erectors are busy with the side-heads *D*, rocker mechanism *J*, and the feed mechanism for the side-heads and cross-rail; each unit is assembled in logical order, and as many operations as possible are carried on simultaneously. The planer is now ready for the cross-rail *K* and table *L*, preliminary work on these members being completed far enough ahead so as to cause no delay at this point.

The operations on the table consist of drilling and reaming the stop-pin holes, drilling and bolting on the rack, and rough scraping the tracks; the oil grooves were cut in the machining process. A large motor-driven multi-spindle drill is used for drilling and reaming the stop-pin holes. This machine carries sixteen spindles, arranged in two rows; one row of spindles carries the drills, and the other the combination mills and countersinks. After the first row of holes is drilled and the table is indexed along the space of one row, the combination mills and countersinks are inserted, and the sixteen tools are used simultaneously, thus producing very rapid work. The table is supported on a special truck running on a track between the drill uprights, and a suitable mechanism for moving and indexing the table completes the equipment. Previous to placing the table on the bed, the ways on the latter are also rough scraped, and then the bearing surfaces receive a coat of red lead which serves the double purpose of marking material and lubricant.

Laying Out and Setting the Cross-rail

The stud holes *A* for the cross-rail gibs are drilled in the manner shown in Fig. 15. As will be seen, spots are planed off at *B* which serve to square jigs *C* and C_1, and the holes for the elevating screw nuts are utilized for clamping the jigs by means of bolts *D*. Endwise location of jig C is determined by matching the end of the cross-rail as shown; then jig C_1 is set by transferring the measurement from the housings by means of the wooden straightedge E. A flat scriber, shown at F and F_1, is used to mark lines on the straightedge which is chalked for this purpose, and when corresponding lines on the jigs coincide with those on the straightedge, jig C_1 is properly set.

When the studs are screwed in place and the back surface of the cross-rail is scraped, the cross-rail is placed in position on the planer and clamped by its gibs. Squaring the cross-rail with the housings is accomplished by holding the bar of a sweep in the angle M, Fig. 10, and applying an indicator to the front housing at N and N_1. The low end of the cross-rail is raised a sufficient amount by either moving the teeth in bevel gear O or O_1 (Fig. 10), as the case may require, in relation to its pinion, or by adjusting one of the nuts on the gear end of the elevating screws, final adjustment being obtained by the

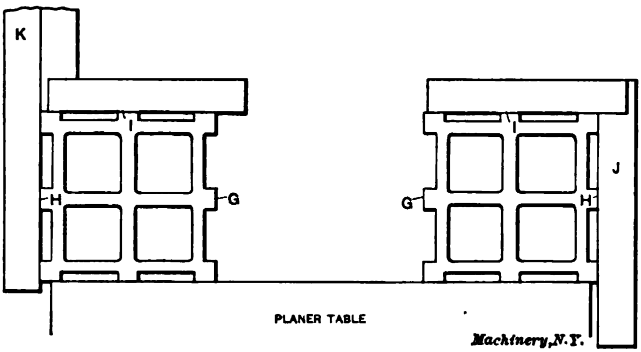

Fig. 21. Method of Testing Accuracy of "Planing Square" when using the Side-heads

latter method. It is always better to raise the low end rather than lower the high end of the cross-rail, on account of the fact that this will take up any lost motion or backlash between the nuts, the feed-screws, and the housings. As the studs have 1/16 inch clearance in the gib, it is necessary to pin the latter after setting the cross-rail.

Final Test and Inspection

With the motors wired up, the belts in place, and the machine thoroughly oiled, the driving works are run for a while before moving the table into mesh with its rack gear, the idea being to prevent possible heating of the bearings by running without load. Next the table is brought into mesh and the bed is again carefully leveled in the same manner as before. When this is accomplished, the ways and tracks are scraped to a bearing, after which the ways are oiled and one or more cuts taken across the table to true it up for the purpose of testing the planer. A straightedge tried on the table crosswise, lengthwise, and across corners, is used to test the truth of the planing.

The side-heads are next tested for "planing square" by the method illustrated in Fig. 21. Two cast iron parallels G are clamped one on each side of the table as shown, and then light cuts are taken down faces *H* with tools in the side-heads. Now, with the faces *H* clamped to the table, cuts are taken down faces *I*, after which the parallels are turned back to their original positions and a square tried as at *J*. To "prove" the square, it is used in connection with a straightedge (on the same parallel) as at *K*, any error detected between the blade of square and the straightedge showing double the amount of actual error.

The accuracy of setting the cross-rail is now determined by taking a light cut across faces *I*, using a tool in one of the cross-rail heads, and testing with the square and straightedge as in the previous case. The object in making these tests is a precautionary measure, for by testing the planer under actual working conditions, the accuracy of the tests made during erection are thus proved.

Final inspection includes running the cross-rail to the top of the housings to test the elevating mechanism and ascertain the fact that there is clearance between the cross-rail and arch. All gearing is tested for quiet and smooth running; the fits of all bearing surfaces are inspected; the slides and saddles are run by hand to test the parallelism of their ways and also the ease with which the slides operate, after which the power feed is applied and tested in various ways; the balance of the driving motor armature, and of the fly-wheels and pulleys also, receives careful inspection; in fact, no part is neglected and all errors must be within allowable limits of variation. All tests are made under the personal supervision of an inspector, who enters all data on a form prepared for the purpose, and this report bears the serial number of the planer, and is filed away for future reference.

As opportunity offers during erection, the filling, rubbing down, and priming has progressed, so that after inspection, all that remains to be done is to give the bright parts their final polish, and apply the last coat of paint.

CHAPTER III

LAYING-OUT AND ALIGNING OPERATIONS*

In general, laying out is the process of placing such lines on castings, forgings, or partially finished surfaces, as will designate the exact location and nature of the operations specified on the drawing; an aligning operation, as its name implies, consists in lining-up a shaft bearing, bracket, or other similar machine element, in its proper place relative to other members. The first-named operation usually is associated with the process of machining, while the last-mentioned is generally included in the work of assembling. Laying-out and aligning operations may be divided into two parts; the preliminary and the final. The preliminary operation consists in approximately locating a machine element in place for the purpose of marking the clamping bolt holes on its supporting member; in the final alignment, the exact location is ascertained for the purpose of drilling the dowel pin holes, the work being held by its clamping bolts. Clearance in the bolt holes permits of this adjustment.

As the ultimate results obtained in assembling are controlled to a large extent by the accuracy of these operations, it is of the utmost importance that means be provided for insuring the refinement that the nature of the case demands. Jigs and fixtures have, of course, been a dominant factor in dispensing with much of the ingenuity and skill required in this work, but owing to special considerations a preclusion of these valuable adjuncts to manufacturing work may be advisable. In this case a simple gage or templet, or even a wooden jig provided with steel bushings, will greatly facilitate the operation of laying out or aligning, and, in fact, when proper care is exercised in using these comparatively crude devices, work may be produced on an interchangeable basis as good as with more expensive tools; although it is to be expected that more skilled labor will be required.

As regards the different methods of laying out and aligning, no definite rules can be given. The machinist must consider the means at hand and the nature of the job; he must then use his ingenuity and be guided by his practical experience. A few special cases are illustrated in the accompanying line-engravings; the methods and processes shown and the remarks made in regard to them are intended only as suggestions of how the work may be accomplished without the employment of drill jigs. It is not to be inferred that the way shown is, in each instance, the best method possible and the only one applicable. Circumstances alter cases; while the methods shown may be eminently suitable for one set of conditions, they may either be too refined or not refined enough for other requirements.

* MACHINERY, February and March, 1910.

Special Tools and Appliances

Aside from the more common laying-out tools such as the dividers, surface gage, steel scale, etc., there are a number of tools of a special form used for laying-out operations, some of which are shown in Fig. 22. The form of center punch shown at *A* will greatly facilitate marking off holes through brackets and templets, or in laying off pin holes for cams. It is necessary to provide a number of different sized center punches of this type, as the body of the punch must fit the

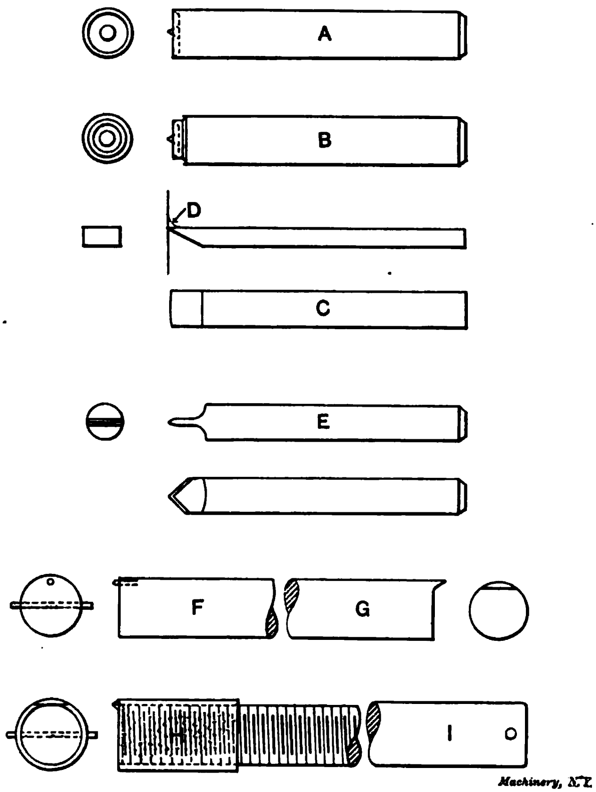

Fig. 22. Special Tools used in Laying-out Operations

clearance hole in the work. For obtaining a circle, the diameter of the tap drill, the punch or marker may take the form shown at *B*, while a combination of the two will provide a guard circle.

A flat scriber *C* is very useful for marking a line on a plane surface at right angles to another plane surface when the corner is rounded as shown at *D*. The form of marker illustrated at *E* is for giving permanence to lines intersecting on surfaces at right angles, as for instance, in marking the relative position of a gear on a shaft. At *F* is shown a special marker for laying out a circle, the center of which must coincide with a hole already bored. The body of the marker fits the bored hole and a circle is scribed in the piece to be marked off by rotating the marker when the point is in contact with the work. Two methods of making the scribing point are clearly indicated in the engraving, the one shown at end *G* producing a circle the diameter of the body. In marking off a hole in alignment with a threaded hole, a bushing *H* having a scribing point is made to fit the threaded arbor *I*. This arbor fits the threaded hole, and the bushing is rotated to mark the circle.

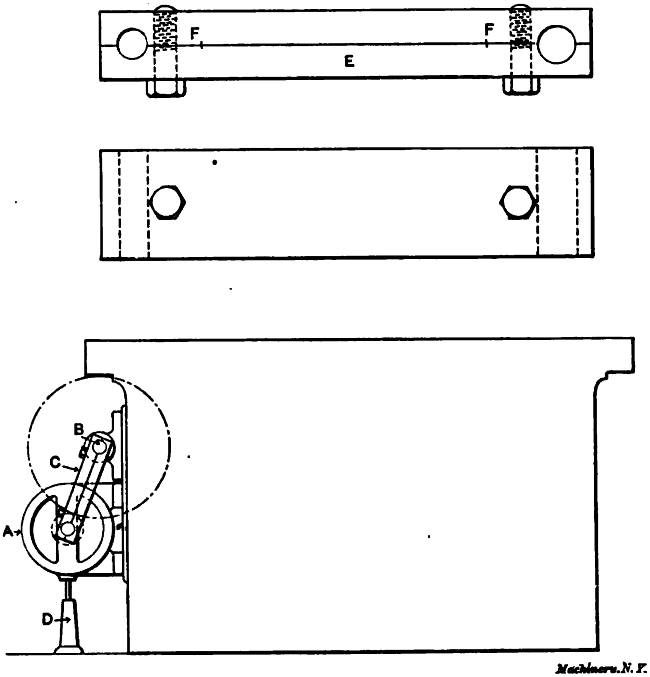

Fig. 23. Locating a Small Motor by the Use of a Link

One of the most convenient and accurate methods of locating gear centers is by the use of links. For drilling or boring operations the link may take the form of a casting provided with hardened steel bushings to guide the cutting tools. Again, a link may be used for cases similar to the one shown in Fig. 23, which illustrates the method of accurately setting a small motor *A* so that its pinion will mesh properly with a gear on shaft *B*. The work is accomplished as follows: With the link *C* and the motor in position as shown, the jack-screw *D* is adjusted until the motor frame just touches the finished seat on the bed. This adjustment is determined by means of tissue paper placed between the motor and bed, after which the bolt holes are marked off on the bed; the special marker *B*, Fig. 22, is used for the purpose. The construction of the link is clearly shown at *E*, Fig. 23; this form, being made of two pieces bolted together, permits of ready application to a shaft supported between bearings, without removing the shaft. Such a case is frequently met with in applying a geared pump to a machine already built. For ordinary cases, however, the link may be made in one casting or forging, as the circumstances require, and provision for clamping may be made by sawing through the ends as far as indicated at *F*.

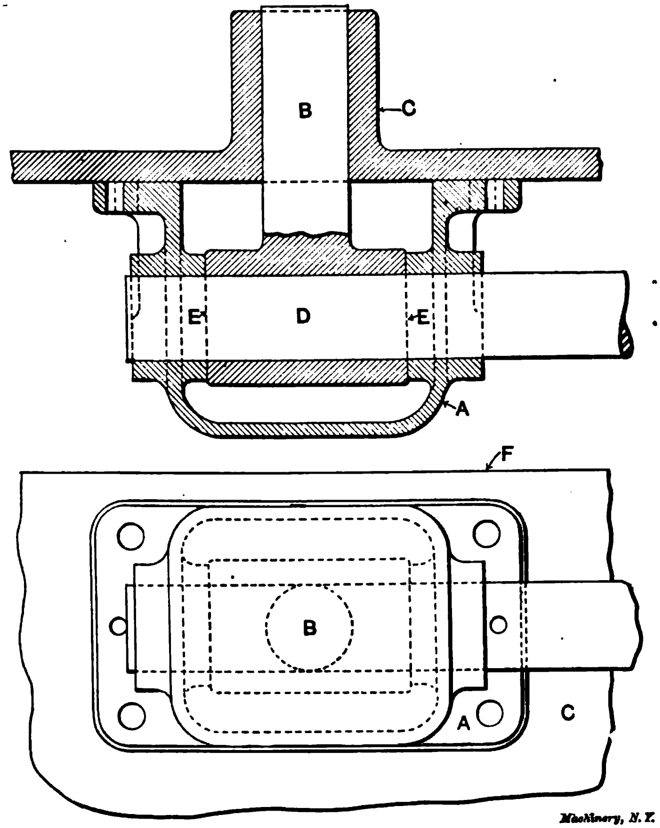

Fig. 24. T-jig for Locating a Bevel Gear Bracket

In Fig. 24 is shown a T-jig for locating a bearing bracket *A* relative to the hole *B* in the main casting *C*. The requirements are that the axes of hole *B* and shaft *D* must intersect, and the faces of hubs *E* must be equidistant from the axis of *B*. It is evident, however, that the T-jig will not take care of the alignment of the shaft *D* with reference to its being parallel with the surface *F*. This may be accomplished by measuring down from surface *F* with either a combination square or surface gage or, in case the adjacent bearing for shaft *D* is already located, bracket *A* will find its own alignment by using this bearing to support the shaft.

The Use of Templets

When a number of pieces are to be made interchangeable without the use of jigs or fixtures, this can be accomplished by the employment of templets for laying out the work. While these devices greatly simplify laying-out and aligning operations, they are not intended for guiding the cutting tools. Templets are particularly well adapted for work

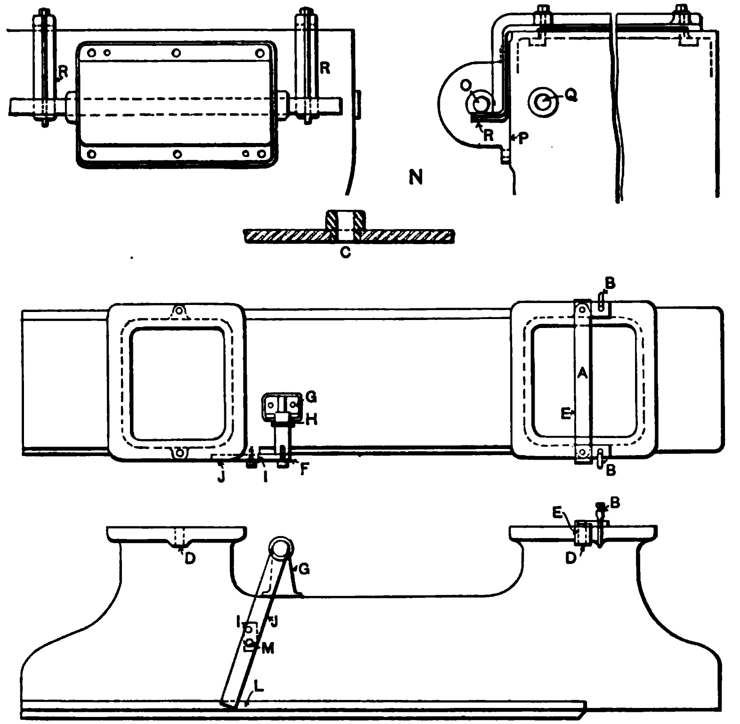

Fig. 25. Special Gages and Templets for Laying-out and Aligning Operations on a Turret Lathe Bed

where the holes to be laid out lie in the same horizontal plane, and owing to this condition, the templet usually takes the form of a flat plate of sheet iron, or a wooden piece, having the same general outline as the work to be laid out. Again, many irregular forms are drawn on work from accurately filed templets, after which permanence is given the lines by dotting them with prick-punch marks placed directly on the line.

In making a templet for the first-mentioned class of work, holes are drilled in the templet to conform to the drawing of the piece to be laid out. In use, the templet is laid on the work and is then clamped to it by suitable and convenient means, so that its outline coincides with that of the work. The layout may be transferred to the work by means of a marker as already explained, or, in the case of comparatively large holes, an ordinary scriber is used to mark the circles, and after the templet is removed from the work, the center of each circle is laid out with dividers, permanence being given the lines by a prick punch. Witness circles are often placed on the work to make sure that the original lines were closely followed in drilling, *i. e.*, a circle is drawn in each case 1/32 inch larger in diameter than the one worked to; then, if the hole is correctly drilled, it will be concentric with this circle.

For a certain class of work where great accuracy is not required, templets may be made provided with hardened steel bushings for guiding the cutting tools independently of the skill of the operator, in which case, however, the templet takes the form of a jig. Owing to the lack of rigidity due to the thin material of which such jigs or templets are constructed, no attempt is made to provide clamping arrangements. The templet may be clamped to the work by means of ordinary C-clamps, or with machinists' clamps. Very frequently, however, it is desirable to provide locating points which may consist of pins extending from one or both sides of the templet, as the case may require, or the locating points may be formed by bending the edges of the metal to a right angle.

The application of a jig such as just described is illustrated in Fig. 25, which shows the method of drilling foundation bolt holes in a turret lathe bed, the holes being drilled from the bottom. As will be seen, the jig or templet *A* consists of three pieces of flat iron riveted together and clamped to the bed by means of clamps *B*. The method of inserting the drill bushings is shown in detail at *C*. To facilitate setting the jig with reference to the bosses *D* on the under side of the bed casting so that the holes when drilled will be concentric with these bosses, jig member *E* is bent to a right angle as each end so as to extend down the casting; the location is determined by matching these ears with the bosses on the bed.

In Fig. 26 is shown the application of sheet-iron templets for laying out cross-rail members for lathe planers. Templet *A* for swivel member *B* is located by the hub *C*, and is lined up to match the end *D*. A separate templet is provided for laying out the swivel clamp *E*; edge *F* of the templet is bent over to form a locating point. But one templet is required for laying out slide *G* and its clapper-box *H*. This is lined up on each member as shown. It is obvious that these templets are more advantageous than cast-iron jigs for this class of work, since very large and heavy jigs would be required, and furthermore, no great accuracy is necessary, as the bolt holes have 1/16 inch clearance.

As already stated, a very cheap and serviceable jig for certain classes of work can be constructed of wood. At *A*, Fig. 27, is shown a jig of this character for drilling the clamping bolt holes in an engine lathe head-stock—in this case for a 30-inch lathe. The jig is located by pin *B* and keys *C*, the latter fitting a keyway in the headstock; having these keys on both sides of the jig as shown at *D*, it is also used for drilling the tap holes in the lathe bed, not shown. Steel lining bushings *E* are provided for the drill bushings. The jig and work are clamped to the drill press table by straps and bolts. The frame consists of four pieces of ash fastened at the corners with glue and wood screws, the joints being made as shown. Ash is the best wood for the purpose, since, if well seasoned, it is less likely to warp than any other, but where this wood is not available, maple is a good substitute.

A slightly more expensive, but more durable jig, for the same purpose is shown at *F* in the same engraving. This jig is made of flat

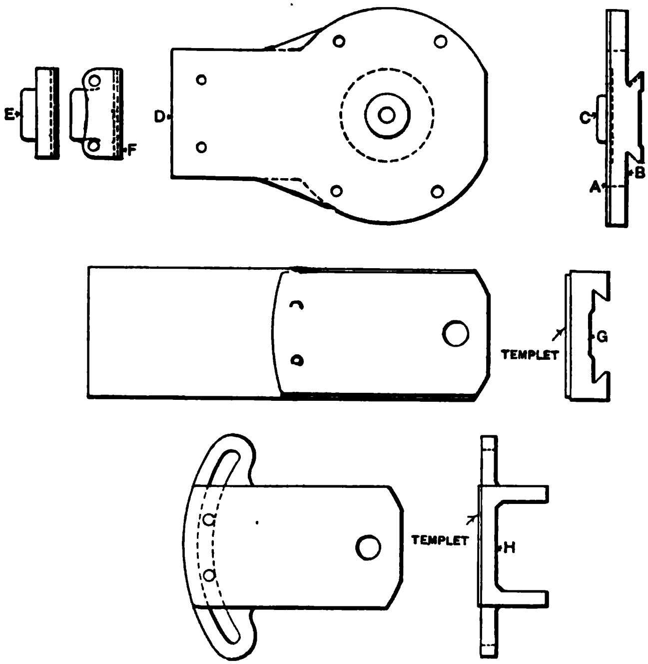

Fig. 26. Sheet-iron Templets for Laying out Planer Cross-rail Members

bar steel riveted together, and is of the same general construction as the wooden one.

Gages for Aligning Operations

A gage may briefly be defined as any standard of comparison; as here used, the term gage will have reference to special devices for aligning work without the employment of ordinary tools such as a combination square, surface gage, etc. Besides greatly facilitating aligning operations, the particular advantage of using gages is that the possibility of error due to carelessness in transferring scale measurements is avoided. It is assumed, however, that the gages here shown are intended only for duplicate work; it would not be economy to make gages for aligning only a few pieces.

Turning back to Fig. 25, *F* represents a simple gage for aligning bracket *G* on the bottom of a turret lathe bed. The requirements are that face *H* of the bracket must be a certain definite distance from seat *I* on the bed, but the alignment in a longitudinal direction is non-essential. The gage merely consists of two pieces; a straightedge *J*, planed only on one side and one end, and a gage which is fastened to the end of the straightedge as shown. As surface *L* on the bed lies in

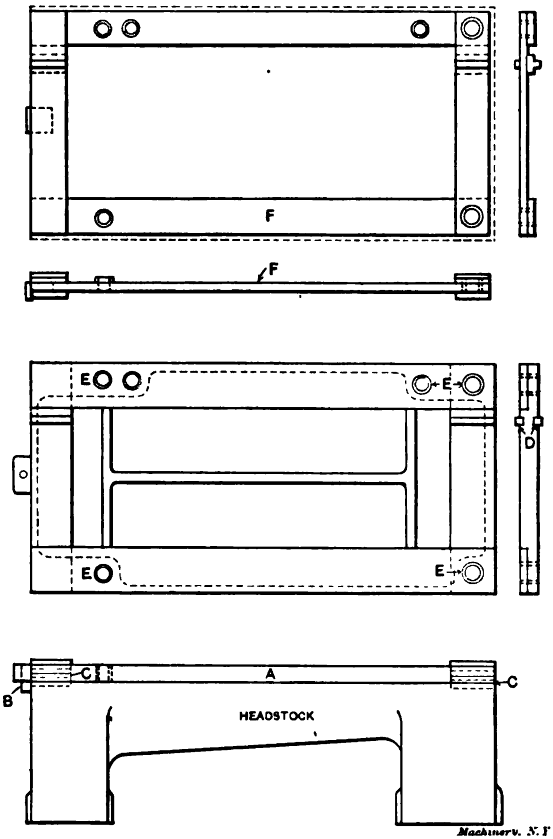

Fig. 27. Application of a Wooden Jig for Drilling a Lathe Head-stock. At F is shown a Similar Jig Constructed of Flat Bar Steel

the same plane with seat *I*, the straightedge is made long enough to reach this surface, thereby obtaining greater accuracy in the alignment. Bolt hole *M*, already tapped, is utilized for clamping the gage. In aligning the bracket, its face is brought into contact with the gage, and the bracket is then set longitudinally to match its seat on the bed.

Another gage, or more properly speaking, a pair of gages, for aligning a feed-box on the turret lathe bed shown in the lower part of the same engraving, is shown at *N*. The requirement of the present case is simply that the feed-box shaft hole *O* be located a certain definite distance from the top of the bed; seat *P* on the bed takes care of the center distance from hole *Q*. It is obvious then, that the gage castings *R* should only provide a positive locating surface with reference to the top of the bed; this is accomplished in the manner shown. The location endwise is determined by scale measurements from the end of the bed.

An aligning operation on a vertical boring mill bed, and the gage

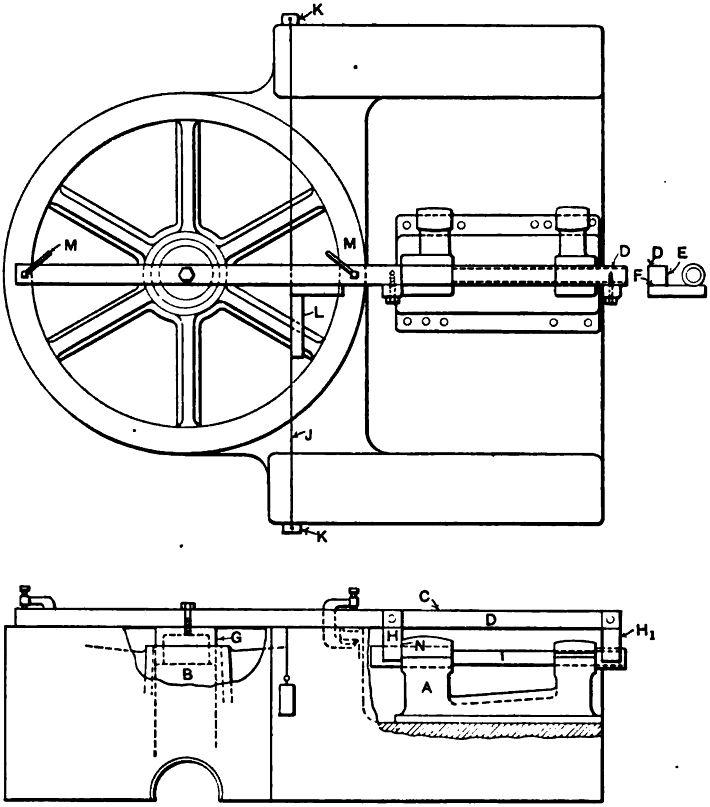

Fig. 28. Gage for Aligning the Driving Shaft Bracket on a Vertical Boring Mill of the Bevel Gear Driven Type

used, is illustrated in Fig. 28. This boring mill is of the bevel gear driven type, in which the pinion meshing into the table bevel gear is carried on the driving shaft in bracket *A*. The problem of aligning this driving shaft bracket with reference to the spindle hole *B* in the bed, is easily solved by using gage *C*. As will be seen, this gage consists of a bar *D* planed on two sides, *E* and *F*; a bushing *G* fitting the spindle hole; and two gage pieces *H* and H_1. A special arbor *I*, having both its ends ground to the same diameter, fits bracket *A*.

When in use, a line wire *J* is stretched across the bed by means of weights *K*. This wire lies in a small groove or mark planed in the bed for the double purpose of squaring the gage and setting the housings. Square *L* is used in setting the gage before it is clamped by means of clamps *M*, so that when the bed members are assembled, the driving shaft will be approximately square with the housing faces. With the gage in this position, bracket *A* is set so that its arbor just touches the gage blocks *H* and H_1. The location with reference to the distance from spindle hole *B* in the bed is determined by simply bringing the hub face *N* on bracket *A* into contact with the side of gage block *H*. After the operation of marking off the tap holes in the bed is accomplished, and the holes are drilled and tapped, bracket *A* is reset in the same manner and clamped by its bolts, for drilling and reaming the dowel pin holes.

Laying-out and Aligning Operations without the Use of Special Tools and Appliances

In the absence of special tools and appliances for laying-out and aligning operations, the principal points to be observed are the selection of a proper starting point from which to lay out all dimensions, the employment of efficient means to compensate for the deflection in horizontal aligning arbors when the bracket seats lie in a vertical plane, *i. e.*, when it is impracticable to place the bed of a machine in such a position that the bracket seats will lie in a horizontal plane and thus carry the bracket members unsupported; and the avoidance of assembling all the correlated members together for the laying-out operations.

The first two points brought out above are exemplified in the aligning operations on a vertical boring mill of the spur pinion driven type, which is illustrated in Fig. 29. The operations involved are the alignment of the driving pinion bracket *A*, the driving shaft bracket *B*, the feed shaft bracket *C*, and the housings; only one housing *D* is shown. The hubs on the brackets have clearance in cored holes in the bed. It is the general practice to bore out the table spindle hole *E*, and table gear pinion hole *F*, by means of a boring jig, previous to aligning the brackets; hole *F* in the bed is then used as a starting point for the aligning operations.

For convenience in setting bracket *A*, which forms the lower bearing for the table driving pinion, and also carries inner bearings *G* and *H* for the driving and feed shafts, the bed casting is turned bottom side up, the process being as follows: The table pinion is placed in position in its hole *F*, for the purpose of centering the bracket; next the driving shaft *I* is put in place in its bearing *G*, and then the bracket is squared with seat *J* by means of a square held against the seat and the shaft. This setting is for marking off the bolt holes in the bed, and after the drilling and tapping is accomplished, the bracket is reset in the same manner as before and clamped by its bolts; then the dowel pin holes are drilled and reamed, and the pins fitted.

The next logical step is to turn the bed right side up and place reference lines on seats *J* and *K* respectively. A separate operation

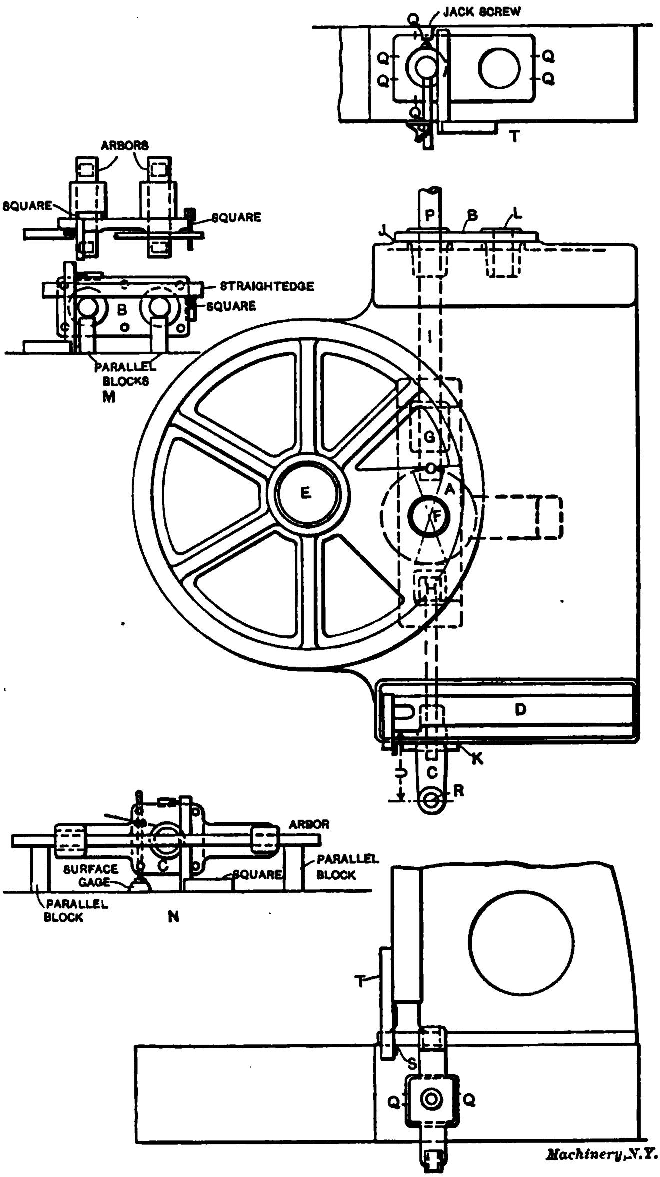

Fig. 29. Laying-out and Aligning Operations on a Vertical Boring Mill of the Spur Gear Driven Type. By the Method shown, no Special Gages or Tools are required

involves placing reference lines on the bracket members *B* and *C*. The reference lines represent, on each separate member, the outside diameter of the shaft used in aligning; then, in assembling the bracket members on the bed for the aligning operations, it is simply necessary to match the lines on the brackets with those corresponding on the bed. During this operation the brackets are supported on screw-jacks. This method of setting brackets illustrates the second point stated above, the employment of efficient means to compensate for the deflection in aligning arbors when the bracket seats lie in a vertical plane; another point of advantage, however, is that this method insures proper alignment of the shafts, any inaccuracy in the machining of the seats being immediately apparent when the members are assembled. For instance, another method of setting the driving shaft bracket would simply be to place the bracket in position on its shaft, support it on screw-jacks, and then shift the bracket around until a feeler indicated that all sides of the bracket were tight against its seat on the bed, attention being paid, of course, to the location of the back-gear hole *L* with reference to the top of the bed. With this method it would be possible to have all sides of the bracket tight against the bed as just explained, but in the event of the bracket seat not being perfectly square with the hole, the shaft would be thrown out of proper alignment with reference to the planing on the bed and also to hole *G*.

Turning back now to the question of reference lines, the method of placing these lines on the driving shaft bracket is clearly illustrated at *M* in Fig. 29, while at *N* is shown the same operation on the feed shaft bracket; both cases are essentially the same. To place lines on the bed seat *J*, a surface gage is first set to the driving shaft at *O*; next the gage is moved to position *P* and the shaft jacked up until the surface gage indicates parallelism with the top of the bed; and then, after testing the shaft with a square on seat *J* to insure that the screw-jack is not holding the shaft out of alignment sideways, a combination square and an ordinary square are used as shown at *T*. Similar operations are involved on seat *K*. Permanence is given all reference lines by driving a thin chisel into the casting directly on the line. These lines are indicated in the engraving by the letter *Q*.

The housings are located on the bed with reference to hole *R* for the vertical feed shaft. Previous to placing the housings on the bed, however, lines are drawn at *S* on each housing and on the bed casting, which lines are in the same plane as the front face of the housings. This is accomplished in the former case by means of a straightedge *T*, and in the latter case by laying off on the bed the correct distance from the center line of brackets *A* and *C*. In placing the housings on the bed the corresponding lines on each member, are matched, and then the housings are moved so that measurement *U* is correct; this measurement is taken with a straightedge and combination square. The housings are set for the pinning operations in the same manner. It may be of interest to state that the housings are first set on a large surface plate for the purpose of bolting on the arch casting and fitting the top-works, thus enabling the accomplishment of several operations simultaneously, as the cross-rail gibs also can be laid out at this time.

By observing the third principle laid down before, the avoidance of assembling all the correlated members for the laying-out operations, it is often possible to advance work that otherwise could not be accomplished. This point is illustrated in Fig. 30, which shows an oil pumping arrangement attached to an engine lathe carriage and driven by a shaft carried in bearings on the back of the bed. This outfit is special in its nature and is furnished as an attachment; considering this fact it will be apparent that the bed member is likely to reach the assemblers last, and therefore the work can be greatly advanced by laying out and fitting up the carriage members independently of the bed.

After bolting the oil tank *A* to carriage *B*, its bolts fitting T-slots in the carriage, the oil tank is set and then dowel-pinned, the operations being so simple as to need no explanation. As bracket *C* is already bored out on a boring mill, the first laying-out operation involves the location of stud hole *D* in the oil tank. Since this hole must be laid out with reference to surfaces *E* and *F* on the bed (surface *G* on the carriage is merely planed for clearance) very effective use can be made of a jig consisting of a short bed section; but if this is not available, a gage block *H* similar to that used in planing the carriage, can be substituted. A graduated try-square is used first on surface *E* and then on surface *F*; the measurements are read directly from the graduations. Next, the hole is drilled and tapped using a pneumatic drill, and then bracket *C* is clamped in place by the intermediate gear *I*.

The next operation involves the alignment of hole *J* with reference to surface *E*, the object being to provide standardization for the planing of shaft bearing *K* and its seat on the bed. This is accomplished by swiveling bracket *C* around, using stud *I* as a pivot, until a combination square and scale indicate that the measurement is correct. The slide of the combination square rests against surface *E*, while its base is held in contact with surface *F*; the scale is then used to measure from the combination square slide to the center in the hole. After the bolt holes *L* and *M* are located, drilled and tapped, bolts are entered and the bracket is reset for pinning; the clearance in the bolt holes permits of adjustment.

To set the oil pump *N* it is merely necessary to bring gears *O* and *P* into proper mesh. The driving shaft brackets on the bed, one of which is shown at *K*, are located as follows: A short arbor representing the driving shaft is inserted in the bearing hole, and then the measurement is taken from surface *F* on the bed. These brackets are located and drilled on the bed while it is under a radial drill for other drilling operations, and the bed is turned over on its side at this time for convenience. The dowel pin holes in the driving shaft brackets are not drilled until all the parts are assembled on the bed; then the final alignment is accomplished by moving the carriage close to each bracket alternately and slackening off the bolts, thus allowing the adjacent bracket to be self-aligned, after which the bolts are tightened for the pinning operations.

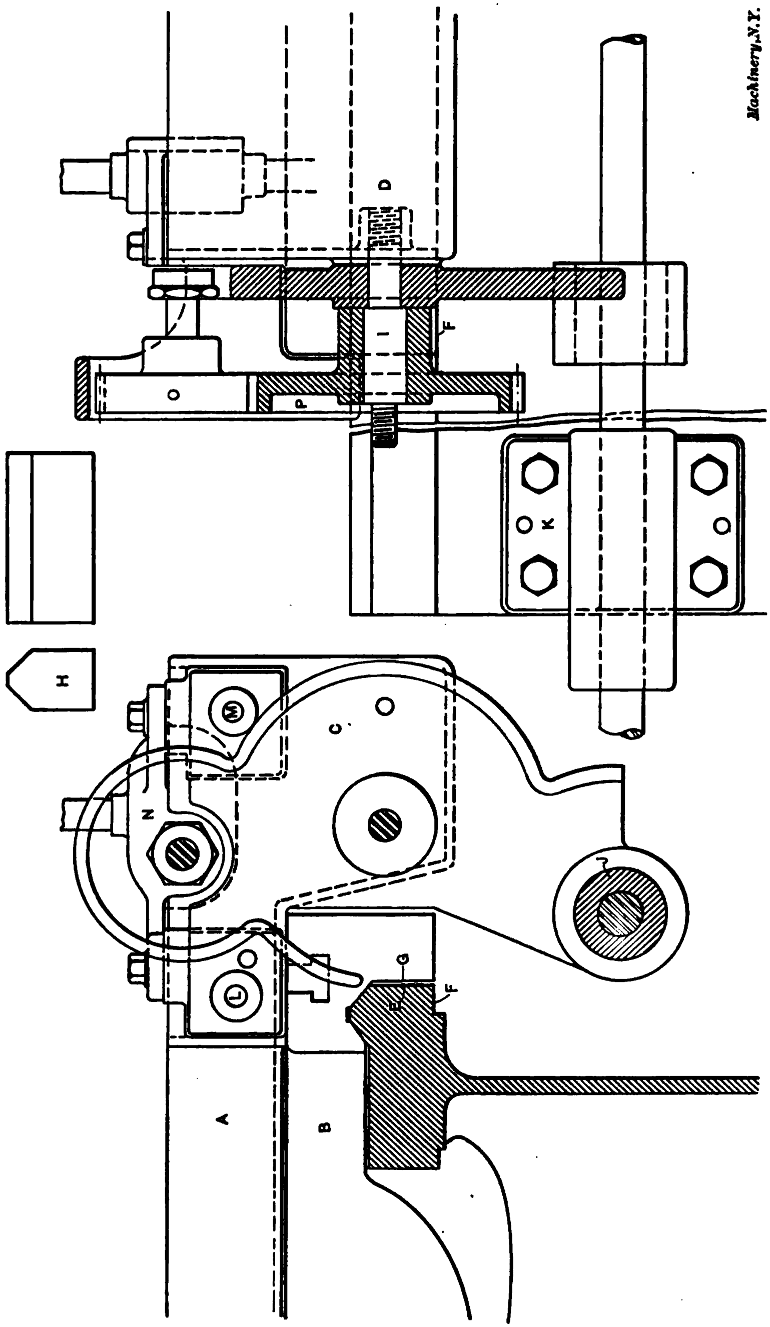

Fig. 30. Method of Laying Out and Aligning an Oil Pump Attachment on an Engine Lathe. Illustrating the Principle of Avoidance of Assembling all the Members together for the Laying-out Operations

Laying Out Angles

In machine tool work it frequently becomes necessary to lay out angles, and as a general rule it may be stated that a much greater degree of accuracy can be obtained by the following methods than is possible by laying off angles with the ordinary bevel protractor made for machine shop work. The correctness with which an angle can thus be produced, however, naturally depends on the skill of the workman in working to the scribed lines and on the accuracy with which they have been located. If it is not convenient to lay off the lines directly on the work, the given angle or taper may be laid off on a piece of sheet steel, which is then carefully filed to the lines scribed thereon.

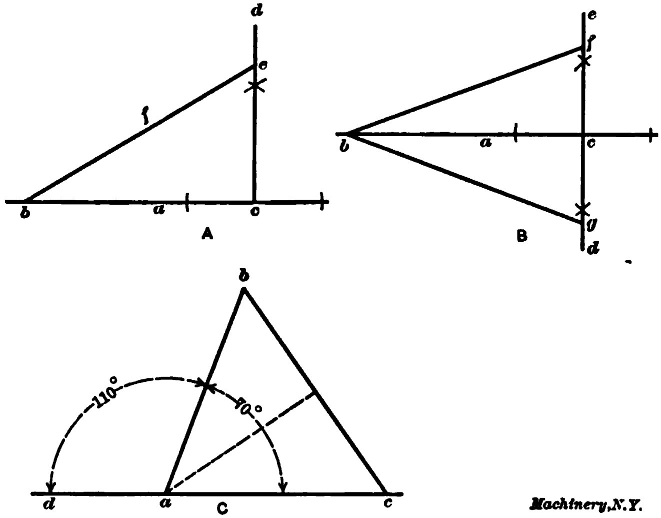

Fig. 31. Graphical Methods of Laying out Angles

Scribe a straight line *a*, as at *A* in Fig. 31; then make two very fine center punch marks, *b* and *c*, on this line, as far apart as circumstances will permit. At *c* erect a perpendicular, as *cd*. The distance *bc* being laid off to some convenient dimension, take the tangent of the required angle and multiply the distance *bc* by this tangent, using a table of natural tangents. Then, on *cd* lay off as accurately as possible the product of *bc* and the tangent, marking it by a fine center punch mark as at *e* on the line *cd*. Scribe a line through *b* and *e*; the angle *ebc* will then be the required angle.

When the required angle is greater than 45 degrees, it is more convenient to use the method shown at *B*. Scribe the line *a* and on it lay off *bc* as long as convenient. At *c* erect the perpendicular line *de*. From a table of natural tangents take the tangent corresponding to one-half the required angle; multiply the distance *bc* by this tangent and lay off the distance thus found on both sides of *c*, marking it at *f* and *g*. Join *f* and *g* to *b* by straight lines. The angle *fbg* is the required angle.

When the required angle is greater than 90 degrees, instead of laying off that angle, its supplement is laid off. Subtract the required angle from 180 degrees and lay off the angle thus formed. Thus, if the required angle is 110 degrees, lay off the angle *bac*, as at *C*, Fig. 31, equal to 180 degrees — 110 degrees = 70 degrees by the method illustrated at *B*. The angle *dab* is then 110 degrees. All other factors remaining as before, the accuracy attainable will be greater as the base line, as *bc*, at *A* and *B*, or *ac*, at *C*, is made longer.

The laying-out and aligning operations on machine tools require, as we have seen, a thorough understanding of the purpose of the various parts making up the machine, and the accuracy required in their alignment. Besides this, a general knowledge of elementary geometry is not only helpful but in many cases almost indispensable. The examples of aligning operations given in the present treatise are, of course, intended to be primarily of suggestive value. Individual judgment will have to be used in each particular case, and definite rules cannot be laid down that would be applicable under all conditions. The general outlines presented above, however, and the simple methods given for the laying out of angles will be found useful in operations of this kind not only on machine tools but on all classes of machinery where the accuracy of the alignment of interdependent parts is necessary for the successful working of the machines.

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By ERIK OBERG

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NUMBER 52

ADVANCED SHOP ARITHMETIC FOR THE MACHINIST

By Erik Oberg

CONTENTS

49-55 Lafayette Street, New York City

Students whose knowledge of elementary arithmetic and its application to simple problems is too limited for intelligent study of this treatise, are advised to first study MACHINERY'S Jig Sheets 5A to 15A, inclusive, Common Fractions and Decimals, and MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist. Not until the principles of elementary arithmetic and its application to simple shop problems are well understood can the student expect to derive the full benefit from the study of the present book.

CHAPTER I

SQUARE ROOT

The square of a number is the product of that number multiplied by itself. The square of 2 is $2 \times 2 = 4$, and the square of 10 is $10 \times 10 = 100$; similarly the square of 177 is $177 \times 177 = 31{,}329$. Instead of writing 2×2 for the square of 2, it is often written 2^2, which is read *two square*, and means that 2 is multiplied by 2. In the same way 128^2 means 128×128. The small figure (^{2}) in these expressions is called *exponent.*

The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number. Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. We may say that the square root is the reverse of the square, so that if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{\ }$, but the *index figure* (^{2}) is generally left out, making the square-root sign simply $\sqrt{\ }$, thus:

$\sqrt{4} = 2$ (the square root of four equals two),

$\sqrt{100} = 10$ (the square root of one hundred equals ten).

The operation of finding the square root of a given number is called *extracting* the square root.

Assume that the square root of 119,716 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will, of course, have only one figure.

11'97'16 | Space for root.

Now find the greatest whole number the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure of the root. In the example this number is 3, the square of which is 9. Subtract this square from the left-hand period, and move down the next period of two figures and annex it to the remainder, thus:

```
           11'97'16 | 3
3 × 3 = 9
        ---
        297
```

Now multiply the figure of the root obtained by the constant 20 which is always used when extracting the square root by this method ($3 \times 20 = 60$), and find how many times this product is contained in the number 297. This gives us a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

```
                 11'97'16 | 34
         3 × 3 = 9
     3 × 20 = 60  297
```

Now subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); (60 + 4) × 4 = 256. If this product were larger than 297 it would indicate that the trial figure is too large, and a figure one unit smaller should be used.

Then move down the next period of two figures and annex it to the remainder.

```
                 11'97'16 | 34
          3 × 3 = 9
     3 × 20 = 60 297
   (60 + 4) × 4 = 256
                  4116
```

Now multiply the figures of the root thus far obtained by 20 (34 × 20 = 680), and find how many times this product is contained in 4116. This gives us a trial figure for the third figure of the root; 680 is contained 6 times in 4116, and 6 is therefore placed as the third figure of the root. Then subtract from 4116 the product of 680 plus the figure of the root just obtained (6), multiplied by the same figure (6).

```
                  11'97'16 | 346
           3 × 3 = 9
      3 × 20 = 60  297
    (60 + 4) × 4 = 256
    34 × 20 = 680   4116
  (680 + 6) × 6 =   4116
```

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716.

If there is a remainder when the last period of figures has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, multiply the number so far obtained in the root by 20, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

```
                    1'25 | 11.18
           1 × 1 = 1
       1 × 20 = 20   25
     (20 + 1) × 1 =  21
     11 × 20 = 220   400
   (220 + 1) × 1 =   221
  111 × 20 = 2220    17900
 (2220 + 8) × 8 =    17824
```

It will be seen from the calculation that when multiplying by the constant 20, the decimal point is disregarded, and the figures obtained in the root considered as a whole number. The decimal point must, however, be placed in the root as already explained before annexing the two first ciphers (not in the given number) to the remainder, in order to give a correct value to the root.

When extracting the square root of a decimal fraction, or when the square root of a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of two figures each, *beginning at the decimal point*, thus:

2'17'63.56'78'5

If the number of decimal places is not an even number, the period to the right will have only one figure instead of two. By placing a cipher after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

2'17'63.56'78'50

It should be borne in mind that the pointing off of periods of two figures each should always be begun at the decimal point, both for the whole numbers and for the decimals. Thus, for instance, the pointing off in the first line below is correct, while the pointing off in the second line is incorrect:

Correctly pointed off:	0.76'34'5	3'26.75'4
Incorrectly pointed off:	0.7'63'45	32'6.7'54

When extracting the square root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

Example:

$$
\begin{array}{rl|l}
 & 5.71'21 & \underline{2.39} \\
2 \times 2 = & \underline{4} & \\
2 \times 20 = 40 & 171 & \\
(40 + 3) \times 3 = & \underline{129} & \\
23 \times 20 = 460 & 4221 & \\
(460 + 9) \times 9 = & \underline{4221} &
\end{array}
$$

When it is found that the next figure in the root is a cipher, place it as usual in the root, and move down the next period of two figures, in all other respects following the procedure already explained.

Example:

$$
\begin{array}{rl|l}
 & 9'12'04 & \underline{302} \\
3 \times 3 = & \underline{9} & \\
\left.\begin{array}{r} 3 \times 20 = 60 \\ 30 \times 20 = 600 \end{array}\right\} & 1204 & \\
(600 + 2) \times 2 = & \underline{1204} &
\end{array}
$$

The square root of a common fraction may be obtained by extracting the square root of both numerator and denominator, thus:

$$\sqrt{\frac{25}{49}}=\frac{\sqrt{25}}{\sqrt{49}}=\frac{5}{7}$$

When the terms of the fraction are not perfect squares (squares of whole numbers), it is preferable to change the common fraction to a decimal fraction, and extract the square root of this.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by multiplying the root by itself, in which case the product must equal the number given, of which the square root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation; if this approximate root is multiplied by itself, the product should *very nearly* equal the given number; if not, an error has been made.

CHAPTER II

CUBE ROOT

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1{,}728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read "two cube." In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in these expressions is called *exponent*. An expression of the form 18^3 may also be read the "third power of 18."

In the same way as square root means the reverse of square, so cube root means the reverse of cube; that is, the cube root of a given number is the number which, if repeated as factor three times, would give the number given. Thus the cube root of 27 is 3, because $3 \times 3 \times 3 = 27$. If the cube of 15 is 3,375, then the cube root of 3,375 is, of course, 15. The mathematical sign for the cube root is $\sqrt[3]{\ }$, thus:

$\sqrt[3]{64} = 4$ (the cube root of sixty-four equals four),

$\sqrt[3]{4096} = 16$ (the cube root of four thousand ninety-six equals sixteen).

In the case of all roots, except the square root, the index, or the small figure in the radical sign ($\sqrt{\ }$), must be given.

Assume that the cube root of 80,621,568 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of *three* figures each. According to the total number of figures in the given number, the last period to the left will, of course, have one, two or three figures.

80'621'568 | Space for root.

Now find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root. The cube of 4 is 64 ($4 \times 4 \times 4 = 64$), and the cube of 5 is 125 ($5 \times 5 \times 5 = 125$). Hence 4 is the greatest whole number, the cube of which does not exceed 80, and 4, therefore, is the first figure of the root. Subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder, thus:

```
                80'621'568 | 4
   4 × 4 × 4 = 64
               -----
                16621
```

Now multiply the square of the figure in the root by the constant 300, which is always used when extracting the cube root by this method ($4^2 \times 300 = 4 \times 4 \times 300 = 4{,}800$), and find how many times this product is contained in the number 16,621. This gives us a trial figure for the second figure of the root; 4,800 is contained three whole times in 16,621, and 3 is therefore placed as the next figure of the root:

```
                     80'621'568 | 43
         4 × 4 × 4 = 64
                    -----
   4² × 300 = 4,800   16621
```

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root, excepting the last one, multiplied by 300, and this product multiplied by the figure just obtained in the root, thus:

$$4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14{,}400.$$

2. The figure or figures already obtained in the root, excepting the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained, thus:

$$4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1{,}080.$$

3. The cube of the last figure obtained, thus:

$$3^3 = 3 \times 3 \times 3 = 27.$$

The method followed will be understood by studying the example and comparing the different quantities with the worded explanations just given. If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used.

Now move down the next period of three figures, and annex it to the remainder.

```
                                       80'621'568 | 43
                           4 × 4 × 4 = 64
                                       -----
                      4² × 300 = 4,800  16621
   4² × 300 × 3 + 4 × 30 × 3² + 3³ = 15507
                                       -------
                                        1114568
```

Multiply the square of the figures of the root thus far obtained by 300 ($43^2 \times 300 = 43 \times 43 \times 300 = 554{,}700$), and find how many times this product is contained in 1,114,568. This gives a trial figure for the third figure of the root; 554,700 is contained two times in 1,114,568, and 2 is therefore placed as the third figure of the root. Now subtract from 1,114,568 a sum made up of the three products previously given, and shown in the example below:

$$80'621'568 \mid 432$$

$$4 \times 4 \times 4 = 64$$

$$4^2 \times 300 = 4{,}800 \qquad 16621$$

$$4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = 15507$$

$$43^2 \times 300 = 554{,}700 \qquad 1114568$$

$$43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 = 1114568$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 432 is the exact cube root of 80,621,568.

If there is a remainder when the last period of three figures has been moved down, place a decimal point after the figures already obtained in the root, annex three ciphers (000) to the remainder, multiply the square of the number thus far obtained in the root by 300, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

$$1'816 \mid 12.2$$

$$1 \times 1 \times 1 = 1$$

$$1^2 \times 300 = 300 \qquad 816$$

$$1^2 \times 300 \times 2 + 1 \times 30 \times 2^2 + 2^3 = 728$$

$$12^2 \times 300 = 43{,}200 \qquad 88000$$

$$12^2 \times 300 \times 2 + 12 \times 30 \times 2^2 + 2^3 = 87848$$

It should be noted in these calculations that when squaring the figures thus far obtained in the root, and multiplying by the constant 300, the decimal point is disregarded and the figures obtained in the root considered as a whole number. The decimal point, must, however, be placed in the root as already explained, before annexing the first three ciphers (not in the given number) to the remainder, in order to give a correct value of the root.

When the cube root of a number containing a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of three figures each, *beginning at the decimal point*, thus:

83'675'731.563'75

If the number of decimal places is not evenly divisible by three, the period to the right will have only one or two figures instead of three. By placing one or two ciphers after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

83'675'731.563'750

It should be borne in mind that the pointing off of periods of three figures each should always be begun at the decimal point, both for the whole number and for the decimals. Thus, for instance, the pointing off in the first line below is correct while the pointing off in the second line is incorrect:

Correctly pointed off:	0.765'354'3	2'765.354'2
Incorrectly pointed off:	0.7'653'543	27'65.3'542

When extracting the cube root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

When it is found that the next figure in the root is a cipher, place it as usual in the root and move down the next period of three figures, in all other respects following the procedure already explained.

The cube root of a common fraction may be obtained by extracting the cube root of both the numerator and denominator, thus:

$$\sqrt[3]{\frac{27}{1000}}=\frac{\sqrt[3]{27}}{\sqrt[3]{1000}}=\frac{3}{10}$$

When the terms of the fraction are not perfect cubes (cubes of whole numbers), it is preferable to change the common fraction to a decimal fraction and then extract the cube root.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by repeating the root as a factor three times, in which case the product must equal the number given, of which the cube root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation. If this approximate root is repeated as a factor three times the product should *very nearly* equal the given number; if not, an error has been made.

CHAPTER III

THE USE OF FORMULAS

In mathematical and mechanical books and treatises, as well as in articles containing calculations published in the engineering journals, formulas are used to a great extent instead of rules expressed in words. In these formulas, signs and symbols are used in order to condense into a small space the essentials of what would otherwise be long and cumbersome rules. The symbols used are generally the letters in the alphabet, and the signs are simply the ordinary signs for arithmetical calculations, with some additional ones necessary for special purposes. Letters from the Greek alphabet are commonly used to designate angles, and the Greek letter π (pi) is always used to indicate the proportion of the circumference of a circle to its diameter; π, therefore, is always, in formulas, equal to 3.1416. The most commonly used Greek letters, besides π, are α (alpha), β (beta), and γ (gamma).

Knowledge of algebra is not necessary in order to make possible the successful use of formulas for the solving of problems such as occur in ordinary shop practice; but a thorough understanding of the rules and processes of arithmetic is very essential. The symbols or letters used in the formulas simply stand in place of the actual figures or numerical values which are inserted in the formula in each specific case, according to the requirements of the problem to be solved. When these values are inserted, the result required may be obtained by simple arithmetical processes.

There are two main reasons why a formula is preferable to a rule expressed in words. Firstly, the formula is more concise, it occupies less space, and it is possible for the eye to catch at a glance the whole meaning of the rule laid down; secondly, it is easier to remember a short formula than a long rule, and it is, therefore, of greater value and convenience, as it is not always possible to carry a handbook or reference book about, but the memory must be relied upon to store up a number of the most frequently occurring mathematical and mechanical rules.

The use of formulas can be explained most readily by actual examples. In the following, therefore, a number of simple formulas will be given, and the values will be inserted so as to show, in detail, the principles involved.

Example 1.—When the diameter of a circle is known, the circumference may be found by multiplying the diameter by 3.1416. This rule, expressed as a formula, is:

$$C = D \times 3.1416$$

in which C = circumference of circle,
D = diameter of circle.

This formula shows at a glance that no matter what the diameter of the circle be, the circumference is always equal to the diameter times 3.1416. Let it be required to find, for example, the circumference of a circle 24 inches in diameter. If, then, we insert 24 in place of D in the formula, we have:

$$C = 24 \times 3.1416 = 75.3984 \text{ inches.}$$

Hence, our formula gives, by means of a simple multiplication, the result required.

Assume that the diameter of a circle is 5.13 inches. The circumference of this circle is found by inserting this value instead of D in the formula:

$$C = 5.13 \times 3.1416 = 16.1164 \text{ inches.}$$

Example 2.—In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the diametral pitch. Then the formula would be:

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter (D) of the gear equals 2 added to the number of teeth (N), this sum divided by the pitch (P).

If the number of teeth in a gear is 26 and the diametral pitch 4, then simply put these figures in the place of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{26 + 2}{4} = \frac{28}{4} = 7.$$

D, or the outside diameter, then, is 7 inches.

In another gear the number of teeth is 62 and the pitch 8; find the outside diameter of the gear.

$$D = \frac{62 + 2}{8} = \frac{64}{8} = 8 \text{ inches.}$$

From the examples given it will be seen that in formulas, each letter stands for a certain dimension or quantity. When using a formula for solving a problem, replace the letters in the formula by the equivalent figures given in a certain problem, and find the result by means of regular arithmetical calculation.

Example 3.—The formula for the horsepower of a steam engine is as follows:

$$\text{H. P.} = \frac{P \times L \times A \times N}{33{,}000}$$

in which H. P. = indicated horsepower of engine,

P = mean effective pressure on piston in pounds per square inch,

L = length of piston stroke in feet,

A = area of piston in square inches,

N = number of strokes of piston per minute.

Assume that $P = 120$, $L = 2$, $A = 320$ and $N = 160$; what would be the horsepower?

If we insert the given values in the formula we have:

$$\text{H. P.} = \frac{120 \times 2 \times 320 \times 160}{33{,}000} = 372.36$$

In formulas the sign for multiplication ($\times$) is often left out between letters, the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$\frac{P \times L \times A \times N}{33{,}000} \text{ can also be written } \frac{PLAN}{33{,}000}$$

Thus, if $A = 6$ and $B = 7$, then:

$$AB = A \times B = 6 \times 7 = 42.$$

If $A = 9$, $B = 6$ and $C = 7$, then:

$$ABC = A \times B \times C = 9 \times 6 \times 7 = 378.$$

It is only the multiplication sign ($\times$) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic.

A parenthesis ( ) or bracket [ ] in a formula means that the expression inside the parenthesis or bracket should be considered as one single symbol, or in other words, that the calculation inside the parenthesis should be carried out by itself, before other calculations are carried out.

Examples:

$5 \times (8 + 4) = 5 \times 12 = 60.$

$7 \times (18 - 6) + 6\ (4.52 - 1.95) = 7 \times 12 + 6 \times 2.57 = 84 + 15.42 = 99.42.$

In the last example above it will be seen that 7 is multiplied by 12, and 6 by 2.57, and then the products of these two multiplications are added. From the order of the numbers $7 \times 12 + 6 \times 2.57$, one might have assumed that the calculation should have been carried out as follows: 7 times 12 = 84, plus 6 = 90, times 2.57 = 231.3. This latter procedure, however, is not correct, as the following rule should be applied:

When several numbers or expressions are connected by the signs $+$, $-$, $\times$ and $\div$, the operations are carried out in the order written, except that *all multiplications should be carried out before the other operations*. The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. Divisions should be carried out before additions and subtractions, if the division is indicated in the same line with these other processes.

Examples:

$4 \times 7 + 9 - 2 \times 9 = 28 + 9 - 18 = 37 - 18 = 19.$

$6 + 7 \times 4 = 6 + 28 = 34.$

$72 \div 3 \times 8 = 72 \div 24 = 3.$

$8.5 + 16.4 \div 4.1 - 2.5 = 8.5 + 4 - 2.5 = 10.$

But $4 \times (7 + 9) - 2 \times 9 = 4 \times 16 - 18 = 64 - 18 = 46.$

$(6 + 7) \times 4 = 13 \times 4 = 52.$

$(72 \div 3) \times 8 = 24 \times 8 = 192.$

$(8.5 + 16.4) \div (4.1 - 2.5) = 24.9 \div 1.6 = 15.56.$

In Chapters I and II the meaning of square and cube, and square root and cube root have already been explained. The squares and square roots as well as the cubes and cube roots of all numbers up to 1,000 (sometimes up to 1,600) are generally given in all standard handbooks.

Example:—Find the value of A in the formula

$$A = \sqrt{B^2 + C^2}$$

if $B = 16$ and $C = 12$.

If we insert the given values in the formula, we have

$$A = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20.$$

In the same way as we write $2^2 = 2 + 2$, and $2^3 = 2 \times 2 \times 2$, we can write $2^4 = 2 \times 2 \times 2 \times 2$; and the expression 2^5 would mean that 2 is repeated as a factor five times, or

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

The expression 2^4 is read "the fourth power of 2" and 6^5, "the fifth power of 6," etc.

In the same way as we may say that the square root means the reverse of square, and the cube root the reverse of cube, so we may say that the fourth root is the reverse of the fourth power; that is, if we want the number which repeated as a factor four times gives a given number, we must obtain the fourth root, or $\sqrt[4]{\ }$. Thus $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Similarly we write the fifth root $\sqrt[5]{\ }$; and $\sqrt[5]{32} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

The examples given indicate the principles involved in the use of formulas, and show also how easily formulas may be employed by anyone who has a general understanding of arithmetic. While it would be possible to express in words all the rules required in ordinary

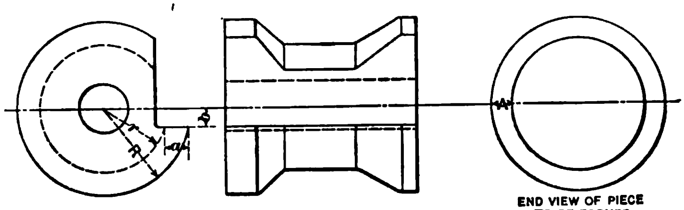


Fig. 1

shop problems, it is very much simpler to make use of formulas; and in the following, formulas will be employed wherever required, and their use in practical work thus made clear.

A useful application both of the use of formulas and of the square and square root of numbers, is found in the problems occurring when figuring forming tools.

Formulas for Circular Forming Tools

When laying out circular forming tools, such as shown in Fig. 1, the cutting edge, as is well known, must be located a certain amount below the horizontal center line of the tool, in order to provide for sufficient clearance for the cut. On account of this, the actual differences of diameters in the piece of work to be formed cannot be directly copied in the forming tool. The distance A in the piece to be formed must equal the distance a on the forming tool, but as this latter distance is measured in a plane a certain distance b below the horizontal plane through the center of the forming tool, it is evident that the differences of diameters in the tool and the piece to be formed are not the same. A general formula may, however, be deduced, by the use of elementary geometry, by means of which the various diameters of the forming tool may be determined if the largest (or smallest) diameter of the tool, the amount that the cutting edge is below the center, and, of course, the diameters of the piece to be formed, are known.

If R = the largest radius of the tool,
a = difference in radii of steps in the work, and
b = amount cutting edge is below center,
then, if r be the radius required,

$$r = \sqrt{\left(\sqrt{R^2 - b^2} - a\right)^2 + b^2}$$

If the smaller radius r is given and the larger radius R sought, the formula takes the form:

$$R = \sqrt{\left(\sqrt{r^2 - b^2} + a\right)^2 + b^2}$$

Suppose, for an example, that a tool is to be made to form the piece in Fig. 2. Assume that the largest diameter of the tool is to be 3

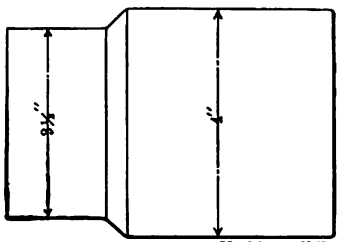


Fig. 2

inches, and that the cutting edge is to be ¼ inch below the center of the tool. Then the diameter next smaller to 3 inches is found from the formulas given by inserting the given values: $R = 1\frac{1}{2}$ inch, $b = \frac{1}{4}$ inch, and $a = \frac{1}{4}$ inch (half the difference between 4 and 3½ inches; see Fig. 2).

Then

$$r = \sqrt{\left(\sqrt{(1\tfrac{1}{2})^2 - (\tfrac{1}{4})^2} - \tfrac{1}{4}\right)^2 + (\tfrac{1}{4})^2} = \sqrt{\left(\sqrt{\tfrac{35}{16}} - \tfrac{1}{4}\right)^2 + \tfrac{1}{16}} = \frac{5.017}{4}$$

$= 1.254$ inch.

While the formula looks complicated, by means of a table of squares the calculations are easily simplified and can be carried out in three or four minutes. The value of r being 1.254 inch, the diameter to make the smaller step of the forming tool will be 2.508 inches, instead of 2½ inches exact, as would have been the case if the cutting edges had been on the center line.

CHAPTER IV

TIME REQUIRED FOR DRILLING, MILLING AND PLANING

In MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, a rule is given for calculating the time required for turning in the lathe, with a given feed. In this chapter, rules and formulas will be given for calculating the time required for drilling, milling and planing.

The feed of a drill in the drill press is the downward motion of the drill per revolution. The feed of a milling cutter is the forward movement of the milling machine table for each revolution of the cutter. Sometimes the feed is expressed as the distance which the drill or the milling machine table moves forward in one minute. In order to avoid confusion, it is, therefore, always best to state plainly in each case whether feed per revolution or feed per minute is meant.

Time Required for Drilling

In order to calculate the time required for drilling a given depth of hole, the number of revolutions per minute of the drill, and the feed per revolution (or the cutting speed, the diameter of the drill and the feed per revolution) must be known.

Assume that a 1⅞-inch drill makes 80 revolutions per minute and that the feed per revolution is 0.008 inch. How long a time will it require to drill a hole 5½ inches deep? To find the number of revolutions required to drill the full depth of the hole, divide 5½ by 0.008, obtaining the quotient 687.5 or approximately 690 revolutions. As the drill makes 80 revolutions in one minute, we find the total number of minutes required by dividing 690 by 80, the quotient 8.6 being the number of minutes required to drill a hole 5½ inches deep under the given conditions. If, in the foregoing,

T = time required for drilling, in minutes,
L = depth of drilled hole, in inches,
N = number of revolutions per minute of the drill,
F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}.$$

Expressed as a rule, this formula would be:

To find the time required to drill a hole to a given depth when the feed per revolution of the drill, the depth of the hole, and the number of revolutions per minute are given, divide the depth of the hole by the number of revolutions per minute multiplied by the feed per revolution.

If the cutting speed of the drill and its diameter are given instead of the number of revolutions, the number of revolutions must first be found before applying the formula given.*

If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

The feed of drills should be about 0.004 inch per revolution for a 1/16-inch drill, 0.005 inch for a ¼-inch drill, 0.008 inch for a ½-inch drill, 0.010 inch for a 1-inch drill, and 0.015 inch for a 2-inch drill. If the drill breaks or chips at the cutting edges, the feed should be reduced.

Time Required for Milling

The time required for milling may be found if the number of revolutions per minute of the cutter, and the feed per revolution (or the cutting speed, the diameter of the cutter and the feed per revolution) are known. If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

If the length of the cut taken in a milling machine is 8⅜ inches and the feed is 1/64 per revolution, how long a time will it take for a cutter making 20 revolutions per minute to traverse the work? As the feed per revolution is 1/64 inch and the cutter makes 20 revolutions per minute, the feed per minute is 20/64 or 5/16 inch. To find the time required for the cutter to traverse the full length of the work, divide the length of the cut, 8⅜ inches, by the feed in one minute; thus:

$$8\frac{3}{8} \div \frac{5}{16} = \frac{67}{8} \times \frac{16}{5} = \frac{134}{5} = 26\frac{4}{5} = 26.8.$$

The time required would thus be 27 minutes, approximately.

If T = time required for the cutter to traverse the work, in minutes,
L = length of cut, in inches,
N = revolutions per minute of the cutter,
F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}.$$

It will be seen that the form of this formula is the same as that of the formula for the time required for drilling.

If the cutting speed and the diameter of the cutter are given instead of the number of revolutions, the latter number must first be found before the formula above is applied.*

The average feed of milling cutters per minute should vary from about 4 inches for a ½-inch mill cutting cast iron, and 1½ inch for the same mill cutting steel, to 1¼ inch for a 6-inch cutter on cast iron and ½ inch for the same cutter on steel. Of course, these feeds must be varied with the depth of the cut.

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 17.

Feed of Planer Tools

The feed of a planer tool is its sidewise motion for each cutting stroke of the table or platen. If for each cutting stroke the tool-carrying head moves 1/16 inch along the cross-rail, we say that the feed is 1/16 inch. Each cutting stroke necessitates a return stroke, and in the following, when the expression "number of strokes" is used, it means number of cutting strokes.

Time Required for Planing

The time required for planing a piece of work can be calculated if the feed per stroke, the number of strokes of the planer table per minute, and the width of the work, are known.

Assume that a planer makes 6 strokes per minute, that the feed per stroke is 3/32 inch, and that the width of the work is 22 inches. Find the time required for planing the work.

As the planer makes 6 strokes per minute and the feed per stroke is 3/32 inch, the feed per minute is 6 × 3/32 or 9/16 inch. The tool must traverse 22 inches to plane the complete work; the traverse in one minute being 9/16 inch, the total number of minutes required to traverse the work is found by dividing 22 by 9/16.

$$22 \div \frac{9}{16} = \frac{22}{1} \times \frac{16}{9} = \frac{352}{9} = 39\frac{1}{9} \text{ minutes.}$$

The time required for planing the work is thus 40 minutes, approximately.

This calculation may be summed up in the following formula, applicable to any case where the feed per stroke, the number of strokes per minute, and the width of the work are known

$$T = \frac{W}{F \times N}.$$

In this formula

T = time required for planing, in minutes,

W = width of work, in inches,

F = feed per stroke, in inches,

N = number of strokes per minute.

The formula expressed as a rule would be as follows:

To find the time required for planing when the width of the work, the feed per stroke and the number of strokes per minute, are known, divide the width of the work by the feed times the number of cutting strokes per minute.

CHAPTER V

PULLEY AND GEAR DRIVES

In MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, the calculations for simple and compound gear drives and simple pulley drives are treated. In this chapter some special cases of compound pulley drives and combined pulley and gear drives will be considered.

Compound Pulley Speeds

In Fig. 3 are shown four pulleys of which the two pulleys *B* and *C* are keyed to the same shaft Pulley *A* is the driving pulley and drives pulley *B*; pulley *C*, on the same shaft as *B*, is also a driving pulley, and pulley *D*, a driven pulley. The rules and formulas for compound

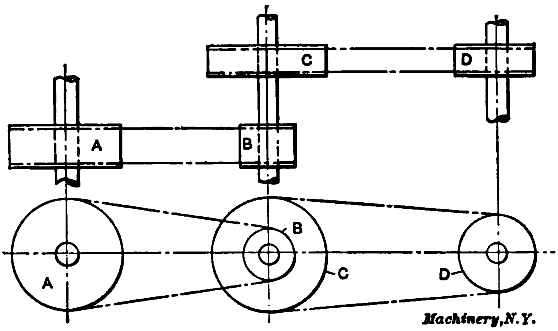


Fig. 3. Compound Pulley Drive

gearing can be directly applied to pulleys arranged in this manner by simply substituting in the formulas the diameters of the pulleys, in inches, for the numbers of teeth in the gears. Thus, to find the revolutions per minute of the driven pulley *D* when the diameters of all the four pulleys and the number of revolutions of pulley *A* are given, the formula below is used:

$$\begin{array}{c}\text{rev. per min.}\\\text{of driven pulley}\end{array} = \begin{array}{c}\text{rev. per min.}\\\text{of driving pulley}\end{array} \times \frac{\begin{array}{c}\text{product of diameters}\\\text{of driving pulleys}\end{array}}{\begin{array}{c}\text{product of diameters}\\\text{of driven pulleys}\end{array}}$$

If the numbers of revolutions of the shafts on which pulley *A* and pulley *D* are mounted, are given, and it is required to find the diameters of four pulleys which will transmit motion from pulley *A* to pulley *D* at the given speed ratio, we proceed in the same way as when finding the number of teeth in gears for transmitting a given motion.*

Find the speed ratio by writing the number of revolutions of the

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 30.

driving pulley as the numerator and the number of revolutions in the driven pulley as the denominator of a fraction, and reduce this fraction to its lowest terms. Then divide both the numerator and denominator in the fraction giving the ratio in two factors, and multiply each "pair" of factors by the same number until pulleys with suitable diameters are found. **(One factor in the numerator and one in the denominator are considered as "one pair.")**

Assume that the number of revolutions per minute of the shaft with pulley *A* is 260, and that it is required to drive the shaft on which pulley *D* is mounted at 720 revolutions. What diameters of pulleys can be used? The fraction $\frac{260}{720}$ reduced to its lowest terms is

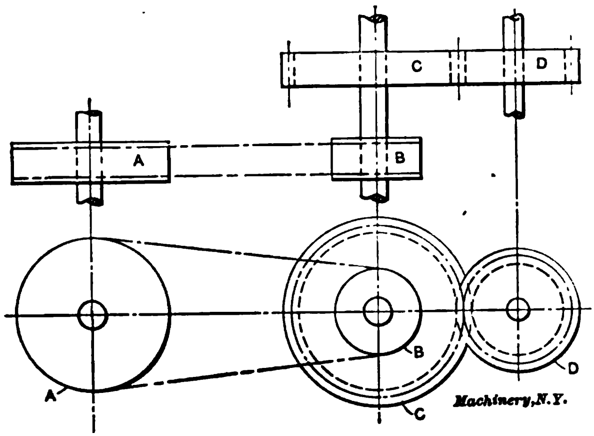


Fig. 4. Combined Pulley and Gear Drive

$\frac{13}{36}$; $\left(\frac{260}{720}=\frac{26}{72}=\frac{13}{36}\right)$. The speed ratio, therefore, is $\frac{13}{36}$. Now, following the rule given above:

$$\frac{13}{36}=\frac{1\times 13}{2\times 18}=\frac{(1\times 12)\times(13\times 1)}{(2\times 12)\times(18\times 1)}=\frac{12\times 13}{24\times 18}$$

The pulleys in the numerator, with 12 and 13 inches diameter, are the driven pulleys *B* and *D*, and the pulleys in the denominator, with 18 and 24 inches diameter, are the driving pulleys. The rule above reduced to a formula would be:

$$\text{ratio of speed of the first driving pulley to the last driven pulley} = \frac{\text{product of diam. of driven pulleys}}{\text{product of diam. of driving pulleys}}$$

Combined Belt and Gear Drive

In Fig. 4 is shown a combined belt and gear drive, where pulley *A* drives pulley *B*, and gear *C*, which is mounted on the same shaft as pulley *B*, drives the gear *D*. Calculations for numbers of revolutions and numbers of teeth and diameters of pulleys are carried out exactly

as in the examples where we have dealt exclusively with gears or exclusively with pulleys. When dealing with the pulleys we use the diameter of the pulley in inches, and when dealing with the gears, the number of teeth in the gears.

Assume that the diameter of pulley *A* is 54 inches and that of pulley *B*, 18 inches, that gear *C* has 112 teeth, and that gear *D* has 78 teeth. If pulley *A* makes 39 revolutions per minute, how many revolutions per minute does gear *D* make? Using the formula for finding the revolutions per minute previously given, we have:

$$\text{rev. per min.} = 39 \times \frac{54 \times 112}{18 \times 78} = 168.$$

If the number of revolutions of the shaft on which pulley *A* is mounted is 60, and the number of revolutions required for the shaft on which gear *D* is mounted, is 110, what diameter pulleys and what size of gears could we employ to transmit the required motion? The speed ratio is $\frac{60}{110} = \frac{6}{11}$. Proceeding as before, we have:

$$\frac{6}{11} = \frac{2 \times 3}{1 \times 11} = \frac{(2 \times 16) \times (3 \times 8)}{(1 \times 16) \times (11 \times 8)} = \frac{32 \times 24}{16 \times 88}$$

The numbers 32 and 24 in the numerator of the last fraction give the diameter of the driven pulley *B* and the number of teeth of the driven gear *D*, respectively, and the numbers 16 and 88 in the denominator of the fraction give the diameter of the driving pulley *A*, and the number of teeth in the driving gear *C*. In this case, then, pulley *A* would be 16 inches in diameter, pulley *B*, 32 inches, gear *C* would have 88 teeth, and gear *D*, 24 teeth.

CHAPTER VI

HORSEPOWER OF BELTING

The horsepower which a belt of a given size can transmit depends on the speed with which the belt travels and the working stress advisable to permit in the belt. The speed with which the belt travels, of course, depends on the diameter and number of revolutions per minute of the pulley over which it travels, it being assumed that there is no appreciable slip between the belt and the pulley. If we are to find the horsepower a belt can safely transmit, we must, therefore, consider in our formulas the diameter of the pulley, its number of revolutions per minute, and the permissible working stress in the belt.

Let d = diameter of driving pulley in inches,
v = velocity of belt in feet per minute,
n = number of revolutions of pulley per minute,

S = working stress of belt per inch of width, in pounds,
w = width of belt in inches.

Then:

$$v = \frac{\pi d n}{12} = \frac{3.1416\, d n}{12} = 0.2618\, d n$$

$$\text{H. P.} = \frac{S v w}{33,000} = \frac{0.2618\, S d n w}{33,000}$$

A commonly used value for the safe working stress per inch of width of single belts is 33 pounds. When this value is adopted, a belt one inch wide, traveling at a rate of 1,000 feet per minute, will transmit one horsepower.

Example:—How many horsepower will a single belt 2½ inches wide, traveling over a pulley 12 inches in diameter, transmit, if the pulley makes 200 revolutions per minute? Assume the working stress at 33 pounds per inch of width of belt.

In this example $d = 12$, $n = 200$, $S = 33$ and $w = 2\frac{1}{2}$. If these values are inserted in the horsepower formula given, we have:

$$\text{H. P.} = \frac{0.2618 \times 33 \times 12 \times 200 \times 2.5}{33,000} = 1.57.$$

A working stress up to 45 pounds per inch of width of belt is permissible for single belts in good condition. If we adopt this latter value for the stress, how many horsepower would the given belt transmit?

We only need to change 33 in the expression above to 45, and then we have:

$$\text{H. P.} = \frac{0.2618 \times 45 \times 12 \times 200 \times 2.5}{33,000} = 2.14.$$

If the horsepower to be transmitted is known, the width of belt required may be found by a transposition of the given formula, as follows:

$$w = \frac{\text{H. P.} \times 33,000}{S v} = \frac{\text{H. P.} \times 33,000}{0.2618\, S d n}$$

in which formula the letters denote the same quantities as previously given.

Example: Find the width of single belt required to transmit 20 horsepower with a belt velocity of 1,800 feet per minute?

In this example H. P. $= 20$, $v = 1,800$, and S may be assumed to be 45. If we insert these values in the given formula for width of belt, we have:

$$w = \frac{20 \times 33,000}{45 \times 1,800} = 8.15 \text{ or, say, } 8\frac{1}{4} \text{ inches.}$$

In order to reduce the width of a single belt when it becomes too wide, a double belt may be used. The working stress of a double belt per inch of width may be assumed at from 65 to 90 pounds, the latter value being only for belts kept in good condition.

Assume that in the example just given, we use a double belt instead of a single, and assume a working stress of 80 pounds per inch of width of belt. How wide, then, would the belt be?

Substituting 80 for 45, we have:

$$w = \frac{20 \times 33{,}000}{80 \times 1{,}800} = 4.58 \text{ or, say, } 4\tfrac{5}{8} \text{ inches.}$$

As the working stress is an assumed quantity, always somewhat uncertain, it is, of course, not necessary to retain in our formulas so exact a quantity as 0.2618. If this number is given in round figures as 0.25 or ¼, we could simplify the given formulas as follows:

$$\text{H. P.} = \frac{S\,d\,n\,w}{4 \times 33{,}000}$$

$$w = \frac{\text{H. P.} \times 33{,}000 \times 4}{S\,d\,n}$$

As a final example, find the horsepower transmitted by a 5-inch wide double belt, working stress 75 pounds per inch width of belt, if the belt transmits power from a 4-foot pulley running at 200 revolutions per minute.

In this example $w = 5$, $S = 75$, $n = 200$, and $d = 4 \times 12 = 48$ inches. If we insert these values in our simplified formula, we have:

$$\text{H. P.} = \frac{75 \times 48 \times 200 \times 5}{4 \times 33{,}000} = 27.3.$$

CHAPTER VII

CHANGE GEARS FOR CUTTING METRIC THREADS

The metric system of length measurement is in use in practically all countries except in the United States, Great Britain and the British colonies. The unit of length in the metric system is the meter, which equals nearly 39.37 inches (or practically 39⅜ inches). The subdivisions of the meter are given below:

1 meter = 10 decimeters,
1 decimeter = 10 centimeters.
1 centimeter = 10 millimeters.

In medium and small machine design the unit employed is almost always the millimeter. One millimeter equals 0.03937 inch; one inch equals $\frac{1}{0.03937}$, or 25.4 millimeters, almost exactly.

When screws are made in accordance with the metric system it is not the usual practice to give the number of threads per millimeter or centimeter in the same way as the number of threads per inch is given in the English system. Instead, the lead of the thread in millimeters is given. A screw thread is said to have 2 millimeters lead, 3 millimeters lead, 4.5 millimeters lead, etc.

Change Gears for Cutting Threads with Metric Pitch

It often happens that screws and taps having threads according to the metric system are required. This thread can be cut on a lathe having an English lead-screw, provided change gears with the required number of teeth are used.

The first step in finding the change gears is to find how many threads per inch there are in the screw to be cut, when the lead is given in millimeters. Assume that a screw is required with 3 millimeters lead. How many threads per inch are there in this screw? As there are 25.4 millimeters in one inch, we can find how many threads there would be in one inch, if we find how many times 3 is contained in 25.4; in other words, we divide 25.4 by 3. It is not necessary to carry out the division; simply write it as a fraction in the form $\frac{25.4}{3}$, which implies that 25.4 is to be divided by 3. This fraction now gives the number of threads per inch to be cut. When this fraction has been obtained, proceed as if change gears were to be found for cutting threads with English pitches.* Place the lathe screw constant in the numerator of a fraction and the number of threads per inch to be cut in the denominator. If the screw constant of a lathe is 6 and the number of threads to be cut $\frac{25.4}{3}$, as previously found, the ratio of the change gearing is

$$\frac{6}{\frac{25.4}{3}} = \text{ratio.}$$

This seems complicated, but remembering that the line between the numerator and denominator in a fraction means that the numerator is to be divided by the denominator, we get, by carrying out this division:

$$6 \div \frac{25.4}{3} = 6 \times \frac{3}{25.4} = \frac{6 \times 3}{25.4}.$$

The fraction $\frac{6 \times 3}{25.4}$ is the ratio of the change gearing required, and all we have to do now is to multiply numerator and denominator of this fraction by the same number until we find suitable numbers of teeth for the change gears. By trial we find that the first whole number by which we can multiply 25.4 so as to get a whole number as a result, is 5. Multiplying 25.4 by 5 gives us 127. Thus we must have one gear with 127 teeth whenever we cut a metric thread by means of

* See MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 31.

an English lead-screw. The other gear required in this case has 90 teeth, because $5 \times 6 \times 3 = 90$. The calculation would be carried out as shown below:

$$\frac{6 \times 3 \times 5}{25.4 \times 5} = \frac{18 \times 5}{127} = \frac{90}{127}.$$

What has just been said can be expressed in the following rule:

To find the change gears for cutting metric pitches with an English lead-screw, place the lathe screw constant multiplied by the number of millimeters lead of the thread to be cut multiplied by 5, in the numerator of the fraction, and 127 as the denominator. The product of the numbers in the numerator give the number of teeth in the gear on the spindle stud, and 127 is the number of teeth in the gear on the lead-screw.

Written as a formula this rule would be:

$$\frac{\text{lathe screw constant} \times \text{lead of thread to be cut, in millimeters} \times 5}{127} = \frac{\text{teeth in spindle stud gear}}{\text{teeth in lead-screw gear}}$$

As an example, assume that a screw with 2.5 millimeters lead is to be cut on a lathe having a screw constant 8. By placing the given figures in the formula we have:

$$\frac{8 \times 2.5 \times 5}{127} = \frac{100 \ldots\text{spindle stud gear}}{127 \ldots\text{lead-screw gear}}$$

Compound Gearing

Sometimes it is necessary to compound the gears because the gear on the spindle stud would have too many teeth, that is, it would be too large to be used in simple gearing. It may also happen that the product of the screw constant × the lead in millimeters × 5, is not a whole number, in which case it would be necessary to compound the gears to get whole numbers of teeth.

The method for finding the change gears is exactly the same as the method for compound gears for cutting regular English pitch threads.*

Assume that a screw of 6 millimeters lead is to be cut on a lathe with a screw constant 8. By first applying the formula just given, and then dividing the numerator and denominator into factors, each "pair" of which are multiplied by the same number, we find the change gears as follows:

$$\frac{8 \times 6 \times 5}{127} = \frac{240}{127} = \frac{60 \times 4}{127 \times 1} = \frac{(60 \times 1) \times (4 \times 25)}{(127 \times 1) \times (1 \times 25)} =$$

$$\frac{60 \times 100 \ldots\text{driving gears}}{127 \times 25 \ldots\text{driven gears.}}$$

In a case when the lead of the metric screw to be cut is not a whole number but a fraction, it sometimes causes difficulty in dividing up the numerator in two factors that can be multiplied by whole num-

* See MACHINERY's Reference Series No. 18, Shop Arithmetic for the Machinist, 3d Edition, page 88.

bers so as to give numbers of teeth for gears which are available. Several trials must often be made.

Assume that the lathe screw constant is 6, and that a screw with 1.25 millimeters lead is to be cut. In this case we would find the change gears as below:

$$\frac{6 \times 1.25 \times 5}{127} = \frac{37.5}{127} = \frac{30 \times 1.25}{127 \times 1} = \frac{(30 \times 1) \times (1.25 \times 40)}{(127 \times 1) \times (1 \times 40)} = \frac{30 \times 50}{127 \times 40}.$$

It would not be necessary to write "30 × 1" and "127 × 1" as has been done in the example above, but these numbers have been multiplied by 1 simply to preserve a systematic appearance.

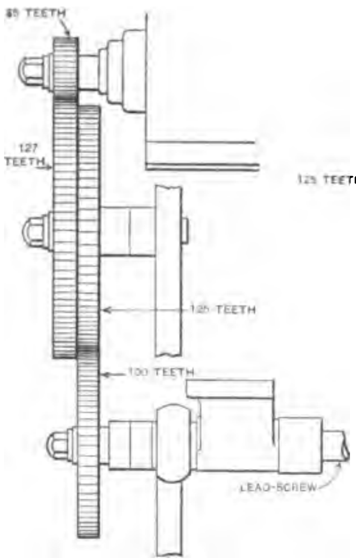


Fig. 5. Cutting a Metric Thread with an English Lead-screw

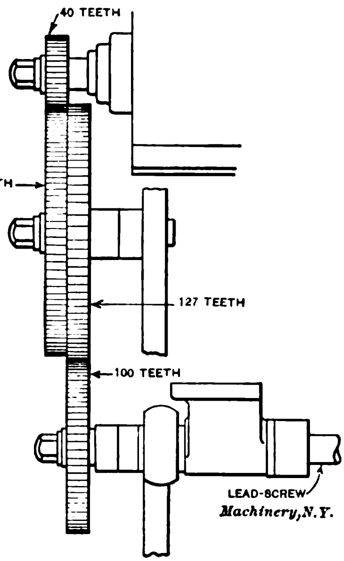


Fig. 6. Cutting an English Thread with a Metric Lead-screw

In Fig. 5 is shown the arrangement of the gearing when cutting a screw of 1.25 millimeters lead on a lathe with a screw constant 7.

$$\frac{7 \times 1.25 \times 5}{127} = \frac{43.75}{127} = \frac{35 \times 1.25}{127 \times 1} = \frac{(35 \times 1) \times (1.25 \times 100)}{(127 \times 1) \times (1 \times 100)} = \frac{35 \times 125}{127 \times 100}.$$

Cutting an English Thread with a Metric Lead-screw

If the lathe has a lead-screw having metric pitch, and it is required to cut a screw with a given number of threads per inch, we must find the "metric screw constant" of the lathe. This is found by placing gears (in simple gearing) with the same number of teeth on the spindle stud and the lead-screw of the lathe, and an idler with any number of teeth, between them, and then cutting a thread on a piece in the lathe. The lead of the thread thus cut, in millimeters, is the metric screw constant of the lathe. Now the method of figuring the change gears when a screw with a given number of threads per inch is to be cut with a lead-screw of metric pitch, is simply the reverse of the method already explained for cutting a metric thread with an English lead-screw.

To find the change gears for cutting English threads with a metric lead-screw, place 127 *in the numerator, and the threads per inch to be cut multiplied by the metric screw constant of the lathe multiplied by* 5 *in the denominator of the fraction;* 127 *is the number of teeth in the gear on the spindle stud, and the product of the numbers in the denominator gives the number of teeth in the gear on the lead-screw.*

This rule expressed as a formula would be:

$$\frac{127}{\text{metric screw constant} \times \text{threads per inch to be cut} \times 5} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}$$

Assume that 5 threads per inch are to be cut in a lathe having a metric screw constant of 4 millimeters. The gears are found directly by using the formula given:

$$\frac{127}{4 \times 5 \times 5} = \frac{127 \ldots\text{spindle stud gear}}{100 \ldots\text{lead-screw gear}}$$

It is sometimes necessary to compound the gears in order to obtain gears which are found in the set of change gears provided with the lathe.

Assume that 10 threads per inch are to be cut in a lathe with a metric screw constant of 4 millimeters. To find the gears we would proceed as follows:

$$\frac{127}{4 \times 10 \times 5} = \frac{127}{200} = \frac{127 \times 1}{100 \times 2} = \frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (2 \times 40)} =$$

$$\frac{127 \times 40 \ldots\text{driving gears}}{100 \times 80 \ldots\text{driven gears.}}$$

In Fig. 6 is shown the arrangement of the gearing when cutting a screw, having 12½ threads per inch in a lathe with a metric lead-screw, the metric screw constant being 5 millimeters.

$$\frac{127}{5 \times 12\frac{1}{2} \times 5} = \frac{127}{312.5} = \frac{127 \times 1}{100 \times 3.125} =$$

$$\frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (3.125 \times 40)} = \frac{127 \times 40}{100 \times 125}$$

CHAPTER VIII

AREAS OF PLANE FIGURES

Squares

The square, Fig. 7, has four sides of equal length, and each of the four angles between the sides is a right or 90-degree angle.

The area of the square equals the length of the side multiplied by itself, or the square of the length of the side. If the side of a square is 14 inches, then the area equals $14 \times 14 = 196$ square inches. If the side is 14 feet, then the area is 196 square feet.

If the area of a square is known, the length of the side equals the square root of the area. Assume that the area of a square equals 1,024 square inches. Then the side equals $\sqrt{1,024} = 32$ inches.

Rectangles

The rectangle, as shown in Fig. 8, has four sides, of which those opposite each other are of equal length, and the four angles between the sides are right or 90-degree angles.

The area of a rectangle is found by multiplying the height or altitude by the length or base. In Fig. 8, B is the altitude and C the base,

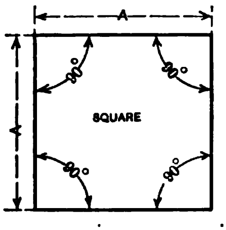


Fig. 7. Square

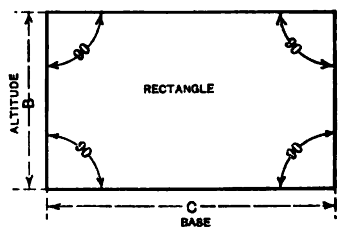


Fig. 8. Rectangle

and the area equals $B \times C$. If $B = 6$ inches, and $C = 11$ inches, then the area equals $6 \times 11 = 66$ square inches.

If the area of a rectangle and the length of its base are known, the height is found by dividing the area by the length of the known base. Either the longer or the shorter side may be considered as the base, the altitude being the side at right angles to the base. If, in Fig. 8, the area of the rectangle is 96 square inches and the side C is 12 inches, then the side $B = 96 \div 12 = 8$ inches.

One square foot equals $12 \times 12 = 144$ square inches. If the area is given in square feet, it can, therefore, be transformed into square inches by multiplying by 144. If the area is given in square inches, it can be transformed into square feet by dividing by 144.

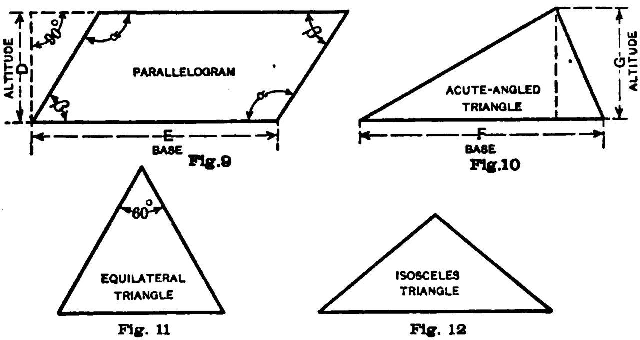
Parallelograms

Two lines are said to be parallel when they have the same direction; when extended, they do not meet or intersect, and the same distance is maintained between the two lines at every point.

Any figure made up of four sides, of which those opposite are parallel, is called a parallelogram. The square and rectangle are parallelograms in which all the angles are right angles. In Fig. 9 is shown a parallelogram where two of the angles are less and two more than 90 degrees. A line drawn from one side of a parallelogram at right angles to the opposite side is called the height or altitude of the parallelogram. In Fig. 9, D is the altitude, and E is the length or base.

The area of a parallelogram equals the altitude multiplied by the base. The area of the parallelogram, in Fig. 9, equals $D \times E$. If D is 16 inches, and E, 22 inches, then the area equals $16 \times 22 = 352$ square inches.

If the area and the base are given, the altitude is found by dividing the area by the base.



Figs. 9 to 12. Parallelogram and Triangles

In parallelograms the angles opposite each other are alike, as indicated in Fig. 9, where the two angles a are equal, and the two angles β also are equal.

Triangles

Any figure bounded by three straight lines is called a triangle. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle. In Fig. 10, if the side F is taken as the base of the triangle, then G is the altitude.

If all three sides of a triangle are of equal length, as in the one shown in Fig. 11, the triangle is called *equilateral.* Each of the three angles in an equilateral triangles equals 60 degrees.

If two sides are of equal length, as shown in Fig. 12, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 13; the side opposite the right angle is called the *hypotenuse.*

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown in Fig. 10. If one of the angles is larger than 90 degrees, as shown in Fig. 14, the triangle is called an *obtuse* or *obtuse-angled* triangle.

The sum of the three angles in every triangle is 180 degrees. The area of a triangle equals one-half the product of the base and the altitude; thus the area of the triangle shown in Fig. 10 equals $\frac{1}{2} \times F \times G$. If F equals 9 inches, and G, 6 inches, then the area equals $\frac{1}{2} \times 9 \times 6 =$ 27 square inches.

If the area and the base of a triangle are known, the altitude can be found by dividing twice the area by the length of the base. If the area and the altitude are known, the base is found by dividing twice the area by the altitude. If the area of a triangle is 180 square inches, and the base is 18 inches, then the altitude equals $(2 \times 180) \div 18 = 20$ inches.

If the length of two sides of a right triangle, Fig. 13, are known, the third side can be found by a simple calculation.

If the base and the altitude are known, the hypotenuse equals the square root of the sum of the squares of the base and the altitude, or

$$\text{Hypotenuse} = \sqrt{(\text{base})^2 + (\text{altitude})^2}.$$

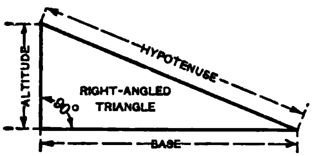


Fig. 13. Right-angled Triangle

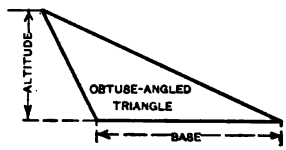


Fig. 14. Obtuse-angled Triangle

The base and the altitude can be found by similar formulas if the hypotenuse is known.

$$\text{Base} = \sqrt{(\text{hypotenuse})^2 - (\text{altitude})^2},$$

$$\text{Altitude} = \sqrt{(\text{hypotenuse})^2 - (\text{base})^2}.$$

Assume that the altitude is 3 feet and the base is 4 feet. Then the hypotenuse $= \sqrt{3^2 + 4^2} = \sqrt{(3 \times 3) + (4 \times 4)} = \sqrt{9 + 16} = \sqrt{25} = 5$ feet.

If the hypotenuse is 10 inches and the altitude 6 inches, then the base equals $\sqrt{10^2 - 6^2} = \sqrt{(10 \times 10) - (6 \times 6)} = \sqrt{100 - 36} = \sqrt{64} = 8$ inches.

Trapezoids

When a figure is bounded by four lines, of which only two are parallel, it is called a trapezoid. The height of a trapezoid is the distance L, Fig. 15, between the two parallel lines H and K. The area of a trapezoid equals one-half the sum of the lengths of the parallel sides multiplied by the height. The area of the trapezoid in Fig. 15 thus equals $\frac{1}{2} \times (H + K) \times L$. If $H = 16$ feet, $K = 24$ feet, and $L = 14$ feet, then the area $= \frac{1}{2}(16 + 24) \times 14 = 280$ square feet.

Trapeziums

When a figure is bounded by four lines, no two of which are parallel, as shown in Fig. 16, it is called a trapezium. The area of a trapezium is found by dividing it into two triangles as indicated by the dash-dotted line in Fig. 16, and finding the area of each of the two triangles, and adding these areas. The dotted lines in Fig. 16 indicate the altitudes of the two triangles into which the trapezium has been divided. If the dimensions of the base and height of the one triangle are R and S, respectively, and of the other T and V, as shown in Fig. 16, then the area of the whole trapezium would be $(\frac{1}{2} \times R \times S) + (\frac{1}{2} \times T \times V)$. Assume that $R = 20$ feet, $S = 17$ feet, $T = 23$ feet, and $V = 9$ feet, then the area of the trapezium $= (\frac{1}{2} \times 20 \times 17) + (\frac{1}{2} \times 23 \times 9) =$ 273.5 square feet.

The Circle

The circle is a plane surface bounded by a curved line called the *periphery* or *circumference*, which is at all points at an equal distance from a point within the circle called the center. The distance from the center of the circle to the periphery is the *radius*, and the distance across the circle through the center is the *diameter*. (See Fig. 17.) It is evident that the radius is one-half of the diameter. If a line is drawn from one point on the periphery to another point, so that it does not pass through the center, it is called a *chord*.

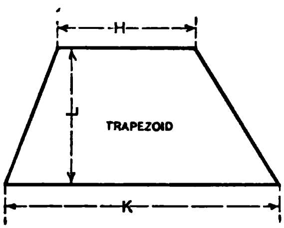


Fig. 15. Trapezoid

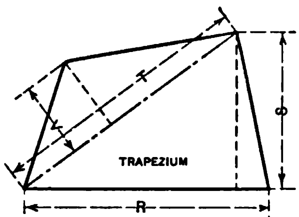


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Fig. 16. Trapezium

If the diameter of a circle is known, the circumference is found by multiplying the diameter by 3.1416. Assume that the circumference of a circle is stretched out into a straight line by the circle rolling upon a flat surface and unfolding itself, as shown in Fig. 20, then the length of the straight line would be three times the diameter plus a distance equal to 0.1416 times the diameter; or the whole length of the circumference would be 3.1416 times the diameter. As the diameter equals $2 \times$ radius, the circumference equals $2 \times$ radius $\times$ 3.1416.

If the circumference of a circle is known, the diameter is found by dividing the circumference by 3.1416; the radius is found by dividing the circumference by 2×3.1416.

If $D =$ diameter, $R =$ radius, $C =$ circumference, then the previous rules can be written as formulas, thus:

$$D = 2 \times R,$$
$$C = 2 \times R \times 3.1416,$$
$$C = D \times 3.1416,$$
$$R = \frac{C}{2 \times 3.1416}, \qquad D = \frac{C}{3.1416}.$$

Instead of writing out the number 3.1416, the Greek letter π (pi) is often used; thus, for example, $3\pi = 3 \times 3.1416$.

Example: The diameter of a circle is 6 inches; find its circumference.

Using the formula given, we have:
Circumference $= 6 \times 3.1416 = 18.8496$ inches.
The circumference of a circle is 13.509 inches; find its radius.

$$\text{Radius} = \frac{13.509}{2 \times 3.1416} = 2.150 \text{ inches.}$$

The area of a circle equals the square of the radius multiplied by 3.1416; or, the square of the diameter multiplied by 0.7854.

If the area of a circle is known, the radius is found by extracting the square root of the quotient of the area divided by 3.1416.

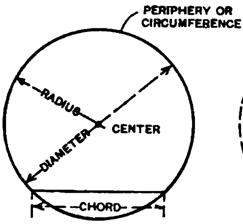

Fig. 17

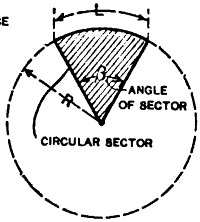

Fig. 18

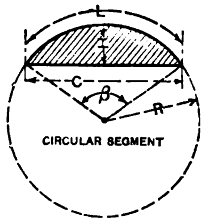

Fig. 19

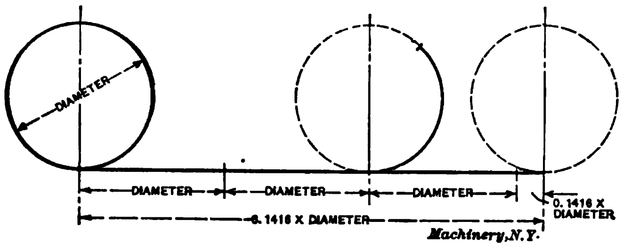

Fig. 20

Figs. 17 to 20. Circles, Sectors and Segments

If D = diameter, R = radius, A = area,
then $A = R^2 \times 3.1416$,

$$A = \frac{D^2 \times 3.1416}{4} = D^2 \times 0.7854,$$

$$R = \sqrt{\frac{A}{3.1416}}.$$

Examples: The diameter of a circle is 6 inches, find the area.
Using the formula given, we have:
Area $= 6^2 \times 0.7854 = 6 \times 6 \times 0.7854 = 28.2744$ square inches.
The area of a circle is 95.033 square inches, find the radius.
Using the formula given, we have:
Radius $= \sqrt{95.033 \div 3.1416} = 5.5$ inches.

Circular Sectors

A figure bounded by a part of the circumference of a circle and two radii, as shown in Fig. 18, is called a circular *sector*. The angle

β (beta) between the radii is called the angle of the sector, and the length L of the circumference of the circle is called the arc of the sector.

If R = radius of circle of which the sector is a part,
β = angle of sector, in degrees,
L = length of arc of sector,
A = area of sector,
then the formulas below are used:

$$L = \frac{R \times \beta \times 3.1416}{180} = \frac{2 \times A}{R},$$

$$\beta = \frac{180 \times L}{R \times 3.1416},$$

$$A = \frac{L \times R}{2},$$

$$R = \frac{2 \times A}{L} = \frac{180 \times L}{\beta \times 3.1416}.$$

If the radius of a circle is 1½ inch, and the angle of a circular sector is 60 degrees, how long is the arc of the sector?

Using the given formula, we have:

$$L = \frac{1\frac{1}{2} \times 60 \times 3.1416}{180} = 1.5708 \text{ inch.}$$

What is the area of the same sector?

From the formula given, we have:

$$A = \frac{1.5708 \times 1\frac{1}{2}}{2} = 1.1781 \text{ square inch.}$$

Circular Segments

A figure bounded by a part of the circumference of a circle and a chord, as shown in Fig. 19, is called a circular *segment*. The distance H from the chord to the highest point of the circular arc is called the height of the segment.

If R = radius, C = length of chord, L = length of arc of segment, H = height of segment, A = area of segment, then the following formulas are used:

$$C = 2 \times \sqrt{H \times (2 \times R - H)},$$

$$R = \frac{C^2 + 4 \times H^2}{8 \times H},$$

$$A = \frac{L \times R - C \times (R - H)}{2}.$$

If the angle, β, Fig. 19, is given, instead of the length of arc L, the length of the arc is found by the previously given formula:

$$L = \frac{R \times \beta \times 3.1416}{180}.$$

Assume that the radius of a segment is 5 feet and the height 8 inches. How long is the chord of this segment?

First transform 5 feet into inches; $5 \times 12 = 60$ inches. Then apply the formula given:

$C = 2 \times \sqrt{8 \times (2 \times 60 - 8)} = 2 \times \sqrt{896} = 2 \times 29.93 = 59.86$ inches.

The length of the chord of a segment is 16 inches and the height 6 inches. How long is the radius of the circle of which the segment is a part?

Applying the formula given, we have:

$$R = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = 8\ 1/3 \text{ inches.}$$

Regular Polygons

Any plane surface or figure bounded by straight lines is called a *polygon*. If all the sides are of equal length and the angles between the sides are equal, the figure is called a *regular* polygon.

A regular polygon having five sides is shown in Fig. 21. The length of each of the five sides equals S, and each of the angles between the sides equals β.

A regular polygon with five sides is called a *pentagon;* one with six sides (Fig. 22), a *hexagon;* one with seven sides (Fig. 28), a *heptagon;* and one with eight sides (Fig. 23), an *octagon.* When a regular polygon has only three sides (Fig. 24), it becomes an equilateral triangle, and when it has four sides (Fig. 25) a square.

A circle may be drawn so that it passes through all the angle-points of a regular polygon, as shown in Figs. 24 to 29 inclusive; such a circle (with the radius R) is said to be *circumscribed* about the polygon. The smaller circle in the same illustrations (with the radius r) which touches or is tangent to the sides of the polygon, is said to be *inscribed* in the polygon. The centers of the circumscribed and inscribed circles are located at the same point. If the angle-points of the polygon are connected by lines with this center, as shown by the dotted lines in Figs. 21, 22 and 23, the polygon is divided up into a number of triangles of equal size and shape. The number of triangles equals the number of sides in the polygon.

The angle α (alpha) of each of these triangles at the center (see Fig. 21) can be determined for any polygon when the number of sides is known. This angle, in degrees, equals 360 divided by the number of sides in the regular polygon, or expressed as a formula, if N equals the number of sides:

$$\alpha = \frac{360}{N}.$$

The angle β between two adjacent sides of the polygon (see Fig. 21) equals α subtracted from 180, or:

$$\beta = 180 - \alpha.$$

The area of a polygon can be found by dividing it into triangles, as shown in Figs. 21, 22 and 23. After having measured the base and height of one triangle and calculated its area, the area of the whole polygon is found by multiplying the area of one triangle by the number of triangles or sides.

For the more commonly used regular polygons, the formulas in the following give the area directly when the length of the side is known.

Equilateral Triangles

The sum of the three angles in any triangle equals 180 degrees, as already mentioned. Each of the angles in an equilateral triangle, therefore, equals 1/3 of 180 degrees, or 60 degrees.

The radius r of the circle inscribed in an equilateral triangle equals the side multiplied by 0.289.

The radius R of the circumscribed circle equals the side multiplied by 0.577.

If the radius of the circumscribed circle is known, the side is found by multiplying the radius by 1.732.

If the radius of the inscribed circle is known, the side is found by multiplying the radius by 3.464.

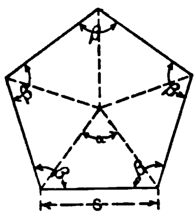

Fig. 21

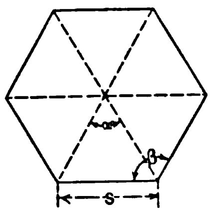

Fig. 22

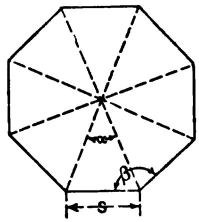

Fig. 23

Regular Polygons

The area of an equilateral triangle equals the square of the side multiplied by 0.433; or, the square of the radius of the circumscribed circle multiplied by 1.299; or, the square of the radius of the inscribed circle multiplied by 5.196.

If r = radius of inscribed circle,
R = radius of circumscribed circle,
S = length of side,
A = area of equilateral triangle,
then the previous rules may be expressed in formulas as follows:

$r = 0.289 \times S,$

$R = 0.577 \times S,$

$S = 1.732 \times R = 3.464 \times r,$

$A = 0.433 \times S^2 = 1.299 \times R^2 = 5.196 \times r^2.$

The Square

Each of the angles between the sides of a square is a 90-degree or right angle.

The radius of the inscribed circle equals one-half of the side.

The radius of the circumscribed circle equals the side multiplied by 0.707.

The side of a square equals twice the radius of the inscribed circle, or 1.414 times the radius of the circumscribed circle.

The area equals the square of the side. The area also equals the square of the radius of the circumscribed circle multiplied by 2; or, the square of the radius of the inscribed circle multiplied by 4.

Using the same meaning for the letters as before, the previous rules may be expressed in formulas as follows:

$r = 0.5 \times S,$

$R = 0.707 \times S,$

$S = 1.414 \times R = 2 \times r.$

$A = S^2 = 2 \times R^2 = 4 \times r^2.$

The Pentagon

In the pentagon (Figs. 21 and 26) the angle β between the sides equals 108 degrees. This is found by the formulas previously given as shown below:

$N =$ number of sides $= 5.$

$$\alpha = \frac{360}{N} = \frac{360}{5} = 72 \text{ degrees.}$$

$\beta = 180 - \alpha = 180 - 72 = 108$ degrees.

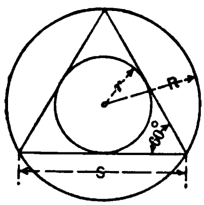

Fig. 24. Equilateral Triangle

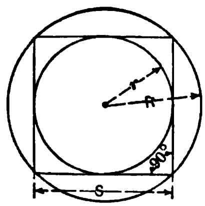

Fig. 25. Square

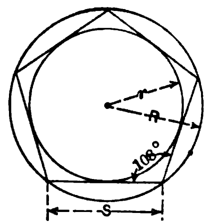

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Fig. 26. Regular Pentagon

The following formulas are used for finding the radii of the circumscribed and inscribed circles, the side and the area of regular pentagons:

$r = 0.688 \times S.$

$R = 0.851 \times S,$

$S = 1.176 \times R = 1.453 \times r,$

$A = 1.720 \times S^2 = 2.378 \times R^2 = 3.633 \times r^2.$

The Hexagon

In the hexagon (Figs. 22 and 27) the length of the side S equals the radius R of the circumscribed circle so that each of the six triangles formed, when lines are drawn from the center to the angle-points, are equilateral triangles. The angle β between two adjacent sides equals the sum of two angles in two of the equilateral triangles and, consequently, equals $60 + 60 = 120$ degrees.

Using the same letters as previously given in the formulas, we have for the hexagon:

$r = 0.866 \times S,$

$R = S,$

$S = R = 1.155 \times r,$

$A = 2.598 \times S^2 = 2.598 \times R^2 = 3.464 \times r^2.$

The Heptagon

The heptagon, Fig. 28, has seven sides, and the angle between two adjacent sides is found by the formulas already given, as shown below:

N = number of sides = 7.

$$a = \frac{360}{N} = \frac{360}{7} = 51\ 3/7 \text{ degrees.}$$

$\beta = 180 - 51\ 3/7 = 128\ 4/7$ degrees.

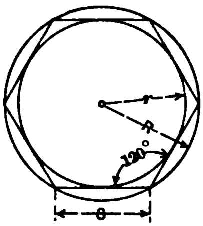

Fig. 27. Regular Hexagon

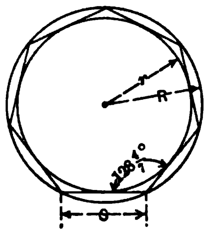

Fig. 28. Regular Heptagon

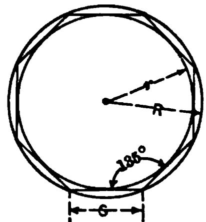

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Fig. 29. Regular Octagon

Using the same letters as in the formulas previously given, we have for the heptagon:

$r = 1.038 \times S$,

$R = 1.152 \times S$,

$S = 0.868 \times R = 0.963 \times r$.

$A = 3.634 \times S^2 = 2.736 \times R^2 = 3.371 \times r^2$.

The Octagon

The angle β between two adjacent sides of the octagon, as shown in Figs. 23 and 29, is 135 degrees.

Using the same meaning for the letters as previously given, the formulas for the octagon are:

$r = 1.207 \times S$,

$R = 1.307 \times S$,

$S = 0.765 \times R = 0.828 \times r$,

$A = 4.828\ S^2 = 2.828 \times R^2 = 3.314 \times r^2$.

CHAPTER IX

VOLUMES OF SOLIDS

Volume of a Cube

The cube, Fig. 30, is a solid body having six surfaces or faces, all of which are squares; as all the faces are squares, all the sides are of equal length.

If the side of a face of a cube equals S, the volume equals $S \times S \times S$ or, as it is commonly written, S^3.

Assume that the length of the side of a cube equals 3 inches; then the volume equals $3 \times 3 \times 3 = 27$ cubic inches.

When the volume of a cube is known, the length of the side is found by extracting the cube root of the volume.

Assume that the volume of a cube equals 343 cubic inches. If we extract the cube root of 343, we find that the side of the cube is 7 inches.

One cubic foot equals $12 \times 12 \times 12 = 1728$ cubic inches; therefore, a volume given in cubic feet can be transformed into cubic inches by

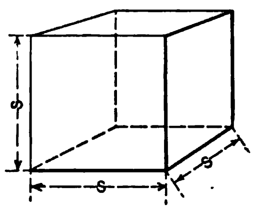

Fig. 30. Cube

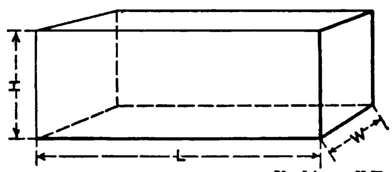

Fig. 31. Square Prism

multiplying by 1728; if the volume is given in cubic inches it can be transformed into cubic feet by dividing by 1728.

Volume of Prisms

A solid body, the sides of which are all rectangles, and the ends of which are either rectangles or squares is commonly called a *square prism*. Opposite surfaces or faces are parallel, and all the angles are right angles. A square prism is shown in Fig. 31, where L is its length, W its width, and H its height. The volume of a square prism equals the length times the width times the height, or, expressed as a formula, if $V =$ volume,

$$V = L \times W \times H.$$

Assume that $L = 20$ inches, $W = 4$ inches, and $H = 5$ inches, then volume $= 20 \times 4 \times 5 = 400$ cubic inches.

A solid body having the end faces parallel, and the lines along which the other faces intersect or meet parallel, is called a *prism*. The two parallel end faces are called *bases*. The length, height, or altitude L, Fig. 32, of a prism is the distance between the bases, measured at right angles to the base surfaces.

The volume of a prism equals the area of the base multiplied by the length or height of the prism. The area of the base must, therefore, first be found before the volume can be obtained. If the base is a triangle, parallelogram, trapezoid, trapezium or a regular polygon, its area is found by the rules given in Chapter VIII. If it is a polygon which is not regular, it can always be divided into triangles, and the area of each of the triangles can be calculated, and these areas added together to obtain the area of the whole polygon.

Assume that it is required to find the volume of a prism, the base of which is a regular hexagon having a side S; the length of the prism is L. The volume of this prism is

$$2.598 \times S^2 \times L$$

[See page 35 for formula for area of hexagon.]

If, in this example, S equals $1\frac{1}{2}$ inch, and L equals 9 inches, then the volume equals

$$2.598 \times 1\frac{1}{2}^2 \times 9 = 2.598 \times 1.5 \times 1.5 \times 9 = 52.6095 \text{ cubic inches.}$$

Volume of a Pyramid

A solid body having a polygon for the base and a number of triangles all having a common vertex for the sides is called a *pyramid.* In Fig. 33 a pyramid is shown where the base has four sides and the side surfaces are made up of triangles having two equal sides. If a line is drawn from the vertex of the pyramid at right angles to the base, the length of this line is the altitude or height H of the pyramid.

The volume of a pyramid equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the base area before the volume can be found.

Assume that it is required to find the volume of a pyramid, the base of which is a regular pentagon, having a side S; the height of the pyramid is H. The volume of the pyramid equals

$1.720 \times S^2 \times 1/3 \times H$ (area of base $\times$ one-third the height).

[See page 35 for formula for area of pentagon.]

If $S = 2$ inches and $H = 9$ inches, then the volume equals

$1.720 \times 2^2 \times 1/3 \times 9 = 1.720 \times 2 \times 2 \times 3 = 20.640$ cubic inches.

A *frustum of a pyramid* is shown in Fig. 34. It is a pyramid from which the top has been cut off, the top surface being parallel to the base. The height of a frustum of a pyramid is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a pyramid can be found when the height, the top area, and the base area are known.

If $V =$ volume of frustum of a pyramid,
$H =$ height of frustum,
$A_1 =$ area of top,
$A_2 =$ area of base,
then

$$V = \frac{H}{3} \times (A_1 + A_2 + \sqrt{A_1 \times A_2}).$$

Assume, for example, that the base of a frustum of a pyramid is a square, and that the side of the square is 5 inches. The top area is, of course, also a square; assume the side of this to be 2 inches. The height of the frustum is 6 inches. By first calculating the base and top areas and then inserting the values in the formula given, the volume is obtained.

$$\text{Volume} = \frac{6}{3} \times (5^2 + 2^2 + \sqrt{5^2 \times 2^2}) = 2 \times (25 + 4 + \sqrt{25 \times 4})$$

$$= 2 \times (25 + 4 + 10) = 78.$$

The Prismoidal Formula

The prismoidal formula is a general formula by which the volume of any prism, pyramid or frustrum of a pyramid, and the volume of any solid body bounded by regular curved surfaces may be found.

If A_1 = area at one end of the body

A_2 = area at other end,

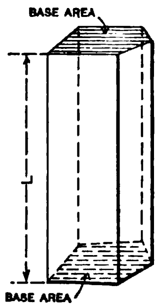

Fig. 32. Prism

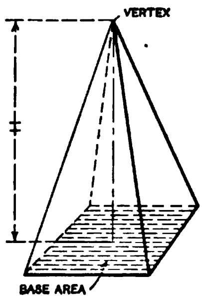

Fig. 33. Pyramid

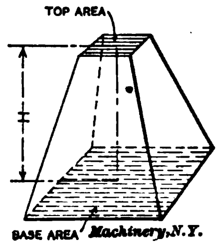

Fig. 34. Frustum of Pyramid

A_m = area of a middle section between the two end surfaces,

H = height of the body,

V = volume of body,

then

$$V = \frac{H}{6} \times (A_1 + 4\,A_m + A_2).$$

As this formula applies to all regular solid bodies, it is useful to remember. For ordinary calculations, however, the formulas given on the two previous pages, for each kind of solid, should be used because of greater simplicity.

Volume of a Cylinder

A solid body, as shown in Fig. 35, having circular and parallel end faces of equal size, is called a *cylinder*. The two parallel faces are called *bases*. The height or altitude H of a cylinder is the distance betwen the bases measured at right angles to the base surfaces.

The volume of a cylinder equals the area of the base multiplied by the height. The area of the base, must, therefore, first be found before the volume can be obtained. If the diameter of the base is D.

the area of the base equals $0.7854D^2$. The volume of the cylinder then equals:

$$0.7854 \times D^2 \times H$$

If $D = 3$ inches and $H = 5$ inches, then the volume equals:

$$0.7854 \times 3^2 \times 5 = 0.7854 \times 3 \times 3 \times 5 = 35.343 \text{ cubic inches.}$$

Volume of a Cone

A solid body having a circular base and the sides inclined so that they meet at a common vertex, the same as in a pyramid, is called a cone. (See Fig. 36.) If a line is drawn from the vertex of the cone at right angles to the base, the length of this line is the altitude or height H of the cone.

The volume of a cone equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the area of the base circle before the volume can be found. If the diameter of the base

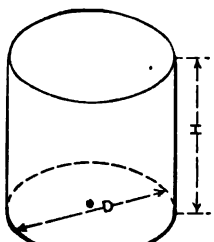

Fig. 35. Cylinder

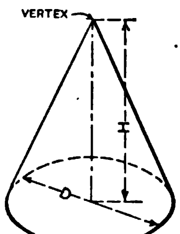

Fig. 36. Cone

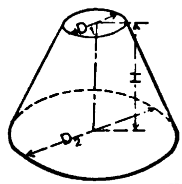

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Fig. 37. Frustum of Cone

area equals D, then the area equals $0.7854D^2$, and this multiplied by one-third of the height H gives us the volume:

$$0.7854 \times D^2 \times 1/3 \times H = 1/3 \times 0.7854 \times D^2 \times H = 0.2618 \times D^2 \times H.$$

If the diameter of the base of a cone equals 4 inches and the height 6 inches, then the volume equals:

$$0.2618 \times 4^2 \times 6 = 0.2618 \times 4 \times 4 \times 6 = 25.1328 \text{ cubic inches.}$$

A frustum of a cone is shown in Fig. 37. It is a cone from which the top has been cut off, the top surface being a circle parallel to the base. The height H of a frustum of a cone is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a cone can be found when the diameters of the top and base circles, and the height are known.

If V = volume of frustum of a cone,
H = height of frustum,
D_1 = diameter of top circle,
D_2 = diameter of base circle,

then

$$V = 0.2618 \times H \times (D_1^2 + D_2^2 + [D_1 \times D_2]).$$

Assume, for example, that the diameter of the base of a frustum of a cone is 5 inches, and that the diameter of the top circle is 2 inches. The height of the frustum is 6 inches. By inserting these values in the formula given we have:

$V = 0.2618 \times 6 \times (2^2 + 5^2 + [2 \times 5]) = 0.2618 \times 6 \times (4 + 25 + 10)$
$= 0.2618 \times 6 \times 39 = 61.2612$ cubic inches.

Volume of a Sphere, Spherical Sector, Segment and Zone

The name *sphere* is applied to a solid body shaped like a ball or globe, that is, bounded by a surface which at all points is at the same distance from a point inside of the sphere called its center. The diameter of a sphere is the length of a line drawn from a point on the surface through the center to the opposite side.

The volume of a sphere equals 3.1416 multiplied by four-thirds of the cube of the radius, or 3.1416 multiplied by one-sixth of the cube of the diameter.

If R = radius of the sphere, D = diameter, and V = volume, this rule given can be written in the form of formulas thus:

$$V = 3.1416 \times 4/3 \times R^3 = 4.1888 \times R^3,$$
$$V = 3.1416 \times 1/6 \times D^3 = 0.5236 \times D^3.$$

If the volume of a sphere is known, the radius can be found by extracting the cube root of the quotient of the volume divided by 4.1888; the diameter can be found by extracting the cube root of the quotient of the volume divided by 0.5236.

Written as formulas, these rules are:

$$R = \sqrt[3]{\frac{V}{4.1888}} \qquad D = \sqrt[3]{\frac{V}{0.5236}}$$

A *spherical sector* is a part of a sphere bounded by a section of the spherical surface and a cone, having its vertex at the center of the sphere, as shown in Fig. 39. The volume of a spherical sector can be found if the radius R and the height H, Fig. 39, are known.

The formula for the volume V is

$$V = 2.0944 \times R^2 \times H.$$

Assume that the length of the radius of a spherical sector is 15 inches and the height is 4 inches. Then the volume equals

$2.0944 \times 15^2 \times 4 = 2.0944 \times 15 \times 15 \times 4 = 1884.96$ cubic inches.

A *spherical segment* is a part of a sphere bounded by a portion of the spherical surface and a plane circular base, as shown in Fig. 40.

The volume of a spherical segment can be found when the radius of the sphere and the height H of the segment, or the diameter C of the base of the segment and its height H, are known.

If V = volume of segment,
H = height of segment,
R = radius of the sphere of which the segment is a part,
C = diameter of the base of the segment,

then,

$$V = 3.1416 \times H^2 \times \left(R - \frac{H}{3}\right)$$

$$V = 3.1416 \times H \times \left(\frac{C^2}{8} + \frac{H^2}{6}\right)$$

Assume that the height of a spherical segment is 6 inches and the radius 8 inches, then the volume is

$$3.1416 \times 6^2 \times (8 - 6 \div 3) = 3.1416 \times 6 \times 6 \times (8 - 2) =$$
$$3.1416 \times 6 \times 6 \times 6 = 678.5856 \text{ cubic inches.}$$

A *spherical zone* is bounded by a part of a spherical surface, and by two parallel circular bases, as shown in Fig. 40, where C_1 and C_2 are the diameters of the circular bases of the zone, and H its height.

The volume of a spherical zone can be found when the height of the segment and the two base diameters are known.

If $V =$ volume of zone,
$C_1 =$ diameter of the smaller base circle,
$C_2 =$ diameter of the larger base circle,
$H =$ height of zone,
then

$$V = 0.5236 \times H \times \left(\frac{3\,C_1^2}{4} + \frac{3\,C_2^2}{4} + H^2 \right)$$

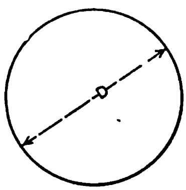

Fig. 38. Sphere

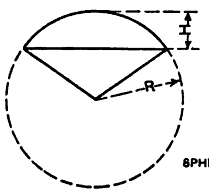

Fig. 39. Spherical Sector

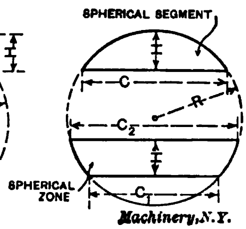

Fig. 40. Spherical Segment and Zone

Assume that the diameter $C_1 = 3$ inches, the diameter $C_2 = 4$ inches, and the height of the segment equals 1 inch, then the volume is

$$0.5236 \times 1 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1^2 \right) =$$

$$0.5236 \times 1 \times \left(\frac{27}{4} + \frac{48}{4} + 1 \right) = 0.5236 \times 1 \times 19.75 = 10.3411 \text{ cubic inches.}$$

[If a plane parallel with the end faces and passing through the center of the sphere intersects the zone, consider the zone as two zones, one zone being on each side of the center. Calculate the volume of each, and add these to find the total volume.]

CHAPTER X

SPECIFIC GRAVITY AND WEIGHTS OF BAR STOCK AND CASTINGS

The expression "specific gravity" indicates how many times a certain volume of a material is heavier than an equal volume of water. If it is found, for example, that one cubic inch of steel weighs 7.8 times as much as one cubic inch of pure water, the specific gravity of steel is 7.8.

As the density of water differs slightly at different temperatures, it is usual to make comparisons on the basis that the water has a temperature of 62 degrees F. The weight of one cubic inch of pure water at 62 degrees F. is 0.0361 pound. If the specific gravity of any material is known, the weight of a cubic inch of the material can, therefore, be found by multiplying its specific gravity by 0.0361.

TABLE OF SPECIFIC GRAVITY AND WEIGHT PER CUBIC INCH OF VARIOUS METALS AND ALLOYS

Metal	Specific Gravity	Weight in Pounds per Cubic Inch
Aluminum	2.56	0.092
Antimony	6.71	0.242
Bismuth	9.80	0.354
Brass	8.00	0.289
Copper	8.82	0.318
Gold	19.32	0.697
Iron, cast	7.20	0.260
" pure	7.77	0.280
" wrought	7.70	0.278
Lead	11.37	0.410
Manganese	8.00	0.289
Mercury	13.58	0.490
Nickel	8.80	0.318
Platinum	21.50	0.776
Silver	10.50	0.379
Steel, machine and tool	7.85	0.283
Tin	7.29	0.263
Tungsten	17.60	0.635
Vanadium	5.50	0.199
Zinc	7.15	0.258

The specific gravity of cast iron, for example, is 7.2. The weight of one cubic inch of cast iron is found by multiplying 7.2 by 0.0361. The product, 0.260, is the weight of one cubic inch of cast iron.

As there are $12 \times 12 \times 12 = 1,728$ cubic inches in one cubic foot, the weight of a cubic foot is found by multiplying the weight of a cubic inch by 1,728.

If the weight of a cubic inch of a material is known, the specific gravity is found by dividing the weight per cubic inch by 0.0361.

The weight of a cubic inch of gold is 0.697 pound. The specific gravity of gold is then found by dividing 0.697 by 0.0361. The quotient, 19.32, is the specific gravity of gold.

If the weight per cubic inch of any material is known, the weight of any volume of the material is found by multiplying the weight per cubic inch by the volume expressed in cubic inches. If brass weighs 0.289 pound per cubic inch, 16 cubic inches of brass, of course, weigh 0.289 × 16 = 4.624 pounds. In an example of this kind, if the specific gravity is known, instead of the weight per cubic inch, this latter weight is first found by the rule previously given for finding the weight per cubic inch from the specific gravity.

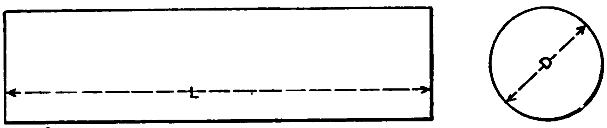

Fig. 41

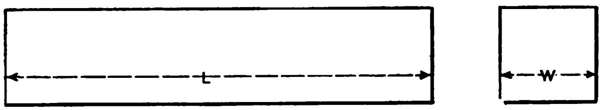

Fig. 42

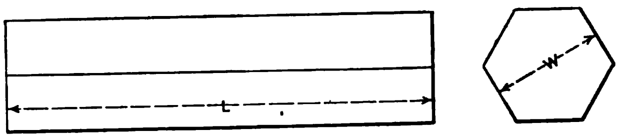

Fig. 43

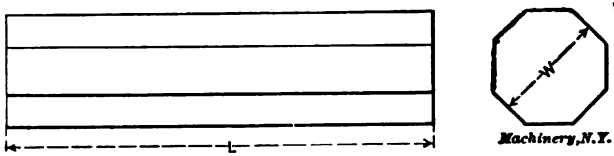

Fig. 44

Figs. 41 to 44. Round, Square, Hexagonal and Octagonal Bar Stock

If the specific gravity of tool steel is 7.85, what is the weight of 12 cubic inches of tool steel? The weight of one cubic inch is found by multiplying 7.85 by 0.0361. The product, 0.283, is then multiplied by 12 to find the weight of 12 cubic inches; 0.283 × 12 = 3.396 pounds.

Weight of Bar Stock

The weight of a piece of round bar stock, as shown in Fig. 41, can be found by first calculating the volume of the piece. When the volume is found in cubic inches, the weight is found by multiplying the volume by the weight of the material per cubic inch, as already explained.

If the diameter D, Fig. 41, of a piece of round tool steel bar is 2 inches, and the length L is 7 inches, the volume of this piece equals

0.7854 × square of diameter × length, or $0.7854 \times 2^2 \times 7 = 0.7854 \times 2 \times 2 \times 7 = 21.991$ cubic inches. The volume in cubic inches having been found, it is multiplied by the weight of tool steel per cubic inch, which is 0.283 pound, as given in the accompanying table of specific gravity and weight per cubic inch of various metals and alloys. The weight of the bar is then $21.991 \times 0.283 = 6.2235$ pounds.

If the specific gravity is given instead of the weight per cubic inch, find the weight per cubic inch as explained on page 43.

The weight of a square bar, as shown in Fig. 42, can be calculated when the width across flats, *W*, the length of the bar, *L*, and the weight of one cubic inch of the material from which the bar is made, are known.

Assume that the width across flats is 2½ inches, that the length is 11 inches, and that the bar is made from brass; the volume of this bar equals the area of its end section multiplied by its length, or, in this case, 2½ × 2½ × 11 = 68¾ cubic inches. The weight of one cubic inch of brass is 0.289 pound, and the weight of the given bar is, therefore, 68¾ × 0.289 = 19.869 pounds.

In order to find the weight of a hexagonal bar, as shown in Fig. 43, when the width across flats, *W*, the length *L*, and the weight per cubic inch of the material from which the bar is made, are known, the area of its end section must first be found so that the volume can be determined by multiplying this area by the length; when the width across flats, *W*, is given, this area equals 0.866 × the square of the width across flats.

Assume that the weight is to be found of a hexagonal piece of machine steel bar stock 3 inches across flats, and 6 inches long. The volume of this piece equals then $0.866 \times 3^2 \times 6 = 0.866 \times 3 \times 3 \times 6 = 46.764$ cubic inches, and the weight equals $46.764 \times 0.283 = 13.234$ pounds. The factor 0.283 is the weight of one cubic inch of machine steel, as given in the table on page 43.

In order to find the weight of a piece of octagonal stock, as shown in Fig. 44, it is first necessary to find the area of the end section; when the width across flats, *W*, is given, this area equals 0.828 × the square of the width across flats.

Assume that the weight of an octagonal piece of tool steel 4 inches across flats and 15 inches long is to be found. The volume of this piece then equals $0.828 \times 4^2 \times 15 = 0.828 \times 4 \times 4 \times 15 = 198.72$ cubic inches, and the weight equals $198.72 \times 0.283 = 56.238$ pounds. The factor 0.283 is the weight of one cubic inch of tool steel as given in the table on page 43.

The Weight of Castings

The weight of a casting can be calculated when the volume of the casting and the specific gravity or the weight per cubic inch of the material from which the casting is made, are known. If the volume is known in cubic inches, the volume is simply multiplied by the weight per cubic inch to obtain the weight of the casting.

The specific gravity of cast iron is 7.2 and the weight per cubic inch is 0.260; the specific gravity of brass is 8 and the weight per cubic inch is 0.289; the specific gravity of cast zinc is 6.86, and the weight per cubic inch 0.248; the specific gravity of gun metal bronze is 8.7 and the weight per cubic inch is 0.314.

With the constants above given, the problem of finding the weight of castings reduces itself to finding the volume of the casting. The multiplication by the weight per cubic inch of the material is then a simple matter.

Assume that it is required to find the weight of a hollow cast iron cylinder, as shown in Fig. 45, where the outside diameter is A, the inside or core diameter B, and the length L. To find the volume, first calculate the volume of a cylinder with the diameter A and the length L and then subtract from this the volume of the cylinder forming the core.

Assume that in a hollow cylinder as shown in Fig. 45, $A = 3$ inches, $B = 2$ inches, and $L = 8$ inches. The volume of a cylinder $= 0.7854 \times$ the square of the diameter $\times$ the height. The volume of a cylinder with 3 inches diameter and a height of 8 inches $= 0.7854 \times 3^2 \times 8 =$

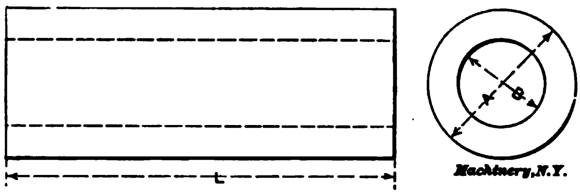

Fig. 45. Hollow Cylinder

$0.7854 \times 3 \times 3 \times 8 = 56.5488$ cubic inches. From this is subtracted the volume of the cylinder forming the core, which has a diameter of 2 inches. The volume of this cylinder is $0.7854 \times 2^2 \times 8 = 25.1328$ cubic inches. This last volume subtracted from the volume 56.5488 gives us 31.416 cubic inches as the volume of the hollow cylinder ($56.5488 - 25.1328 = 31.416$). As the weight per cubic inch of cast iron is 0.260 pound, the total weight of the hollow cylinder will be $31.416 \times 0.260 = 8.168$ pounds.

If the cylinder had been cast from gun metal bronze instead of cast iron, the volume should be multiplied by 0.314, in order to find the weight.

If the outside diameter of a hollow cylinder is A, the inside diameter B, and the length L, the following formula may be used for finding the volume of the cylinder:

$$\text{Volume} = 0.7854 \times (A^2 - B^2) \times L.$$

In Fig. 46 is shown a knee made from cast iron, all the necessary dimensions for calculating the weight being given. To calculate the volume of a casting of this shape, it is divided into prisms or other simple geometric shapes, and the volume of each of the parts is found, after which these volumes are added together to find the total volume of the casting. The piece shown in Fig. 46 can be divided up into three parts, the volume of each of which can be calculated by simple means. One part has for base the rectangle *HMLK*, another the rectangle *PFMN*, and the base of the third is bounded by two straight lines *EF* and *FG* and the circular arc *EG*. The length of all the parts in this case equals the length of the casting, or 5 inches, as shown.

The area of the rectangle *HMLK* equals $6 \times 2 = 12$ square inches. This area multiplied by 5 gives us the volume of this part in cubic inches; $12 \times 5 = 60$ cubic inches.

The length of the line *NM* is 4 inches ($6 - 2 = 4$), and, therefore, the area of the rectangle *PFMN* is $4 \times 2 = 8$ square inches. This area multiplied by 5 gives us the volume of this section of the casting in cubic inches; $8 \times 5 = 40$ cubic inches.

It now remains to find the volume of the section having for base the area bounded by the two straight lines *EF* and *FG* and the circular arc *EG*. The area of the base is found by first finding the area of the square *DEFG* and subtracting from this area the area of the circular

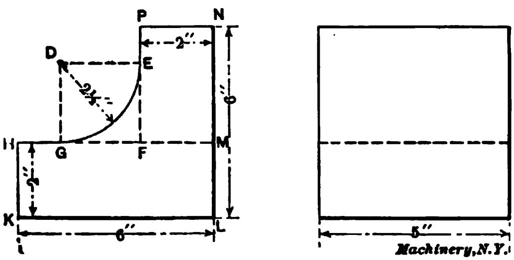

Fig. 46. Bracket or Knee

sector *DEG*. The area of the square is $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$ square inches. The area of the circular sector which is one-fourth of a complete circle is $\frac{2\frac{1}{2}^2 \times 3.1416}{4} = 4.909$ square inches. This subtracted from the area of the square equals 1.341 square inch ($6.25 - 4.909 = 1.341$). This is then the area of the base of the third part into which the casting is divided, and this area multiplied by 5 gives the volume of the third part of the casting ($1.341 \times 5 = 6.705$). Now adding the volumes of the three parts together we have $60 + 40 + 6.705 = 106.705$ cubic inches. This total volume multiplied by the weight per cubic inch of cast iron gives us the total weight: $106.705 \times 0.260 = 27.743$ pounds.

When the pattern for a casting contains no core-prints, but is in all respects an exact duplicate of the casting to be made, the weight of the casting may be approximately found by multiplying the weight of the pattern by a constant which varies for different kinds of woods used for the pattern. When the pattern is made from white pine, multiply the weight of the pattern by 13 to obtain the weight of a cast iron casting; if the pattern is made from cherry, multiply by 10.7; if made of mahogany, multiply by 10.28. When an aluminum pattern is used, the weight of the aluminum pattern may be multiplied by 2.88 to obtain the weight of a cast iron casting.

Assume that the weight of a cast iron bracket, as shown in Fig. 47, is required. All the required dimensions are here given by the letters *A*, *B*, *C*, *D*, *E*, *F*, and *G*. The casting is divided up into sections, and the volume of each section is calculated separately; then the volumes are added together and the total volume multiplied by the weight per cubic inch of cast iron. Very small fillets, like those shown at *N* and *R*, are not considered, and the area *NRST* is regarded as a perfect rectangle.

In the example given, the casting is divided up in five parts; one is a hollow cylinder with an outside diameter *A*; two parts have for bases the rectangles *NRST* and *KMTU*; and two parts have for bases the areas *HKL* and *OML*, respectively, each being bounded by two straight lines and a circular arc.

For an example, assume that in Fig. 47, $A = 7$ inches, $B = 4$ inches, $C = 3$ inches, $D = 4$ inches, $E = 12$ inches, $F = 10$ inches, $G = 8$ inches.

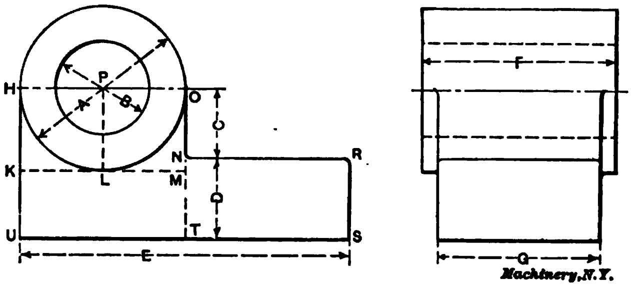

Fig. 47. Bearing Bracket

The volumes of the different parts will then be found as follows:

Volume of hollow cylinder having an outside diameter of 7 inches, an inside diameter of 4 inches, and length of 10 inches:

$$0.7854 \times (7^2 - 4^2) \times 10 = 0.7854 \times (49 - 16) \times 10$$
$$= 0.7854 \times 33 \times 10 = 259.18 \text{ cubic inches.}$$

Volume of section having for base the rectangle *NRST*:

$$4 \times 5 \times 8 = 160 \text{ cubic inches.}$$

Volume of section having for base the rectangle *KMTU*:

$$3\tfrac{1}{2} \times 7 \times 8 = 196 \text{ cubic inches.}$$

Volume of section having for base the area *HKL*:

$$\left(3\tfrac{1}{2} \times 3\tfrac{1}{2} - \frac{3\tfrac{1}{2}^2 \times 3.1416}{4}\right) \times 8 = (12.25 - 9.62) \times 8$$
$$= 2.63 \times 8 = 21.04 \text{ cubic inches.}$$

The volume of the section having for base *OML* equals the volume of the section having for base *HKL* and is consequently 21.04 cubic inches.

The total of the five sections then equals

$$259.18 + 160 + 196 + 21.04 + 21.04 = 657.26 \text{ cubic inches.}$$

The total weight of the casting equals $657.26 \times 0.260 = 170.89$ pounds.

CHAPTER XI

USE OF TABLES OF SINES, COSINES, TANGENTS AND COTANGENTS

The figuring of angles the average mechanic usually looks upon as something above his capacity; but simple cases of the figuring of angles from given formulas are often much easier than many ordinary arithmetical problems in the shop which he successfully solves. All that is necessary is a table of sines, cosines, tangents, and cotangents; after having found the figures corresponding to a given angle from the table, the whole thing resolves itself into a case of simple multiplication or division.

Often, in technical papers, the reader will find himself confronted by such formulas as, for instance,

$$A = \frac{27}{\cos 36 \text{ deg.}}.$$

Of course, it is impossible to figure out how much A is from this formula, unless the expression "cos 36 deg." (read: cosine of 36 degrees) can be transformed and expressed in plain figures. But if we know how much "cos 36 deg." is expressed in plain figures, then we can immediately divide 27 by this value, and thus find the value of A. Suppose that A stands for the length of one side in a triangle and that the expression "cos 36 deg." equals 0.80901. Then,

$$A = \frac{27}{0.80901} = 33.37.$$

The tables of sines, cosines, tangents, and cotangents simply serve the purpose of giving in figures the values of these expressions for different angles. The four expressions: sine, cosine, tangent, and cotangent, which are used to designate certain numerical values, to be found from the tables, are called the *functions of the angle*. These functions or numerical values equal a definite amount for each different angle. On pages 52, 53, 54 and 55 will be found tables giving the values referred to for all degrees and for every ten minutes (1/6 of a degree). From these tables, when the angle is given, the angular function can be found, and when the function is given, the angle can be determined. The four expressions, sine, cosine, tangent, and cotangent are abbreviated "sin," "cos," "tan," and "cot," respectively.

The tables of sines, cosines, etc., are read in the same way as a railroad time-table. At the top of Tables I and II the heading reads "Table of Sines," and at the bottom is the legend "Table of Cosines." At the top of Tables III and IV the heading reads "Table of Tangents," and at the bottom is the legend "Table of Cotangents." At the top of all the tables the heading of the extreme left-hand column reads

"Deg." and the following columns are headed 0′, 10′, 20′, etc. At the bottom of the tables the same legends are placed under the columns, but reading from right to left.

When the sine or tangent of a given angle is to be found, first find the number of degrees in the extreme left-hand column in the respective tables, and then locate the number of minutes at the top of the table. Then follow the column, over which the number of minutes is given, downwards until arriving at the figure in line with the given number of degrees. This figure is the numerical value of the sine or tangent, as the case may be, for the given angle. If the angle is given in even degrees with no minutes, the corresponding function will be found opposite the number of degrees in the column marked 0′ at the top.

The cosines and cotangents of angles are found in the same tables as the sines and tangents, but the tables in this case are read *from the bottom up*. The number of degrees is found in the extreme right-hand column and the number of minutes at the bottom of the columns.

If the number of minutes given is not an even multiple of 10, as 10′, 20′, 30′, etc., but 27′, for example, it is, for nearly all shop calculations near enough to take the figures for the nearest number of minutes given, being in this case, for 30′.

Examples of the Use of the Tables

Example 1.—Find from the tables given the sine of 56 degrees, or, as it is written in formulas, sin 56°.—The "sines" are found by reading Tables I and II from the top; the number of degrees, 56, is found in Table II in the left-hand column, and opposite 56 in the column 0′, read off 0.82903.

Example 2.—Find sin 56° 20′.—In the column marked 20′ at the top, follow downwards until opposite 56 in the left-hand column. The value 0.83227 is the sine of 56° 20′.

Example 3.—Find cos 36° 20′.—To find the cosines, read the tables from the bottom, and locate 36 in the right-hand column in Table II. Then follow the column marked 20′ at the bottom upwards until opposite 36, and read off 0.80558.

Example 4.—Find tan 56° 40′.—The tangents are found in Tables III and IV by locating the number of degrees in the left-hand column and reading the value in the column under the specified number of minutes. In Table IV then we find tan 56° 40′ to be 1.5204.

Example 5.—Find the cotangent of 56° 40′.—Read the tables from the bottom, locating 56 in the right-hand column, and find the required value in line with this figure in the column marked 40′ at the bottom. Thus, cot 56° 40′ = 0.65771.

Example 6.—Find sin 20° 48′.—For shop calculations it is almost always near enough to find the value of the angular functions for the nearest 10 minutes. Therefore in this case find sin 20° 50′, which is 0.35565.

Example 7.—The sine for a certain angle, which may be called a, equals 0.53238. Find the angle.—In the body of the tables of sines find the number 0.53238. It will be seen that this number is opposite

32 degrees and in the column headed 10′ at the top. The angle a, therefore, equals 32° 10′.

Example 8.—Cot $\beta = 0.77195$. Find β.—The cotangents are read from the bottom in Tables III and IV. The value 0.77195 is located opposite 52 in the right-hand column and in the column marked 20′ at the bottom. Angle β, then, is 52° 20′.

Example 9.—Sin $\beta = 0.31190$. Find β.—It will be found that the value 0.31190 is not given in the table of sines; the nearest value in the table is 0.31178. For shop calculations it is near enough to consider the angle β equal to the angle corresponding to this latter value; the angle then is 18° 10′.

Right-angled Triangles*

In right-angled triangles the remaining sides and angles can be found when either two sides, or one side and one of the acute angles, are known. As previously mentioned, the side opposite the right angle, or side a in Fig. 48, is called the *hypotenuse*. Side b is the side

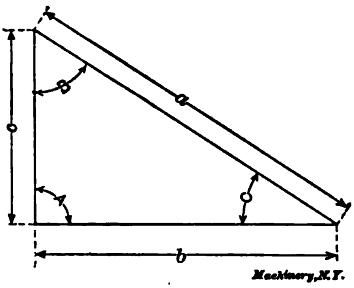

Fig. 48. Right-angled Triangle

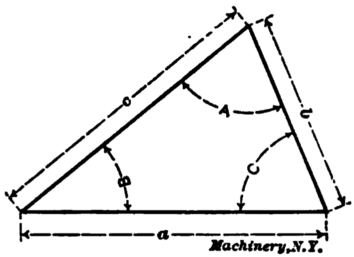

Fig. 49. Oblique-angled Triangle

adjacent to angle C and side c is the side *opposite* to the same angle. In the same way, c is the side *adjacent* to angle B, and b is the side *opposite* angle B.

The problems in right-angled triangles may be divided into five classes, for which the following formulas are given:

1. The hypotenuse and one of the sides forming the right angle are given. Call the hypotenuse a, and the other given side b. Then (see Fig. 48):

$$c = \sqrt{a^2 - b^2} \qquad \sin B = \frac{b}{a} \qquad C = 90° - B$$

2. The two sides forming the right angle are given. Call these two sides b and c. Then (see Fig. 48):

$$a = \sqrt{b^2 + c^2} \qquad \tan B = \frac{b}{c} \qquad C = 90° - B$$

3. The hypotenuse and one of the acute angles are given. Call the hypotenuse a and the given angle B. Then (see Fig. 48):

$$C = 90° - B \qquad b = a \times \sin B \qquad c = a \times \cos B$$

* For a more complete treatment of the solution of triangles, see MACHINERY'S Reference Series No. 54, Solution of Triangles.

I. TABLE OF SINES

Read degrees in left-hand column and minutes at top
Example: sin 7° 10′ = .12475

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom
Example: cos 56° 20′ = .55436

II. TABLE OF SINES

Read degrees in left-hand column and minutes at top

Example: sin 56° 20′ = .83227

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom

Example: cos 7° 10′ = .99218

III. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top

Example: tan 7° 10′ = .12573

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24933	76
14	.24933	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34433	71
19	.34433	.34758	.35085	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40403	68
22	.40403	.40741	.41080	.41421	.41763	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44523	66
24	.44523	.44871	.45221	.45573	.45924	.46277	.46631	65
25	.46631	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50953	63
27	.50953	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54673	.55051	.55431	61
29	.55431	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60483	.60881	.61280	.61681	.62083	.62487	58
32	.62487	.62892	.63299	.63707	.64117	.64528	.64941	57
33	.64941	.65355	.65771	.66189	.66608	.67028	.67451	56
34	.67451	.67875	.68301	.68728	.69157	.69588	.70021	55
35	.70021	.70455	.70891	.71329	.71769	.72211	.72654	54
36	.72654	.73100	.73547	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76272	.76733	.77196	.77661	.78129	52
38	.78129	.78598	.79070	.79544	.80020	.80498	80978	51
39	.80978	.81461	.81946	.82434	.82923	.83415	.83910	50
40	.83910	.84407	.84906	.85408	.85912	.86419	.86929	49
41	.86929	.87441	.87955	.88473	.88992	.89515	.90040	48
42	.90040	.90569	.91099	.91633	.92170	.92709	.93252	47
43	.93252	.93797	.94345	.94896	.95451	.96008	.96569	46
44	.96569	.97133	.97700	.98270	.98843	.99420	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom

Example: cot 56° 20′ = .66607

IV. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top
Example: tan 56° 20′ = 1.5013

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1.2349	39
51	1.2349	1.2423	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom
Example: cot 7° 10′ = 7.9530

4. One acute angle and its adjacent side are given. Call the given angle B and its adjacent side c. Then (see Fig. 48):

$$C = 90° - B \qquad a = \frac{c}{\cos B} \qquad b = c \times \tan B$$

5. One acute angle and its opposite side are given. Call the given angle B and the side opposite it b. Then (see Fig. 48):

$$C = 90° - B \qquad a = \frac{b}{\sin B} \qquad c = b \times \cot B$$

Formulas for Solving Oblique Triangles

Below are given a summary of all the generally required formulas, and the methods of procedure in solving oblique triangles. In all the formulas reference is made to Fig. 49, in which the sides and angles are given the same names as in the formulas below.

1. When two angles and one side are given, call the given side a, the angle opposite it A, and the other angle B. Then if A is known:

$$C = 180° - (A + B) \qquad b = \frac{a \times \sin B}{\sin A} \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being as above.

2. When two sides and the included angle are given, call the given sides a and b and the given angle between them C. Then:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} \qquad B = 180° - (A + C) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

3. When two sides and the angle opposite one of the sides are given, call the given angle A, the side opposite it a and the other given side b. Then:

$$\sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

4. When the three sides of a triangle are given, call them a, b and c and the angles opposite them A, B and C respectively. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c} \qquad \sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B)$$

$$\text{Area} = \frac{a \times b \times \cos C}{2}$$

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SOLUTION OF TRIANGLES

PART I

By ERIK OBERG

CONTENTS

CHAPTER I

THE USE OF FORMULAS*

In mathematical and mechanical books and treatises, as well as in articles containing calculations published in the engineering journals. formulas are used to a great extent instead of rules. In these formulas, signs and symbols are used in order to condense into a small space the essentials of what would otherwise be long and cumbersome rules. The symbols used are generally the letters in the alphabet, and the signs are simply the ordinary signs for arithmetical calculations, with some additional ones necessary for special purposes. Letters from the Greek alphabet are commonly used to designate angles, and the Greek letter π (pi) is always used to indicate the proportion of the circumference of a circle to its diameter; π, therefore. is always, in formulas, equal to 3.1416. The most commonly used Greek letters, besides π, are α (alpha), β (beta), and γ (gamma).

Knowledge of algebra is not necessary in order to make possible the successful use of formulas for the solving of problems such as occur in the solution of triangles; but a thorough understanding of the rules and processes of arithmetic is very essential. The symbols or letters used in the formulas simply stand in place of the actual figures or numerical values which are inserted in the formula in each specific case, according to the requirements of the problem to be solved. When these values are inserted, the result required may be obtained by simple arithmetical processes.

There are two main reasons why a formula is preferable to a rule expressed in words. Firstly, the formula is more concise, it occupies less space, and it is possible for the eye to catch at a glance the whole meaning of the rule laid down; secondly, it is easier to remember a short formula than a long rule, and it is, therefore, of greater value and convenience, as it is not always possible to carry a handbook or reference book about, but the memory must be relied upon to store up a number of the most frequently occurring mathematical and mechanical rules.

The use of formulas can be explained most readily by actual examples. In the following, therefore, a number of simple formulas will be given, and the values will be inserted so as to show, in detail, the principles involved.

Example 1.—When the diameter of a circle is known, the circumference may be found by multiplying the diameter by 3.1416. This rule, expressed as a formula, is:

$$C = D \times 3.1416$$

in which C = circumference of circle,
D = diameter of circle.

* This chapter has been practically reproduced from MACHINERY'S Reference Series No. 52, "Advanced Shop Arithmetic for the Machinist," in order to make the present treatise complete in itself.

This formula shows at a glance, that no matter what the diameter of the circle be, the circumference is always equal to the diameter times 3.1416. Let it be required to find, for example, the circumference of a circle 22 inches in diameter. If then we insert 22 in the place of D in the formula, we have:

$$C = 22 \times 3.1416 = 69.1152 \text{ inches.}$$

Hence, our formula gives, by means of a simple multiplication, the result required.

Assume that the diameter of a circle is 3.72 inches. The circumference of this circle is found by inserting this value instead of D in the formula:

$$C = 3.72 \times 3.1416 = 11.6867 \text{ inches.}$$

Example 2.—In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the pitch. Then the formula would be

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter (D) of the gear equals 2 added to the number of teeth (N), and this sum divided by the pitch (P).

If the number of teeth in a gear is 16 and the pitch 6, then simply put these figures in the place of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{16 + 2}{6} = \frac{18}{6} = 3.$$

D, or the outside diameter, then, is 3 inches.

In another gear the number of teeth is 96 and the pitch 7; find the outside diameter of the gear.

$$D = \frac{96 + 2}{7} = \frac{98}{7} = 14 \text{ inches.}$$

From the examples given it will be seen that in formulas, each letter stands for a certain dimension or quantity. When using a formula for solving a problem, replace the letters in the formula by the figures given in a certain problem, and find the result as in a regular arithmetical calculation.

Example 3.—The formula for the horse-power of a steam engine is as follows:

$$\text{H. P.} = \frac{P \times L \times A \times N}{33{,}000}$$

in which H. P. = indicated horse-power of engine,

P = mean effective pressure on piston in pounds per square inch,

L = length of piston stroke in feet,
A = area of piston in square inches,
N = number of strokes of piston per minute.

Assume that $P = 90$, $L = 2$, $A = 320$, and $N = 110$; what would be the horse-power?

If we insert the given values in the formula we have:

$$\text{H. P.} = \frac{90 \times 2 \times 320 \times 110}{33{,}000} = 192.$$

In formulas, the sign for multiplication ($\times$) is often left out between letters the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$\frac{P \times L \times A \times N}{33{,}000} \text{ can also be written } \frac{PLAN}{33{,}000}$$

Thus, if $A = 3$, and $B = 5$, then:

$$AB = A \times B = 3 \times 5 = 15.$$

If $A = 12$, $B = 2$, and $C = 3$, then:

$$ABC = A \times B \times C = 12 \times 2 \times 3 = 72.$$

It is only the multiplication sign ($\times$) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic.

A parenthesis ( ) or bracket [ ] in a formula means that the expression inside the parenthesis or bracket should be considered as one single symbol, or in other words, that the calculation inside the parenthesis or bracket should be carried out by itself, before other calculations are carried out.

Examples:

$6 \times (8 + 3) = 6 \times 11 = 66.$

$5 \times (16 - 14) + 3\ (2.25 - 1.75) = 5 \times 2 + 3 \times 0.5 = 10 + 1.5 = 11.5.$

In the last example above it will be seen that 5 is multiplied by 2 and 3 by 0.5, and then the products of these two multiplications are added. From the order of the numbers $5 \times 2 + 3 \times 0.5$, one might have assumed that the calculation should have been carried out as follows: 5 times 2 = 10, plus 3 = 13, times 0.5 = 6.5. This latter procedure, however, is not correct.

When several numbers or expressions are connected by the signs $+$, $-$, $\times$ and $\div$, the operations are carried out in the order written, except that *all multiplications should be carried out before the other operations.* The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. Divisions should be carried out before additions and subtractions, if the division is indicated in the same line with these other processes.

Examples:

$5 \times 6 + 4 - 6 \times 4 = 30 + 4 - 24 = 34 - 24 = 10.$

$5 + 3 \times 2 = 5 + 6 = 11.$

$100 \div 2 \times 5 = 100 \div 10 = 10.$

$3.5 + 16.5 \div 3 - 1.75 = 3.5 + 5.5 - 1.75 = 7.25.$

But $5 \times (6 + 4) - 6 \times 4 = 5 \times 10 - 24 = 50 - 24 = 26.$

$(5 + 3) \times 2 = 8 \times 2 = 16.$

$(100 \div 2) \times 5 = 50 \times 5 = 250.$

$(3.5 + 16.5) \div (3 - 1.75) = 20 \div 1.25 = 16.$

Formulas Containing Square and Cube Roots

The square of a number is the product of that number multiplied by itself. The square of 2 is $2 \times 2 = 4$, and the square of 10 is $10 \times 10 = 100$; similarly the square of 177 is $177 \times 177 = 31{,}329$. Instead of writing 4×4 for the quare of 4, it is often written 4^2 which is read *four square*, and means that 4 is multiplied by 4. In the same way 128^2 means 128×128. The small figure (^{2}) in these expressions is called *exponent*.

The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number. Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. We may say that the square root is the reverse of the square, so that if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{\ }$, but the *index figure* (^{2}) is generally left out, making the square-root sign simply $\sqrt{\ }$, thus:

$\sqrt{4} = 2$ (the square root of four equals two),

$\sqrt{100} = 10$ (the square root of one hundred equals ten).

The operation of finding the square root of a given number is called *extracting* the square root.* Squares and square roots as well as bubes and cube roots of all numbers up to 1,000 (sometimes up to 1,600) are generally given in all standard handbooks.

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1{,}728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read "two cube." In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in these expressions is called *exponent*, the same as in the case of the figure (^{2}) indicating the square of a number. An expression of the form 18^3 may also be read the "third power of 18."

In the same way as square root means the reverse of square, so cube root means the reverse of cube; that is, the cube root of a given number is the number which, if repeated as factor three times, would give the number given. Thus the cube root of 27 is 3, because $3 \times 3 \times 3 = 27$. If the cube of 15 is 3,375, then the cube root of 3,375 is, of course, 15. The mathematical sign for the cube root is $\sqrt[3]{\ }$, thus:

$\sqrt[3]{64} = 4$ (the cube root of sixty-four equals four),

$\sqrt[3]{4096} = 16$ (the cube root of four thousand ninety-six equals sixteen).

* See MACHINERY'S Reference Series No. 52, "Advanced Shop Arithmetic for the Machinist", Chapter I.

Assume, for an example, that a formula is given as follows

$$A = \frac{\sqrt{B} \times C}{D}$$

Let $B = 36$, $C = 3.5$, and $D = 10.5$. Find the value of A. If we insert these values in the formula, we have:

$$A = \frac{\sqrt{36} \times 3.5}{10.5} = \frac{6 \times 3.5}{10.5} = \frac{21}{10.5} = 2.$$

As another example, find the value of A in the formula

$$A = \frac{B^2 + C^2}{D^2}, \text{ if } B = 5,\ C = 7, \text{ and } D = 2.$$

If we insert these values in the formula, and carry out the calculation, remembering that $5^2 = 5 \times 5$, $7^2 = 7 \times 7$, etc., we have:

$$A = \frac{5^2 + 7^2}{2^2} = \frac{25 + 49}{4} = \frac{74}{4} = 18.5.$$

Find the value of A in the formula

$$A = \sqrt{B^2 + C^2}, \text{ if } B = 8 \text{ and } C = 6.$$

If we insert the given values in the formula, we have:

$$A = \sqrt{8^2 + 6^2} = \sqrt{8 \times 8 + 6 \times 6} = \sqrt{64 + 36} = \sqrt{100} = 10.$$

The examples given indicate the principles involved in the use of formulas, and show, as well, how easily formulas may be employed by anyone who has a general understanding of arithmetic.

CHAPTER II

ANGLES AND ANGULAR MEASUREMENTS

When two lines meet as shown in Fig. 1, they form an angle with each other. The point where the two lines meet or intersect is called the *vertex* of the angle. The two lines forming the angle are called the sides of the angle.

Angles are measured in degrees and subdivisions of a degree. If the circumference (periphery) of a circle is divided into 360 parts, each part is called one degree, and the angle between two lines from the center to the ends of this small part of the circle is a one-degree angle, as shown in Fig. 2. As the whole circle contains 360 degrees, one-half of a circle contains 180 degrees, and one-quarter of a circle, 90 degrees, as shown in Fig. 9.

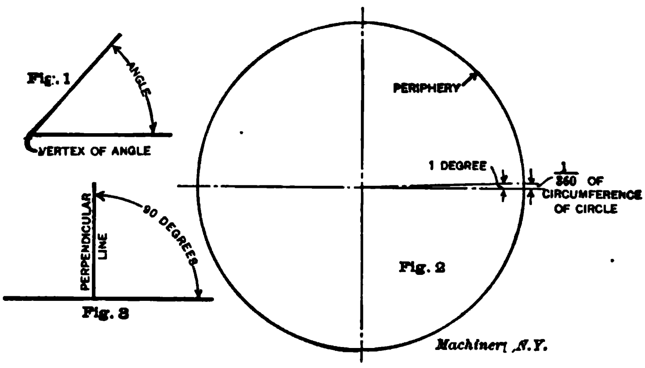
A 90-degree angle is called a *right* angle. An angle larger than 90 degrees is called an *obtuse* angle, and an angle less than 90 degrees is called an *acute* angle. (See Fig. 10.) Any angle which is not a right angle is called an *oblique* angle.

When two lines form a right or 90-degree angle with each other, as shown in Fig. 3, one line is said to be *perpendicular* to the other.

Angles are said to be equal when they contain the same number of degrees. The angle in Fig. 4 and the angle in Fig. 5 are equal, because they are both 60 degrees; that the sides of the angle in Fig. 5 are longer than the sides of the angle in Fig. 4 has no influence on the angle because of the fact that an angle is only the *difference in direction* of two lines. The angle in Fig. 6 which contains only 30 degrees is only one-half of the angle in Fig. 4.

One-half of a right angle is 45 degrees, as shown in Fig. 7. In Fig. 8 is shown an angle which is 120 degrees, and which can be divided into a right or 90-degree angle, and a 30-degree angle.

In order to obtain finer subdivisions for the measurement of angles than the degree, one degree is divided into 60 minutes, and one minute into 60 seconds.



Figs. 1 to 3

Any part of a degree can be expressed in minutes and seconds, for instance, 1/2 of a degree = 30 minutes, 1/3 of a degree = 20 minutes; and since 1/4 of a degree = 15 minutes, 3/4 of a degree = 45 minutes. In the same way 1/2 minute = 30 seconds, 1/4 minute = 15 seconds, and 3/4 minute = 45 seconds.

The word degree is often abbreviated "deg." or the sign (°) is used to indicate degrees; thus, 60° = 60 degrees. In the same way 60′ = 60 min. = 60 minutes, and 60″ = 60 sec. = 60 seconds; and 60° 50′ = 60 degrees 50 minutes.

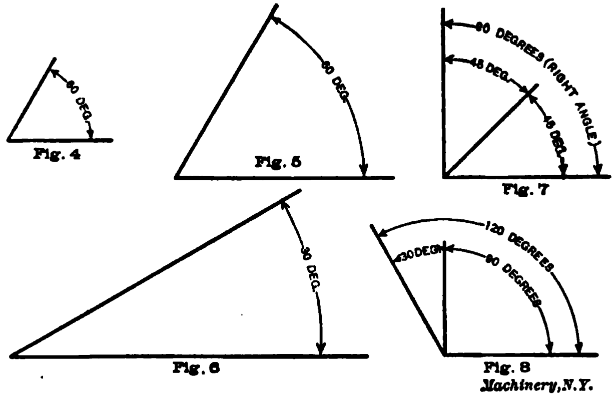
When adding and subtracting degrees and minutes, care must be exercised not to make mistakes on account of there being but 60 minutes in a degree, instead of the usual 100 units met with when adding, for example, dollars and cents.

Example 1.—Add the two angles 60 deg. 32 min. and 35 deg. 16 min.

60 deg. 32 min.
35 deg. 16 min.
———————
95 deg. 48 min.

Example 2.—Add 15 deg. 43 min. to 12 deg. 27 min.

15 deg. 43 min.
12 deg. 27 min.
28 deg. 10 min.



Figs. 4 to 8

In this example the total sum of 43 and 27 minutes is 70 minutes; as 70 minutes, however, contains one whole degree (60 minutes), this is carried over and added to the degrees, leaving 10 minutes in the minute column, and 15 + 12 + 1 = 28 degrees in the degree column.

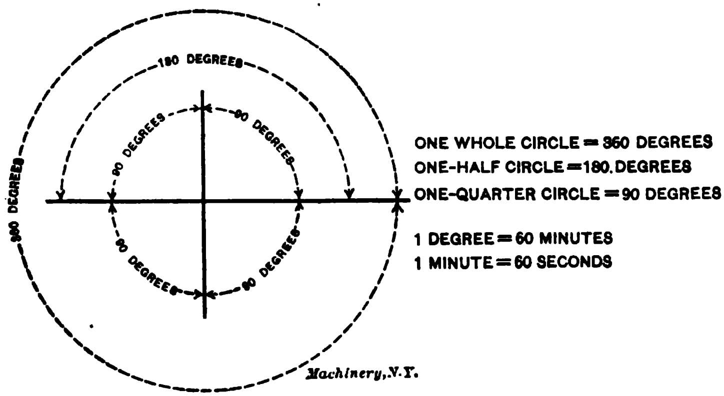


Fig. 9

Example 3.—Add 59 deg. 12 min., 16 deg. 53 min., and 103 deg. 55 min.

59 deg. 12 min.
16 deg. 53 min.
103 deg. 55 min.
180 deg. 0 min.

In adding the minutes (12 + 53 + 55 = 120 min.) we find that their sum equals 2 whole degrees. These are then carried over to the degree column and the total sum equals 59 + 16 + 103 + 2 = 180 deg.

Example 4.—Subtract 12 deg. 17 min. from 21 deg. 39 min.

21 deg. 39 min.
12 deg. 17 min.
9 deg. 22 min.

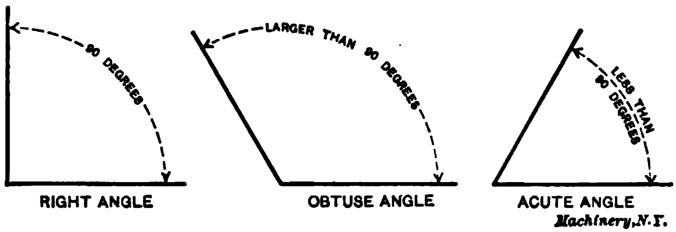


Fig. 10

Example 5.—Subtract 31 deg. 43 min. from 106 deg. 12 min.

106 deg. 12 min.
31 deg. 43 min.
74 deg. 29 min.

In this case we must borrow from the degrees. One deg. = 60 min. and 60 + 12 = 72; then 72 — 43 = 29 min. Having borrowed one degree from 106, we have 105 — 31 = 74 deg.

CHAPTER III

POSITIVE AND NEGATIVE QUANTITIES

In order to be able to use correctly the formulas for the solution of triangles under certain conditions, a working knowledge of the principles of positive and negative numbers or quantities is required. In this chapter, therefore, an explanation of the meaning of these expressions will be given, together with the rules for calculations with negative numbers, and examples to make the rules thoroughly understood.

On the thermometer scale, as is well known, the graduations extend upward from zero, the degrees being numbered 1, 2, 3, etc. Graduations also extend downward and are numbered in the same way: 1, 2, 3, etc. The degrees on the scale extending upward from the zero point may be called *positive* and preceded by a plus sign, so that, for instance, + 5 degrees means 5 degrees above zero. The degrees below zero may be called *negative* and may be preceded by a minus sign, so that — 5 degrees means 5 degrees below zero.

The ordinary numbers may also be considered positive and negative in the same way as the graduations on a thermometer scale. When we count 1, 2, 3, etc., we refer to the numbers that are larger than 0 (corresponding to the degrees *above* the zero point), and these numbers are called positive numbers. We can conceive, however, of numbers extending in the other direction of 0; numbers that are, in fact, less than 0 (corresponding to the degrees below the zero point on the thermometer scale). As these numbers must be expressed by the same figures as the positive numbers, they are designated by a minus sign placed before them. For example, — 3 means a number that is as much less than, or beyond 0 in the negative direction as 3 (or, as it might be written, + 3) is larger than 0 in the positive direction.

A negative value should always be enclosed within a parenthesis whenever it is written in line with other numbers; for example:

$$17 + (-13) - 3 \times (-0.76)$$

In this example — 13 and — 0.76 are negative numbers, and by enclosing the whole number, minus sign and all, in a parenthesis, it is shown that the minus sign is part of the number itself, indicating its negative value.

It must be understood that when we say 7 — 4, then 4 is not a negative number, although it is preceded by a minus sign. In this case the minus sign is simply the sign of subtraction, indicating that 4 is to be subtracted from 7. But 4 is still a positive number or a number that is larger than 0.

It now being clearly understood that positive numbers are all ordinary numbers greater than 0, while negative numbers are conceived of as less than 0, and preceded by a minus sign which is a part of the number itself, we can give the following rules for calculations with negative numbers.

A negative number can be added to a positive number by subtracting its numerical value from the positive number.

Examples:

$4 + (-3) = 4 - 3 = 1.$

$16 + (-7) + (-6) = 16 - 7 - 6 = 3.$

$327 + (-0.5) - 212 = 327 - 0.5 - 212 = 114.5.$

In the last example 212 is not a negative number, because there is no parenthesis indicating that the minus sign is a part of the number itself. The minus sign, then, indicates only that 212 is to be subtracted in the ordinary manner.

As an example illuminating the rule for adding negative numbers to positive ones, the case of a man having $12 in his pocket, but owing $9, may be taken. His debt is a negative quantity, we may say, and equals (— 9). Now if he adds his cash and his debts, to find out how much he really has, we have:

$12 + (-9) = 12 - 9 = 3.$

Of course, in a simple case like this, it is obvious that 9 would be subtracted directly from 12, but the example serves the purpose of illustrating the method used when a negative number is added to a positive number.

A negative number can be subtracted from a positive number by adding its numerical value to the positive number.

Examples:

$4 - (-3) = 4 + 3 = 7.$

$16 - (-7) = 16 + 7 = 23.$

$327 - (-0.5) - 212 = 327 + 0.5 - 212 = 115.5.$

In the last example, note that 212 is subtracted, because the minus sign in front of it does not indicate that 212 is a negative number.

As an illustration of the method used when subtracting a negative number from a positive one, assume that we are required to find how many degrees difference there is between 37 degrees above zero and 24 degrees below; this latter may be written (-24). The difference between the two numbers of degrees mentioned is then:

$37 - (-24) = 37 + 24 = 61.$

A little thought makes it obvious that this result is right, and the example shows that the rule given is based on correct reasoning.

When a positive number is multiplied or divided by a negative number, multiply or divide the numerical values as usual; but the product or quotient, respectively, becomes negative. The same rule holds true if a negative number is divided by a positive number.

Examples:

$4 \times (-3) = -12.$ $(-3) \times 4 = -12.$

$\frac{15}{-3} = -5.$ $\frac{-15}{3} = -5.$

When two negative numbers are multiplied by each other, the product is positive. When a negative number is divided by another negative number the quotient is positive.

Examples:

$(-4) \times (-3) = 12.$ $\frac{-4}{-3} = 1.333.$

If, in a subtraction, the number to be subtracted is larger than the number from which it is to be subtracted, the calculation can be carried out by subtracting the smaller number from the larger, and indicating that the remainder is negative.

Examples:

$3 - 5 = -(5 - 3) = -2.$

In this example 5 cannot, of course, be subtracted from 3, but the numbers are reversed, 3 being subtracted from 5, and the remainder indicated as being negative by placing a minus sign before it.

$227 - 375 = -(375 - 227) = -148.$

The examples given, if carefully studied, will enable the student to carry out calculations with negative numbers when such will be required in solving triangles.

CHAPTER IV

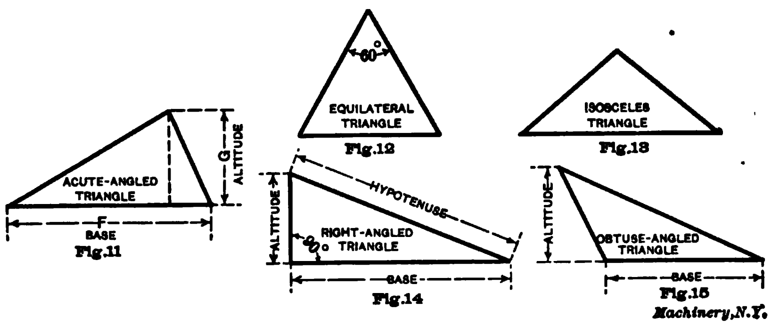
FUNCTIONS OF ANGLES

Any figure bounded by three straight lines is called a triangle. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle. In Fig. 11, if the side F is taken as the base of the triangle, then G is the altitude.

If all the three sides of a triangle are of equal length, as in the one shown in Fig. 12, the triangle is called *equilateral*. Each of the three angles in an equilateral triangle equals 60 degrees.

If two sides are of equal length, as shown in Fig. 13, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 14; the side opposite the right angle is called the *hypotenuse*.



Figs. 11 to 15

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown in Fig. 11. If one of the angles is larger than 90 degrees, as shown in Fig. 15, the triangle is called an *obtuse* or *obtuse-angled* triangle. The sum of the three angles in every triangle is 180 degrees.

Object of Trigonometry and Trigonometric Functions

The object of that part of mathematics called trigonometry is to furnish the methods by which the unknown sides and angles in a triangle may be determined when certain of the sides and angles are given.

The sides and angles of any triangle, which are not known, can be found when:

1. All the three sides,
2. Two sides and one angle, or
3. One side and two angles,

are given. In other words, if the triangle is considered as consisting of six parts, three angles and three sides, the unknown parts can be determined when any three of the parts are given, provided at least one of the given parts is a side.

In order to introduce the values of the angles in calculations of triangles, use is made of certain expressions called *trigonometrical functions* or *functions of angles*. The names of these expressions are: *sine, cosine, tangent, cotangent, secant, and cosecant.* These expressions are usually abbreviated as follows:

sin = sine,	cot = cotangent,
cos = cosine,	sec = secant,
tan = tangent,	cosec = cosecant.

In Fig. 16 is shown a right-angled triangle. The lengths of the three sides are represented by a, b and c, respectively, and the angles opposite each of these sides are called A, B and C, respectively. Angle

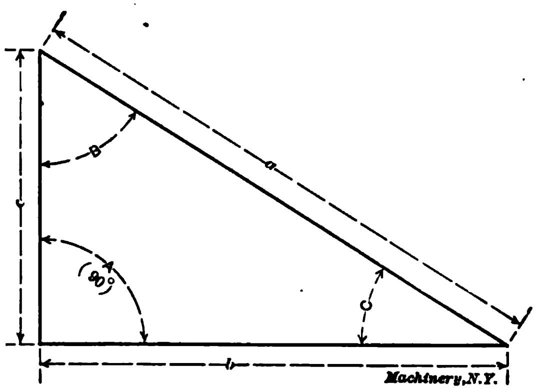


Fig. 16

A is the right angle in the triangle. The side a opposite the right angle is the *hypotenuse*. The side b is called the side *adjacent* to the angle C, but is of course also the side *opposite* to angle B. In the same way, the side c is called the side adjacent to angle B, and the side opposite to angle C. The reason for these names is made clear by studying the figure.

The meanings of the various functions of angles previously named can be explained by the aid of a right-angled triangle.

The sine of an angle equals the opposite side divided by the hypotenuse.

The sine of angle B thus equals the side b, which is opposite to the angle, divided by the hypotenuse a. Expressed as a formula we have:

$$\sin B = \frac{b}{a}.$$

If $a = 16$, and $b = 9$, then $\sin B = \frac{9}{16} = 0.5625$.

The cosine of an angle equals the adjacent side divided by the hypotenuse.

The cosine of angle B thus equals the side c, which is adjacent to this angle, divided by the hypotenuse a, or, expressed as a formula,

$$\cos B = \frac{c}{a}.$$

If $a = 24$, and $c = 15$, then $\cos B = \frac{15}{24} = 0.625$.

The tangent of an angle equals the opposite side divided by the adjacent side.

The tangent of angle B thus equals the side b divided by side c, or,

$$\tan B = \frac{b}{c}.$$

If $b = 28$, and $c = 25$, then $\tan B = \frac{28}{25} = 1.12$.

The cotangent of an angle equals the adjacent side divided by the opposite side.

The cotangent of angle B thus equals the side c divided by the side b, or, $\cot B = \frac{c}{b}$.

If $b = 28$, and $c = 25$, then $\cot B = \frac{25}{28} = 0.89286$.

The secant of an angle equals the hypotenuse divided by the adjacent side.

The secant of angle B thus equals the hypotenuse a divided by the side c adjacent to the angle, or $\sec B = \frac{a}{c}$.

If $a = 24$, and $c = 15$, then $\sec B = \frac{24}{15} = 1.6$.

The cosecant of an angle equals the hypotenuse divided by the opposite side.

The cosecant of angle B thus equals the hypotenuse a divided by the side b opposite the angle, or $\operatorname{cosec} B = \frac{a}{b}$.

If $a = 16$, and $b = 9$, then $\operatorname{cosec} B = \frac{16}{9} = 1.77778$.

The rules given above are very easily memorized, and the student should go no further before he can see at a glance the various functions in a given right-angled triangle.

If the functions of the angle C were to be found instead of the functions of angle B, as given above, they would be as follows:

$$\sin C = \frac{c}{a} \qquad \cos C = \frac{b}{a} \qquad \tan C = \frac{c}{b}$$

$$\cot C = \frac{b}{c} \qquad \sec C = \frac{a}{b} \qquad \text{cosec}\ C = \frac{a}{c}$$

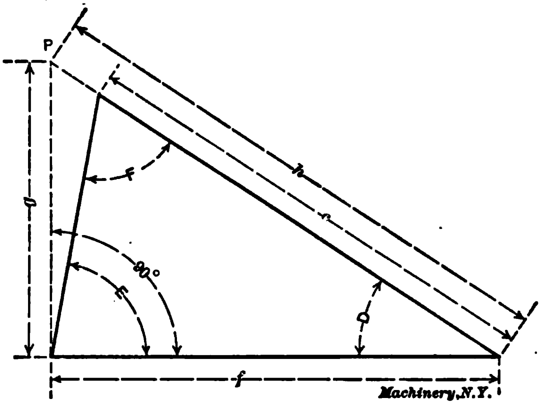


Fig. 17

It must be remembered that the functions of the angles can be found in this manner only when the triangle is right-angled. If the triangle has the shape shown by the full lines in Fig. 17, the sine of angle *D*, for instance, cannot be expressed by any relation between two sides of

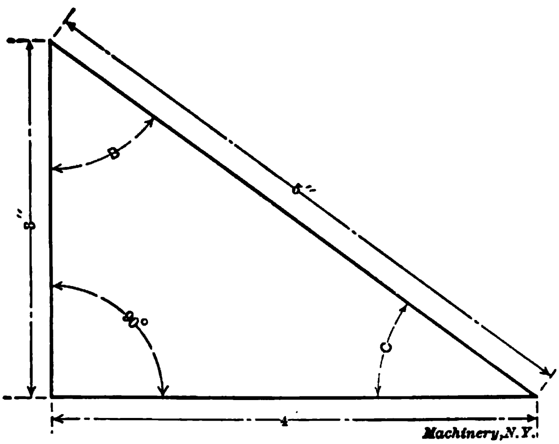


Fig. 18

this triangle. The sine of angle *D*, however, can be found by constructing a right-angled triangle by extending the side *e* to the point *P*, from where a line can be drawn at right angles to the vertex or point of angle *E*, as shown by the dotted line. The sine of angle *D* would then be the length of the dotted line *g* divided by the length of the line *h*, these two lines being, respectively, the side opposite angle D, and the hypotenuse, in a right-angled triangle. In the same way, the tangent of angle D would be the side g divided by the side f.

Examples for Finding the Values of the Functions of Angles

In Fig. 18 is shown a right-angled triangle where the side opposite angle B is four inches, the side opposite angle C is 3 inches, and the hypotenuse is 5 inches. Find the values of the functions of the angles B and C.

Following the rules previously given for finding the sine, cosine, tangent, etc., we have:

$$\sin B = \frac{4}{5} = 0.8 \qquad \cos B = \frac{3}{5} = 0.6$$

$$\tan B = \frac{4}{3} = 1.333 \qquad \cot B = \frac{3}{4} = 0.75$$

$$\sec B = \frac{5}{3} = 1.667 \qquad \text{cosec } B = \frac{5}{4} = 1.25$$

The functions for angle C are as follows:

$$\sin C = \frac{3}{5} = 0.6 \qquad \cos C = \frac{4}{5} = 0.8$$

$$\tan C = \frac{3}{4} = 0.75 \qquad \cot C = \frac{4}{3} = 1.333$$

$$\sec C = \frac{5}{4} = 1.25 \qquad \text{cosec } C = \frac{5}{3} = 1.667$$

The secant and cosecant, being merely the values of 1 divided by the cosine and sine, are not often used in calculations, or included in tables of angular functions.

By studying the results obtained in the calculations above it will be noted that in a right-angled triangle there is a definite relation between the functions of the two acute angles. The sine of angle B equals the cosine of angle C; the tangent of angle B equals the cotangent of angle C, etc. This is true of all right-angled triangles.

As the sum of the three angles in a triangle always equals 180 degrees, and as a right angle equals 90 degrees, it follows that the sum of the two acute angles in a right-angled triangle equals $180 - 90 = 90$ degrees. The angle B (Fig. 18) which together with angle C forms a 90-degree angle, is called the *complement* of angle C. In the same way angle C is the complement of angle B. When any two angles together make 90 degrees, the one is the complement of the other, and in all such cases, the sine of the one equals the cosine of the other, and *vice versa*, the tangent of the one equals the cotangent of the other, etc.

CHAPTER V

TABLES OF TRIGONOMETRIC FUNCTIONS

When using formulas of the type

$$A = \frac{16 \times \sin 36 \text{ deg.}}{2}$$

it is, of course, not possible to find the value of A unless we have some means of transforming the expression "sin 36 deg." (read: sine of 36 degrees) into plain figures. In other words, we must know the *numerical value* of "sin 36 deg.," before we can calculate A. Assume that "sin 36 deg." equals 0.58779. Then, if we insert this value in the formula, we have:

$$A = \frac{16 \times 0.58779}{2} = 4.70232.$$

The numerical values for the natural or trigonometric functions which must thus be found before a formula containing an expression with a trigonometric function can be calculated, are given in the tables in Part II of this treatise, MACHINERY'S Reference Series No. 55. In the following, when reference to "the tables" is made, these tables are always referred to. From these tables, when the angle is given in degrees and minutes, the corresponding numerical value of any of the trigonometric functions can be found; and if the numerical value of the function is known, the corresponding angle can be determined.

It will be seen in the tables that the number of degrees from 0 degree (0°) to 44 degrees (44°) are given above the tables, and the number of minutes in the left-hand column headed with the minute sign ('), reading downward from 0 to 60. The number of degrees from 45 degrees (45°) to 89 degrees (89°), inclusive, are given at the bottom of the tables, and the minutes for the latter degrees are given in the extreme right-hand column, reading from below and up, from 0 to 60. The four main columns in the tables are headed "Sin," "Cos," "Tan," and "Cot," at the top of the tables, and at the bottom of the same tables are the main legends "Cos," "Sin," "Cot," and "Tan." This indicates that when the sine of an angle is required the number of degrees of which angle is given at the top of the table, the sine will be found in the column headed "Sin" at the top; but when the sine of an angle, the number of degrees of which is given at the bottom, is to be found, the sine is found in the second main column, having the word "Sin" at the bottom. The same, of course, applies to the other functions, cosine, tangent, and cotangent.

By referring to the tables it will be seen further that there are two columns of figures in each of the main columns, one headed "Nat."

(natural function) and one "Log." (logarithm). For the present, we are concerned *with the figures given in the column under "Nat." only*, and will treat the subject as if the logarithms of the functions and the columns headed "d." and "c. d." did not exist. Later, we will return to the use of these.

Assume now that the sine, cosine, tangent or cotangent of an angle between 0 and 45 degrees is to be found. First find the given number of degrees at the top of the table; then find the given number of minutes in the extreme left-hand column. Then, read off the figures in the column of the natural sine, cosine, tangent or cotangent, as the case may be, which is opposite the given number of minutes. This value, just read off, is now the numerical value of the function which was to be found.

In reading off these values, care must be taken to place the decimal point properly, as this point is not always given in the tables. The sine and cosine of angles are never over 1, so that when the table gives the figures 99949 as the cosine of 1 degree 50 minutes, the decimal point should be placed in front of these figures, the value being 0.99949. The same refers to the other functions when no decimal point is given. A decimal point should then always be placed in front of the figures given in the tables.

When the sine, cosine, tangent or cotangent of an angle between 45 and 90 degrees is to be found, first find the given number of degrees at the bottom of the table; then find the number of minutes in the extreme right-hand column. Then read off the required function opposite the number of minutes, in the column marked with the required function at the bottom.

Examples of the Use of Trigonometric Tables

Example 1.—Find from the tables the sine of 56 degrees, or, as it is commonly written, sin 56°.

Find first "56°" at the bottom of its page, and then (as in this case there are no minutes) locate 0′ (0 minutes) in the extreme right-hand column, reading from the bottom up. Then, in the column "Nat. Sin." marked at the bottom, read off 0.82904 opposite 0 minutes, which is the required value of the sine of 56 degrees. (Note that the two first figures (82) in the number 82904 are not given opposite every number but only at every fifth number of minutes, but these two figures are to be prefixed, as is easily understod from the table.)

Example 2.—Find sin 50° 20′.

Find first "50°" at the bottom of its page, and then locate 20′ in the right-hand column, reading from the bottom up. Then, in the column "Nat. Sin." marked at the bottom, read off 0.76977 opposite 20 minutes. This is the required value of sin 50° 20′.

Example 3.—Find tan 36° 26′.

Locate 36° at the top of its table, and 26′ in the left-hand column. Then read off 0.73816 in the column "Nat. Tan." This is the required value of tan 36° 26′.

Example 4.—Find cos 36° 19′.

In the same manner as in the examples above, cos 36° 19′ is found to equal 0.80576.

The student should find the following functions from the tables and then compare the result found with the values given, to check the accuracy of the work:

sin 12° 10′ = 0.21076	cos 60° 0′ = 0.50000
sin 15° 50′ = 0.27284	sin 65° 10′ = 0.90753
tan 1° 20′ = 0.02328	sin 12° 3′ = 0.20877

Trigonometric Functions for Angles greater than 90 Degrees

The tables in Part II, Reference Series No. 55, give the angular functions only for angles up to 90 degrees (or 89 degrees 60 minutes, which, of course, equals 90 degrees). In obtuse triangles one angle, however, is greater than 90 degrees, and the tables can be used for finding the functions for angles larger than 90 degrees also.

The sine of an angle greater than 90 degrees but less than 180 degrees equals the sine of an angle which is the difference between 180 degrees and the given angle.

Example: Sin 118° = sin (180° — 118°) = sin 62°. In the same way sin 150° 40′ = sin (180° — 150° 40′) = sin 29° 20′.

The cosine, tangent and cotangent for an angle greater than 90 but less than 180 degrees equals, respectively, the cosine, tangent and cotangent of the difference between 180 degrees and the given angle, but in this case the angular function found has a *negative* value (is preceded by a minus sign).

Example 1.—Find tan 150°.

Tan 150° = — tan (180° — 150°) = — tan 30°. From the tables we have tan 30° = 0.57735; thus tan 150° = — 0.57735.

Example 2.—Find sin 155° 50′.

As explained above sin 155° 50′ = sin (180° — 155° 50′) = sin 24° 10′ = 0.40939.

Example 3.—Find tan 123° 20′.

As explained above tan 123° 20′ = — tan (180° — 123° 20′) = — tan 56° 40′ = — 1.5204.

[In calculations of triangles it is very important that the minus sign is not omitted in the cosines, tangents and cotangents of angles between 90 and 180 degrees.]

Finding the Angle when the Function is Given

When the value of the function of an angle is given, and the angle required in degrees and minutes, the function is located in the tables and the corresponding angle found by a process the reverse of that employed for finding the functions when the angle is given. If the value of the function cannot be found exactly in the tables, use the nearest value found.

Example 1.—The sine of a certain angle, which may be called a, equals 0.53238. Find the angle.

The function 0.53238 is located in the columns marked "Sin." either at the top or at the bottom. When located, the degrees and minutes of the angle are read off directly. If the function is located in the column marked "Sin" at the top, the number of degrees is read off at the top and the number of minutes in the left-hand column; if the function is located in the column marked "Sin." at the bottom, the degrees are read off at the bottom and the minutes in the right-hand column. Following these rules, we find the required angle to be 32° 10′.

Example 2.—The cotangent of an angle is 0.77196. Find the angle.

By observing the rules given in the previous example we find that the required angle is 52° 20′.

Example 3.—The tangent of angle $a = -3.3402$. Find a.

The positive value 3.3402 is first located and the corresponding angle found. This angle is 73° 20′. As the tangent is negative (preceded by a minus sign) the angle a, however, is not 73° 20′ but $(180° - 73° 20') = 106° 40'$.

Example 4.—If $\sin a = 0.29381$, what is the value of angle a?

It will be seen that the function 0.29381 cannot be found exactly in the tables. The nearest value to be found in the sine columns is 0.29376. For practical purposes in machine construction and shop calculations it is near enough to find the angle corresponding to this nearest value. Hence, $a = 17° 5'$.

CHAPTER VI

PRACTICAL APPLICATIONS OF TRIGONOMETRIC FORMULAS

In the following are given a few problems solved by the use of formulas of which trigonometric functions are a part. These examples will show the use of these functions; as obtained from the tables, in cases where it is only required to insert their value in the given formulas.

Example 1.—The depth of the thread in the United States standard screw thread system is expressed by the formula:

$$d = 3/4 \times p \times \cos 30°$$

in which d = depth of thread,

$$p = \text{pitch of thread} = \frac{1}{\text{No. of threads per inch}}$$

Assume that it is required to find the depth of thread for 14 threads per inch. Then $p = \frac{1}{14}$, and

$$d = \frac{3}{4} \times \frac{1}{14} \times \cos 30° = \frac{3}{56} \times 0.86603 = 0.0464 \text{ inch.}$$

Example 2.—In spiral gearing, the pitch diameter of a gear is found by the formula:

$$D = \frac{N}{P \times \cos a}$$

in which D = pitch diameter of spiral gear,
N = number of teeth in gear,
P = normal diametral pitch,
a = tooth angle of gear.

Assume that in a specific case we know that $N = 20$, $P = 8$, and angle $a = 24$ degrees; find the pitch diameter. Then:

$$D = \frac{20}{8 \times \cos 24°} = \frac{20}{8 \times 0.91355} = 2.7366 \text{ inches.}$$

Example 3.—The formula for finding the lead for which to gear up the milling machine when cutting spiral gears is:

$$L = 3.1416 \times D \times \cot a$$

in which L = the lead for which to gear up the machine,
D = pitch diameter,
a = tooth angle.

Assume that in a specific case we know that $D = 5$, and angle $a = 24$ degrees. Then

$$L = 3.1416 \times 5 \times \cot 24° = 15.708 \times 2.246 = 35.28 \text{ inches.}$$

Example 4.—In a radial ball bearing, if the diameter of the balls, d, and the number of balls, N, are known, the diameter D of the outside or enveloping ball race may be found by the following formula:

$$D = \frac{d}{\sin\left(\frac{180}{N}\right)^{\circ}} + d$$

Assume that $d = \frac{1}{4}$ inch, and $N = 15$. Then:

$$D = \frac{0.25}{\sin\left(\frac{180}{15}\right)^{\circ}} + 0.25 = \frac{0.25}{\sin 12°} + 0.25 = \frac{0.25}{0.20791} + 0.25$$

$$= 1.2025 + 0.25 = 1.4525 \text{ inch.}$$

Example 5.—In a sprocket wheel for ordinary link chain, the pitch diameter D can be determined when the number of teeth required, N, the length of the inside oval of the chain link, r, and the diameter of the stock from which the chain link is made, d, are known. The formula used is:

$$D = \sqrt{\left(\frac{r}{\sin (90 \div N)°}\right)^2 + \left(\frac{d}{\cos (90 \div N)°}\right)^2}$$

If $r = \frac{3}{4}$ inch, $d = \frac{1}{4}$ inch, and $N = 20$ teeth, then:

$$D = \sqrt{\left(\frac{0.75}{\sin 4° 30'}\right)^2 + \left(\frac{0.25}{\cos 4° 30'}\right)^2} = \sqrt{9.559^2 + 0.251^2}$$

$$= \sqrt{91.437} = 9.562 \text{ inches.}$$

Example 6.—In a Bush roller chain wheel the pitch diameter D of the sprocket wheel can be found if the number of teeth in the sprocket, N, and the pitch P of the chain are decided upon. The formula is:

$$D = \frac{P}{\sin\left(\frac{180}{N}\right)^\circ}$$

Assume that the pitch diameter of a sprocket with 72 teeth, for a chain of ¾ inch pitch, is required. Then $P = ¾$, and $N = 72$; hence $\frac{180}{N} = 2½$, and $D = \frac{0.75}{\sin 2° 30'} = \frac{0.75}{0.04362} = 17.194$ inches.

Example 7.—The following formula may be used for finding the angle to which to set the dividing head of the milling machine when cutting teeth in the ends of end mills:

$$\cos a = \tan \frac{360}{N} \times \cot \beta$$

in which a = angle to which to set dividing head,
β = included angle of cutter with which teeth are milled,
N = number of teeth in end mill.

Assume that it is required to cut the teeth in the end of an end mill having 12 teeth with a 70-degree angular milling cutter.

$$\cos a = \tan \frac{360}{12} \times \cot 70° = \tan 30° \times \cot 70°$$

$$= 0.57735 \times 0.36397 = 0.21014.$$

Having found that $\cos a = 0.21014$, we find that $a = 77° 52'$.

Example 8.—The angle to which to set the planer head when planing an Acme threading tool having no side clearance, but 15 degrees front clearance, can be determined by the formula:

$$\tan x = \frac{\tan 14° 30'}{\cos 15°}$$

in which x = angle to which to set planer head.

Carrying out the calculation, we have:

$$\tan x = \frac{\tan 14° 30'}{\cos 15°} = \frac{0.25862}{0.96593} = 0.26774$$

Having found that $\tan x = 0.26774$, we find from the tables that $x = 14° 59'$, or practically 15 degrees.

CHAPTER VII

RIGHT-ANGLED TRIANGLES

If the lengths of two sides of a right-angled triangle are known, the third side can be found by a simple calculation. In every right-angled triangle the hypotenuse equals the square root of the sum of the squares of the two sides forming the right angle. If the hypotenuse equals a, and the sides forming the right angle b and c, respectively, as shown in Fig. 19, then:

$$a = \sqrt{b^2 + c^2}$$

Each of the sides b and c can also be found if the hypotenuse and one of the sides are known. The following formulas would then be used:

$$b = \sqrt{a^2 - c^2}$$
$$c = \sqrt{a^2 - b^2}$$

Assume that side b is 18 inches, and side c, 7.5 inches. What is the length of the hypotenuse a?

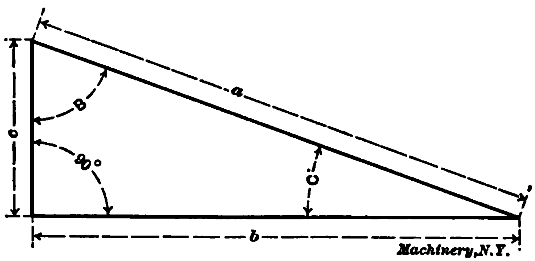


Fig. 19

If we insert the values of b and c in the formula given above for a, we have:

$$a = \sqrt{18^2 + 7.5^2} = \sqrt{18 \times 18 + 7.5 \times 7.5} = \sqrt{324 + 56.25} = \sqrt{380.25} = 19.5$$

Assume that the length of the hypotenuse is 10 inches and that the side c is 6 inches. What is the length of the side b?

Using the formula given above for b, and inserting the values of a and c we have:

$$b = \sqrt{10^2 - 6^2} = \sqrt{10 \times 10 - 6 \times 6} = \sqrt{100 - 36} = \sqrt{64} = 8.$$

Thus whenever two sides of a right-angled triangle are given, the third side can always be found by a simple arithmetical calculation. To find the angles, however, it is necessary to use the tables of sines, cosines, tangents and cotangents, as given in Part II, MACHINERY'S Reference Series No. 55; and if only one side and one of the acute angles are given, the natural trigonometric functions must be used for finding the lengths of the other sides, as explained in the following.

Solution of Right-angled Triangles by Means of the Functions of Angles

In Chapter IV it is stated that the sides and angles of any triangle, which are not known, can be found when:

1. All the three sides,
2. Two sides and one angle, or
3. One side and two angles

are given. In every right-angled triangle one angle, the right or 90-degree angle is, of course, always known. In a right triangle, therefore, the unknown sides and angles can be found when either two sides, or one side and one of the acute angles are known.

The methods of solution of right-angled triangles may be divided into four classes, according to which sides and angles are given or known:

1. Two sides known.
2. The hypotenuse and one acute angle known.
3. One acute angle and its adjacent side known.
4. One acute angle and its opposite side known.

Case 1.—When two sides are known, the third side is found by one of the formulas:

$$a = \sqrt{b^2 + c^2} \qquad (1)$$
$$b = \sqrt{a^2 - c^2} \qquad (2)$$
$$c = \sqrt{a^2 - b^2} \qquad (3)$$

which formulas are given in the first part of this chapter, and in which a is the hypotenuse, and b and c the sides forming the right angle.

The acute angles B and C, Fig. 19, are found by determining either the sine, cosine, tangent or cotangent for the angles, as explained in Chapter IV, and obtaining the angles, expressed in degrees and minutes, from the trigonometric tables. When one angle has been found, the other can also be found directly without reference to the tables, because the sum of the acute angles in a right-angled triangle equals 90 degrees, and if one of them is known, the other must equal 90 degrees minus the known angle. Expressed as formulas this would be:

$$B = 90° - C$$
$$C = 90° - B$$

As an example, assume that the hypotenuse of a right-angled triangle is 5 inches and one of the sides 4 inches, as shown in Fig. 20. Find angles B and C and the length of side c.

The side c is first found by Formula (3) given above, a and b being inserted in this formula as below:

$$c = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3.$$

As explained in Chapter IV, the side opposite an angle divided by the hypotenuse, gives the sine of the angle.

Hence

$$\sin C = \frac{3}{5} = 0.6.$$

By referring to the trigonometric tables, it will be found that the nearest value to 0.6 in the columns of sines is 0.59995, and the angle corresponding to this value is 36° 52′. Angle *C*, then equals, 36° 52′.

In the same way

$$\sin B = \frac{4}{5} = 0.8.$$

From the tables we find the nearest value in the columns of sines to be 0.80003, which is the sine of 53° 8′.

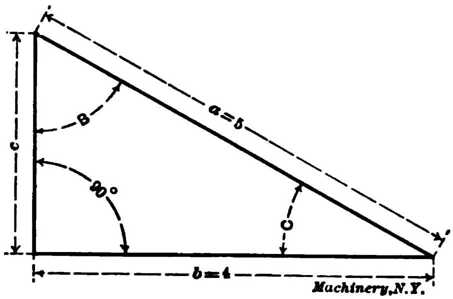


Fig. 20

This last calculation would not have been necessary, because, as has already been mentioned, angle *B* could have been found directly when angle *C* was known, by the formula

$$B = 90° - C = 90° - 36°\ 52' = 53°\ 8'.$$

It will be noted that either method for finding angle *B* gives the same result.

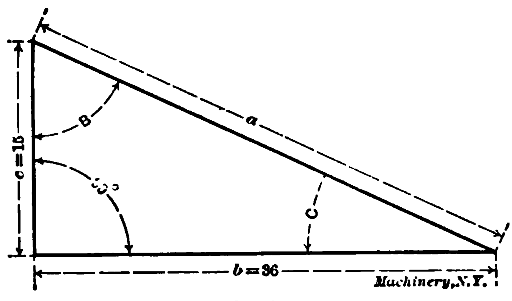


Fig. 21

As a further example, assume that the sides forming the right angle are given as shown in Fig. 21; one is 15 inches and the other is 36 inches. Find the hypotenuse and the angles *B* and *C*.

The hypotenuse is found by Formula (1), on page 25, the values of *b* and *c* being inserted.

$$a = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39.$$

As explained in Chapter IV, the side opposite an angle divided by the side adjacent, equals the tangent of the angle.

Hence

$$\tan B = \frac{36}{15} = 2.4.$$

By referring to the tables, it will be found that the nearest value to 2.4 in the columns of tangents is 2.4004, which is the tangent of 67° 23′. Hence $B = 67° 23'$, and

$$C = 90° - B = 90° - 67° 23' = 22° 37'.$$

Case 2.—If the hypotenuse and one acute angle are known, the side adjacent to the known angle is found by multiplying the hypotenuse by the cosine of the known angle; the side opposite the known angle is found by multiplying the hypotenuse by the sine of the known angle; and the other acute angle is found by subtracting the known angle from 90 degrees.

We can express this rule by simple formulas. Referring to Fig. 19, if a is the hypotenuse, and B the known angle, then:

$$c = a \times \cos B$$
$$b = a \times \sin B$$
$$C = 90° - B$$

If C is the known angle, then:

$$b = a \times \cos C$$
$$c = a \times \sin C$$
$$B = 90° - C$$

As an example, assume that the hypotenuse $a = 22$ inches, and angle $B = 41° 36'$. Find sides b and c, and angle C. (See Fig. 19.)

By referring to the tables, it will be found that the nearest value to case when angle B is known, we have:

$c = a \times \cos B = 22 \times \cos 41° 36' = 22 \times 0.74780 = 16.4516$ inches.
$b = a \times \sin B = 22 \times \sin 41° 36' = 22 \times 0.66393 = 14.6065$ inches.
$C = 90° - 41° 36' = 48° 24'$.

Case 3.—When one acute angle and its adjacent side are known, the hypotenuse is found by dividing the known side by the cosine of the known angle; the side opposite the known angle is found by multiplying the known adjacent side by the tangent of the known angle; and the other acute angle is found by subtracting the known angle from 90°.

Referring to Fig. 19, we can express this rule by simple formulas. If B is the known angle, and c the known side, adjacent to angle B, then:

$$a = \frac{c}{\cos B} \qquad b = c \times \tan B \qquad C = 90° - B$$

If C is the known angle, and b the known side, adjacent to angle C, then:

$$a = \frac{b}{\cos C} \qquad c = b \times \tan C \qquad B = 90° - C$$

As an example, assume that angle $B = 25° 12'$, and its adjacent side $c = 12$ inches. Find the hypotenuse a, opposite side b, and angle C.

By inserting the known values in the formulas just given for the case where angle B is known, we have:

$$a = \frac{c}{\cos B} = \frac{12}{\cos 25^\circ\ 12'} = \frac{12}{0.90483} = 13.262 \text{ inches.}$$

$$b = c \times \tan B = 12 \times 0.47056 = 5.6467 \text{ inches.}$$

$$C = 90^\circ - 25^\circ\ 12' = 64^\circ\ 48'.$$

Case 4.—When one acute angle and the side opposite it are known, the hypotenuse is found by dividing the known side by the sine of the known angle; the side adjacent to the known angle is found by multiplying the known opposite side by the cotangent of the known angle; and the other acute angle is found by subtracting the known angle from 90°.

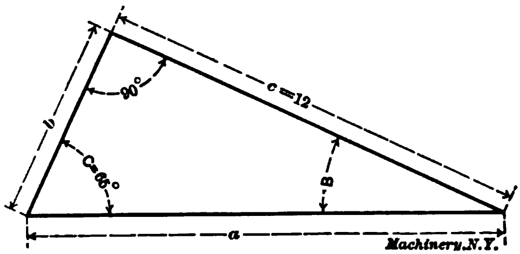


Fig. 22

By referring to Fig. 19, we can express this rule by simple formulas. If B is the known angle, and b the side opposite, which is also known, then:

$$a = \frac{b}{\sin B} \qquad c = b \times \cot B \qquad C = 90^\circ - B$$

If C is the known angle, and c the known side, opposite to angle C, then:

$$a = \frac{c}{\sin C} \qquad b = c \times \cot C \qquad B = 90^\circ - C$$

As an example, assume that angle C equals 65 degrees, and that the length of side c is 12 feet, as shown in Fig. 22. Find the lengths of sides a and b and angle B.

By inserting the known values in the formulas just given for the case when angle C is known, we have:

$$a = \frac{c}{\sin C} = \frac{12}{\sin 65^\circ} = \frac{12}{0.90631} = 13.2405 \text{ inches.}$$

$$b = c \times \cot C = 12 \times 0.46631 = 5.5957 \text{ inches.}$$

$$B = 90^\circ - 65^\circ = 25^\circ.$$

CHAPTER VIII

PROBLEMS FROM PRACTICE

The calculations required in the design of bevel gearing offer abundant examples of the use of the trigonometric functions and the solution of right-angled triangles. The student who is anxious to obtain additional practice, and to whom the practical applications of the formulas given are of especial interest, is, therefore, referred to MACHINERY'S Reference Series No. 37, Bevel Gearing, for practical applications. In the following, however, a number of practical examples, selected for the purpose of illustration, will also be given.

Example 1.—Fig. 23 shows a section of a United States standard thread. Find a formula for the depth of the thread in terms of the pitch, and calculate the depth of screw threads with 12 and 16 threads per inch.

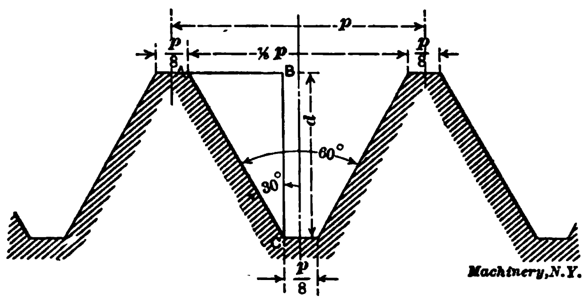


Fig. 23

In the illustration, p is the pitch of the thread. The pitch, of course, equals $\frac{1}{\text{No. of threads per inch.}}$.* It is required to find the depth BC of the thread, expressed in terms of the pitch. This depth can be found if we can solve the triangle ABC.

In the U. S. standard thread system there is a flat at the top and bottom of the thread as shown in Fig. 23. The width of this flat is one-eighth of the pitch, as indicated. Hence, side AB of the right-angled triangle ABC equals one-half of $\frac{7}{8}$ pitch minus one-half of $\frac{1}{8}$ pitch, or $\left(\frac{7}{16} - \frac{1}{16}\right)$ pitch $= \frac{3}{8}$ pitch. The angle opposite this side is also known; it is one-half of the total thread angle, or 30 degrees. According to the rules and formulas in the previous chapter, therefore,

$$BC = AB \times \cot 30°.$$

* See MACHINERY'S Reference Series No. 18, "Shop Arithmetic for the Machinist," third edition, Chapter IV.

If we insert in this formula $BC = d$, $AB = \frac{3}{8} p$, and $\cot 30° = 1.7321$, we have:

$$d = \frac{3}{8} p \times 1.7321 = 0.6495\ p$$

in which d = depth of thread,
p = pitch of thread.

We will now find the depth of the thread for 12 and 16 threads per inch. As $p = \frac{1}{\text{No. of threads per inch}}$, we have, by inserting the known values in the general formula just found:

$$d = 0.6495 \times \frac{1}{12} = 0.0541 \text{ inch, for 12 threads,}$$

$$d = 0.6495 \times \frac{1}{16} = 0.0406 \text{ inch, for 16 threads.}$$

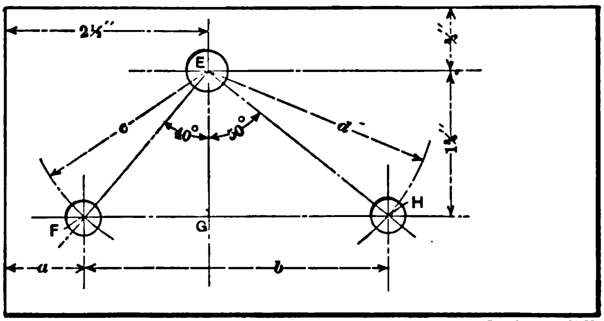


Fig. 24

Example 2.—In laying out a master jig plate, it is required that holes *F* and *H*, Fig. 24, shall be on a straight line which is 1¾ inch distant from hole *E*. The holes must also be on lines making, respectively, 40- and 50-degree angles with line *EG*, drawn at right angles to the sides of the jig plate through *E*, as shown in the engraving. Find the dimensions necessary for the toolmaker.

The dimensions which ought to be given the toolmaker in addition to those already given are indicated by *a*, *b*, *c*, and *d*. The two latter are the radii of the arcs which if struck with *E* as a center will pass through the centers of *F* and *H*. We have here two right-angled triangles *EFG* and *EGH*. We know one acute angle in each, and also the length of side *EG* (1¾ inch) which is mutual to both triangles and which is the side adjacent to the known angle. From the formulas in the preceding chapter we, therefore, have:

$$FG = 1.75 \times \tan 40° = 1.75 \times 0.83910 = 1.4684 \text{ inch.}$$

$$FE = \frac{1.75}{\cos 40°} = \frac{1.75}{0.76604} = 2.2845 \text{ inches.}$$

$$GH = 1.75 \times \tan 50° = 1.75 \times 1.1918 = 2.0856 \text{ inches.}$$

$$EH = \frac{1.75}{\cos 50°} = \frac{1.75}{0.64279} = 2.7225 \text{ inches.}$$

But, by referring to Fig. 24 it will be seen that $FE = c$, $EH = d$, $2\frac{1}{2} - FG = a$, and $FG + GH = b$. Hence

$a = 2.5 - 1.4684 = 1.0316$ inch,
$b = 1.4684 + 2.0856 = 3.5540$ inches,
$c = 2.2845$ inches,
$d = 2.7225$ inches.

Example 3.—If the pitch p of a Bush roller chain is ¾ inch, and the sprocket wheel is to have 32 teeth, what will be the pitch diameter of the gear? (See Fig. 25.)

By referring to the engraving, it will be seen that $AD = p = \frac{3}{4}$ inch, and $AC = \frac{1}{2} AD = \frac{3}{8}$ inch, in this case. Line AB is the pitch radius or one-half the pitch diameter. Angle a is the angle for one

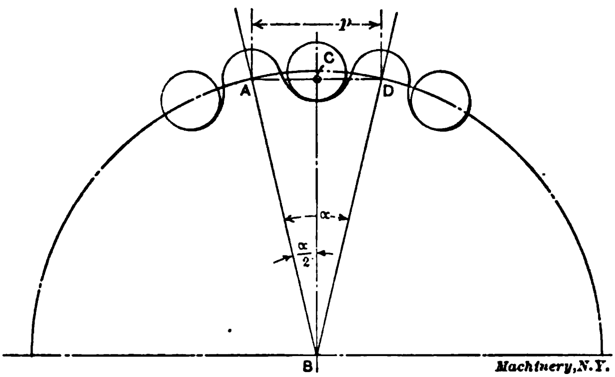


Fig. 25

tooth, and as the whole circle is 360 degrees, a in this case equals $\frac{360}{32} = 11\frac{1}{4}$ degrees, or 11 degrees 15 minutes. One-half of a, then, equals 5 degrees 37 minutes, approximately. We, therefore, have here a right-angled triangle in which we know the length of side AC and the angle opposite it. We want to find the hypotenuse AB. From the formulas in the preceding chapter, we have:

$$AB = \frac{AC}{\sin \frac{a}{2}} = \frac{0.375}{\sin 5° 37'} = \frac{0.375}{0.09787} = 3.832 \text{ inches.}$$

The pitch diameter, then, equals $2 \times 3.832 = 7.664$ inches.

Example 4.—A common method for measuring the width of machine slide dove-tails is indicated diagrammatically in Fig. 26. At A and B are shown carefully ground cylindrical gages of standard dimensions. In the example shown it is required to find what the distance d, measured by micrometers over the gages when these are pushed into the V's of the dove-tail as shown, should be, in order to make sure that the piece is planed to the dimensions given. The diameters of the gages are 0.750 inch.

In order to find the dimension *d* measured over the gages, we must find dimension *KG*, Fig. 27, and add twice this length to the distance 3" from *L* to *M* in Fig. 26. It will be seen that $KG = KE + EG$; $KE = \frac{1}{2} \times$ diam. of gage $= \frac{3}{8}$; *EG* is solved from the right-angled triangle

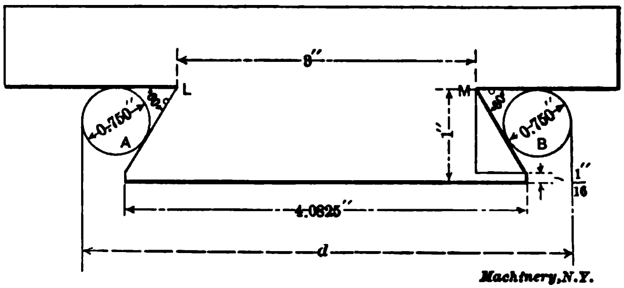


Fig. 26

EGH, in which angle *EHG* equals 60 degrees, and side *HG* equals one-half of the diameter of the gage, or 3/8 inch.

Hence,

$$EG = HG \times \tan 60° = \tfrac{3}{8} \times 1.7321 = 0.6495 \text{ inch.}$$

$$KE + EG = 0.375 + 0.6495 = 1.0245 \text{ inch.}$$

$$d = 2 \times 1.0245 + 3 = 5.049 \text{ inches.}$$

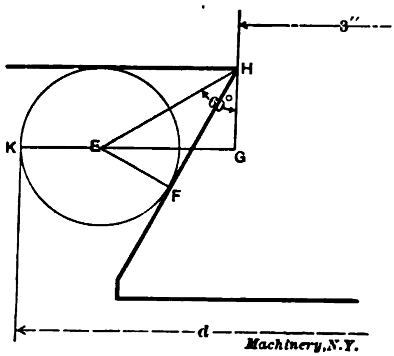


Fig. 27

Example 5.—Small reamers are sometimes provided with flats instead of actual flutes. The diameter of the reamer is, of course, measured over the sharp corners; if the reamer tapers, the taper of the flats will not be the same as the taper of the sharp corners , and the milling machine dividing head must be set to a different angle from that which the cutting edge makes with the center line. A simple formula may be deduced by the aid of trigonometry for finding the angle to which to set the dividing head when milling the flats.

Referring to Fig. 28, in which the reamer is imagined as continued to a sharp point at the end, let

a = angle made by cutting edge with center line,

a_1 = angle made by flat with center line,

N = number of sides of reamer,

T = taper per foot.

Angle β, as shown in the engraving, can be determined by the formula

$$\beta = \frac{360}{2N}$$

as is evident from the illustration.

Angle α_1 is the angle sought. It will be seen that if FE and HE were known, then

$$\tan \alpha_1 = \frac{FE}{HE}$$

But $FE = AE \times \cos \beta$. If we insert this value we have:

$$\tan \alpha_1 = \frac{AE \times \cos \beta}{HE}$$

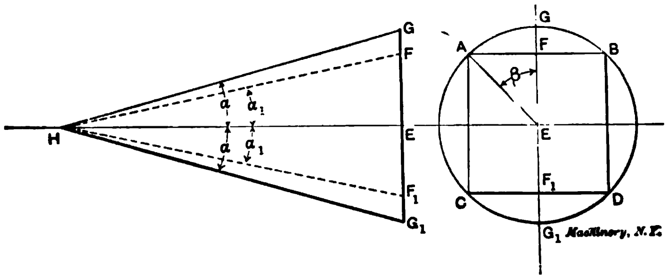


Fig. 28

As $\cos \beta = \cos \frac{360}{2N}$, we have further

$$\tan \alpha_1 = \frac{AE}{HE} \times \cos \frac{360}{2N}$$

The distance AE, however, is one-half of the taper in the distance HE. The taper per inch then is $\frac{2\,AE}{HE}$, and the taper per foot

$$T = 12 \times \frac{2\,AE}{HE} = \frac{24\,AE}{HE}, \text{ or } \frac{T}{24} = \frac{AE}{HE}.$$

If we insert $\frac{T}{24}$ in the formula above, we have

$$\tan \alpha_1 = \frac{T}{24} \times \cos \frac{360}{2N}$$

Assume that the taper per foot is ¼ inch, and that a four-sided reamer is required. Find the angle to which to set the index-head.

$$\tan \alpha_1 = \frac{\frac{1}{4}}{24} \times \cos 45° = 0.00736,$$

which gives $\alpha_1 = 25$ minutes.

Example 6.—In Fig. 29 are shown two pulleys of 6 and 12 inches diameter, with a fixed center distance of 5 feet. Find the length of belt required to pass over the two pulleys. The belt is assumed to be perfectly tight.

The length of the belt is made up of the two straight portions *AC* and *BD*, tangent to the circles as shown in Fig. 29, and of the arc *AEB* of the larger pulley and the arc *CFD* of the smaller pulley. *AC* and *BD* are equal. We will first find the length *AC*. By drawing a line *HG* from *H*, the center of the smaller pulley, parallel to *AC*, we can construct a triangle *HGK* in which $HG = AC$, and $GK = AK - HC$. That $HG = AC$ is clear from the fact that *HC* and *KA* are parallel, both being perpendicular or at right angles to the tangent line *AC*. The figure *HGAC* is, therefore, a rectangle, and, hence, opposite sides are equal. *HG*, therefore, equals *AC*, and $HC = GA$.

That $GK = AK - HC$ is evident from the fact that $GK = AK - GA$, but as $GA = HC$, it follows that $GK = AK - HC$.

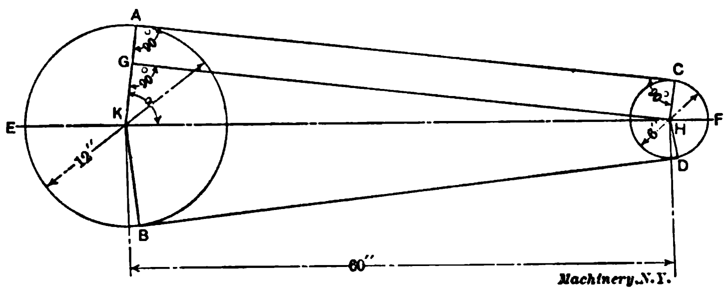


Fig. 29

Now, *AK* is the radius of the larger pulley, which is one-half its diameter, or 6 inches, and *HC* is the radius of the smaller pulley or 3 inches. Hence, $GK = 6 - 3 = 3$ inches. $HK = 5$ feet or 60 inches, as given in the problem. We then have here a right-angled triangle in which the hypotenuse $HK = 60$ inches, and one of the sides forming the right angle is 3 inches. Hence, side *GH* is found by the formula given for this case in the previous chapter, and by inserting the known values we have:

$$GH = \sqrt{60^2 - 3^2} = \sqrt{3600 - 9} = \sqrt{3591} = 59.925.$$

As $GH = AC$, we, therefore, have $AC = 59.925$, and as $AC = BD$, we have $AC + BD = 119.85$ inches. It now remains to find the lengths of the circular arcs *AEB* and *CFD*. In order to find these lengths we must first find the number of degrees in these arcs, and to find this, the first step is to find angle a. According to the rules given in Chapter IV,

$$\cos a = \frac{GK}{KH} = \frac{3}{60} = 0.05.$$

From this we find from the trigonometric tables that $a = 87° 8'$.

It will be seen from Fig. 29 that angle $AKE = 180° - a = 180° -$

$87° 8' = 92° 52'$. Angle *EKB* = angle *AKE*, so that the arc *AEB*, therefore, is equal to twice angle *AKE* or

$$\text{arc } AEB = 2 \times 92° 52' = 185° 44'.$$

The whole circumference of the larger pulley equals $3.1416 \times 12 =$ 37.699 inches. As the whole circumference is 360 degrees, its length in inches is to the length of arc *AEB* as 360° is to 185° 44′, or

$$\frac{37.699}{\text{arc } AEB} = \frac{360°}{185° 44'}$$

Transposing this expression, we have

$$\text{arc } AEB = \frac{37.699 \times 185° 44'}{360°}$$

Before we can carry out this calculation we must transform 44 minutes to decimals of a degree. As 44 minutes equals 44/60 of a degree, this, changed to a decimal fraction equals $\frac{44}{60} = 0.73$, and 185° 44′ equals 185.73 degrees. Then:

$$\text{arc } AEB = \frac{37.699 \times 185.73}{360} = 19.45 \text{ inches.}$$

Now, to find arc *CFD*, angle *CHF* is first determined. This angle equals angle *GKH* or a, because *AK* and *CH* are parallel lines. Hence arc $CFD = 2 \times$ angle $a = 2 \times 87° 8' = 174° 16'$. Now, proceeding as before we have:

$$3.1416 \times 6 = 18.8496 = \text{circumference of small pulley.}$$

$$\frac{18.8496}{\text{arc } CFD} = \frac{360°}{174° 16'}$$

Transposing this and changing 16 minutes to decimals of a degree, gives us:

$$\text{arc } CFD = \frac{18.8496 \times 174.27}{360} = 9.12 \text{ inches.}$$

The total length of the belt, then, equals

$$119.85 + 19.45 + 9.12 = 148.42 \text{ inches.}$$

CHAPTER IX

SOLUTION OF OBLIQUE-ANGLED TRIANGLES

The methods used in the solution of oblique triangles—that is, triangles, no one of whose angles is a right angle—differ according to which parts are known and which are to be found. The problems which present themselves may be divided into four classes:

1. Two angles and one side known.
2. Two sides and the angle included between them known.
3. Two sides and the angle opposite one of them known.
4. The three sides known.

1. Two Angles and One Side Known

Assume that the angles A and B in Fig. 30 are given as shown, and that side a is 5 inches. Find angle C, sides b and c, and the area of the triangle.

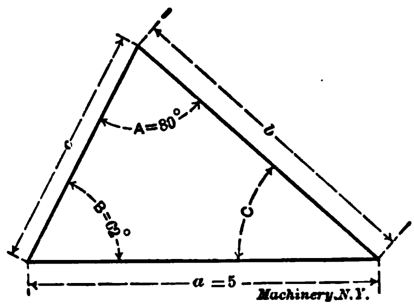


Fig. 30

As the sum of the three angles in a triangle always equals 180 degrees, angle C can be found directly when angles A and B are given, by subtracting the sum of these angles from 180 degrees. Angle $A =$ 80 degrees and $B = 62$ degrees; therefore,

$$C = 180° - (80° + 62°) = 180° - 142° = 38°.$$

For finding the sides b and c the following rule is used: *The side to be found equals the known side multiplied by the sine of the angle opposite the side to be found, and the product divided by the sine of the angle opposite the known side.*

To find side b, for example, multiply the known side a by the sine of angle B, and divide the product by the sine of angle A. Written as a formula this would be:

$$b = \frac{a \times \sin B}{\sin A} \qquad (4)$$

In the same way

$$c = \frac{a \times \sin C}{\sin A} \qquad (5)$$

If we insert the known values for side a and the angles in these formulas, we have:

$$b = \frac{5 \times \sin 62°}{\sin 80°} = \frac{5 \times 0.88295}{0.98481} = 4.483 \text{ inches.}$$

$$c = \frac{5 \times \sin 38°}{\sin 80°} = \frac{5 \times 0.61566}{0.98481} = 3.126 \text{ inches.}$$

Now all the sides and angles are known, and it only remains to find the area of the triangle. This is found by the following rule: *The area of a triangle equals one-half the product of two of its sides multiplied by the sine of the angle between them.* (This rule gives the same result as that given in MACHINERY'S Reference Series Book, No. 52, Advanced Shop Arithmetic for the Machinist, Chapter VIII.)

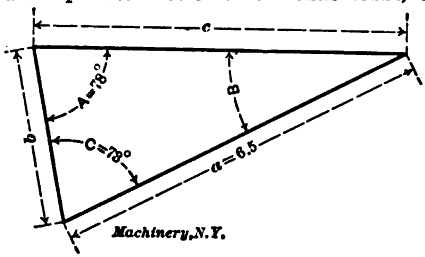


Fig. 31

In the example in Fig. 30, the area, then, equals one-half the product of sides a and b multiplied by the sine of angle C, or, expressed as a formula:

$$\text{Area} = \frac{a \times b \times \sin C}{2} \qquad (6)$$

Inserting the known values for a, b, and C in this formula we have:

$$\text{Area} = \frac{5 \times 4.483 \times \sin 38°}{2} = \frac{5 \times 4.483 \times 0.61566}{2} =$$

$$\frac{13.8000}{2} = 6.9 \text{ square inches.}$$

All the required quantities in this triangle have now been found.

Examples for Practice

Example 1.—In Fig. 31 is shown a triangle of which one side is 6.5 feet, and the two angles A and C (78 and 73 degrees, respectively) are given. Call the sides a, b and c, as shown. Find angle B, sides b and c, and the area.

First find angle B. Using the same method as explained for finding angle C in the previous example, we have:

$$B = 180° - (78° + 73°) = 180° - 151° = 29°.$$

For finding sides b and c use the rule or formulas previously given, inserting the values given in this example:

$$b = \frac{a \times \sin B}{\sin A} = \frac{6.5 \times \sin 29°}{\sin 78°} = \frac{6.5 \times 0.48481}{0.97815}$$

$$= \frac{3.151265}{0.97815} = 3.222 \text{ feet.}$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{6.5 \times \sin 73°}{\sin 78°} = \frac{6.5 \times 0.95630}{0.97815}$$

$$= \frac{6.21595}{0.97815} = 6.355 \text{ feet.}$$

According to the given rule and formula, the area is finally found as below:

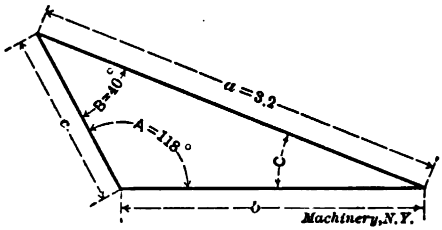


Fig. 32

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{6.5 \times 3.222 \times \sin 73°}{2}$$

$$= \frac{6.5 \times 3.222 \times 0.95630}{2} = \frac{20.027}{2} = 10.013 \text{ square feet.}$$

Example 2.—In Fig. 32, side a equals 3.2 inches, angle A, 118 degrees, and angle B 40 degrees. Find angle C, sides b and c, and the area.

First find angle C.

$$C = 180° - (118° + 40°) = 180° - 158° = 22°.$$

Now find side b.

$$b = \frac{3.2 \times \sin 40°}{\sin 118°} = \frac{3.2 \times 0.64279}{0.88295} = 2.330 \text{ inches.}$$

Note, when finding sin 118° from the tables, that sin 118° = sin (180° — 118°) = sin 62° as explained in Chapter V.

Next, find side c.

$$c = \frac{3.2 \times \sin 22°}{\sin 118°} = \frac{3.2 \times 0.37461}{0.88295} = 1.358 \text{ inch.}$$

Finally,

$$\text{Area} = \frac{3.2 \times 2.33 \times \sin 22^\circ}{2} = 1.396 \text{ square inch.}$$

Example 3.—In Fig. 33, side $b = 0.3$ foot, angle $B = 35^\circ\ 40'$, and angle $C = 24^\circ\ 10'$. Find angle A, sides a and c, and the area.

$$A = 180^\circ - (35^\circ\ 40' + 24^\circ\ 10') = 180^\circ - 59^\circ\ 50' = 120^\circ\ 10'.$$

To find side a, use the rule already given, from which we get the formula below:

$$a = \frac{b \times \sin A}{\sin B} = \frac{0.3 \times \sin 120^\circ\ 10'}{\sin 35^\circ\ 40'} = \frac{0.3 \times 0.86457}{0.58307} = 0.445 \text{ foot.}$$

To find side c, use again the same rule, from which we then get:

$$c = \frac{b \times \sin C}{\sin B} = \frac{0.3 \times \sin 24^\circ\ 10'}{\sin 35^\circ\ 40'} = \frac{0.3 \times 0.40939}{0.58307} = 0.211 \text{ foot.}$$

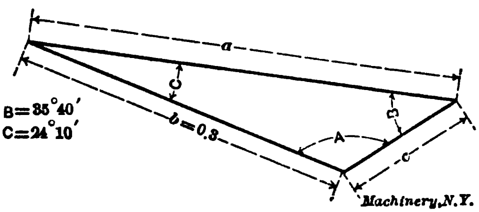


Fig. 33

Note that in this example the formulas for a and c have the same *form* as Formulas (4) and (5) on pages 36 and 37, but as the side b is the known side, instead of a, the side b is brought into the formula instead of a, and angle B instead of angle A. The formulas for a and c in this example are directly deduced from the rule on page 36, for finding the unknown sides.

To find the area, use Formula (6):

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{0.445 \times 0.3 \times \sin 24^\circ\ 10'}{2} =$$

$$\frac{0.445 \times 0.3 \times 0.40939}{2} = 0.027 \text{ square foot.}$$

Summary of Formulas

If the angles of a triangle are called A, B and C, and the sides opposite each of the angles, a, b and c, respectively, as shown in Fig. 30, then, if two angles and one side are known, the remaining angle, the two unknown sides and the area may be found by the formulas below:

$$A = 180^\circ - (B + C) \qquad (7)$$

$$B = 180^\circ - (A + C) \qquad (8)$$

$$C = 180^\circ - (A + B) \qquad (9)$$

$$a = \frac{b \times \sin A}{\sin B} \qquad b = \frac{a \times \sin B}{\sin A} \qquad c = \frac{b \times \sin C}{\sin B}$$

$$a = \frac{c \times \sin A}{\sin C} \qquad b = \frac{c \times \sin B}{\sin C} \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{b \times c \times \sin A}{2} = \frac{a \times c \times \sin B}{2}$$

2. Two Sides and the Included Angle Known

Assume that the sides a and b in Fig. 34 are 9 and 8 inches, respectively, as shown, and that the angle C formed by these two sides is 35 degrees. Find angles A and B, side c, and the area of the triangle.

The tangent of angle A is found by the following formula:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} \qquad (10)$$

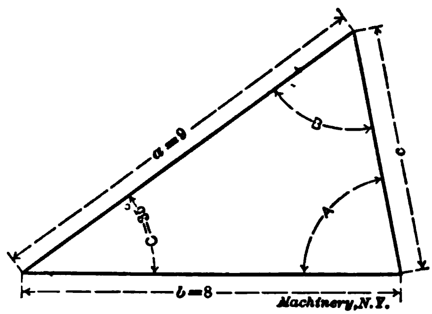


Fig. 34

If the given values of a, b and C are inserted in this formula, we have:

$$\tan A = \frac{9 \times \sin 35°}{8 - 9 \times \cos 35°} = \frac{9 \times 0.57358}{8 - 9 \times 0.81915} = \frac{5.16222}{0.62765} = 8.22468.$$

Having now obtained the tangent of angle $A = 8.22468$, we find from the tables that the angle equals 83° 4′.

Now when both angles A and C are known, angle B is found by Formula (8) already given:

$$B = 180° - (A + C) = 180° - (83° \; 4' + 35°) = 180° - 118° \; 4' = 61° \; 56'$$

Side c is found by Formula (5):

$$c = \frac{a \times \sin C}{\sin A} = \frac{9 \times \sin 35°}{\sin 83° \; 4'} = \frac{9 \times 0.57358}{0.99269} = 5.2 \text{ inches.}$$

The area is found by Formula (6):

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{9 \times 8 \times 0.57358}{2} = 20.649 \text{ square inches.}$$

All the required quantities of this triangle have now been found.

Example 1.—In Fig. 35, $a = 4$ inches, $b = 3$ inches, and $C = 20$ degrees. Find A, B, c, and the area.

According to Formula (10), we have:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{4 \times \sin 20°}{3 - 4 \times \cos 20°} = \frac{4 \times 0.34202}{3 - 4 \times 0.93969}$$
$$= \frac{1.36808}{3 - 3.75876}$$

It will be seen that in the denominator of the fraction above, the number to be subtracted from 3 is greater than 3; the numbers are

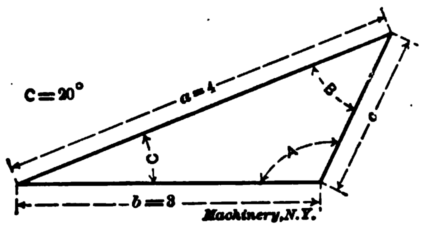


Fig. 35

therefore reversed as explained in Chapter III, 3 being subtracted from 3.75876, the remainder then being negative. Hence:

$$\tan A = \frac{1.36808}{3 - 3.75876} = \frac{1.36808}{-0.75876} = -1.80305$$

The final result is negative because a positive number (1.36808) is divided by a negative number (— 0.75876).

In Chapter V it is stated that the tangents of angles greater than 90 degrees and smaller than 180 degrees are negative. In an example in the same chapter is shown how to find an angle whose tangent is negative. Proceeding in the same manner, find in this case the value nearest to 1.80305 in the columns of tangents in the tables. It will be seen that the nearest value is 1.8028, which is the tangent of 60° 59′. As the tangent here is negative, angle A, however, is not 60° 59′, but equals 180° — 60° 59′ = 119° 1′.

Now angle B is found by the formula

$$B = 180° - (A + C) = 180° - (119° 1' + 20°) =$$
$$180° - 139° 1' = 40° 59'.$$

Side c and the area are now found by the same formulas and in the same manner as previously shown.

Example 2.—In Fig. 36, $a = 7$ feet, $b = 4$ feet, and $C = 121$ degrees. Find A, B, c and the area.

Proceeding as in the previous example we have

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{7 \times \sin 121^\circ}{4 - 7 \times \cos 121^\circ}$$

As explained in Chapter V:

$$\sin 121^\circ = \sin (180^\circ - 121^\circ) = \sin 59^\circ, \text{ and}$$
$$\cos 121^\circ = -\cos (180^\circ - 121^\circ) = -\cos 59^\circ.$$

Therefore

$$\tan A = \frac{7 \times \sin 121^\circ}{4 - 7 \times \cos 121^\circ} = \frac{7 \times \sin 59^\circ}{4 - 7 \times (-\cos 59^\circ)} =$$

$$\frac{7 \times 0.85717}{4 - 7 \times (-0.51504)} = \frac{6.00019}{4 - (-3.60528)} =$$

$$\frac{6.00019}{4 + 3.60528} = \frac{6.00019}{7.60528} = 0.78895.$$

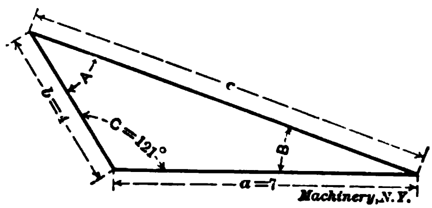


Fig. 36

The calculation with the negative number (— 0.51504) will become clear by comparing the processes above with the rules given in Chapter III. When multiplied by 7, the product 7 × (— 0.51504) becomes negative, and equals — 3.60528. As subtracting a negative quantity from a positive quantity is equal to adding the numerical value of the negative number we have:

$$4 - (-3.60528) = 4 + 3.60528 = 7.60528.$$

Having found $\tan A = 0.78895$, we find angle A from the tables: $A = 38^\circ 16'$.

Angle B, side c and the area are now found in the same way as previously explained.

Summary of Formulas

If the angles of a triangle are called A, B and C and the sides opposite each of the angles a, b and c, respectively, as shown in Fig. 34, then, if any two sides and the included angle are known, the other angles, the remaining side and the area may be found. One of the angles is first found by any of the formulas below:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} \qquad \tan A = \frac{a \times \sin B}{c - a \times \cos B}$$

$$\tan B = \frac{b \times \sin C}{a - b \times \cos C} \qquad \tan B = \frac{b \times \sin A}{c - b \times \cos A}$$

$$\tan C = \frac{c \times \sin B}{a - c \times \cos B} \qquad \tan C = \frac{c \times \sin A}{b - c \times \cos A}$$

The third angle, the remaining side, and the area are then found by using Formulas (4), (5), (6), (7), (8) and (9).

If the unknown angles are not required, but merely the unknown side of the triangle, the following formulas may be employed:

$$a = \sqrt{b^2 + c^2 - 2\,bc \times \cos A}$$
$$b = \sqrt{a^2 + c^2 - 2\,ac \times \cos B}$$
$$c = \sqrt{a^2 + b^2 - 2\,ab \times \cos C}$$

3. Two Sides and One of the Opposite Angles Known

When two sides and the angle opposite one of the given sides are known, two triangles can be drawn which have the sides the re-

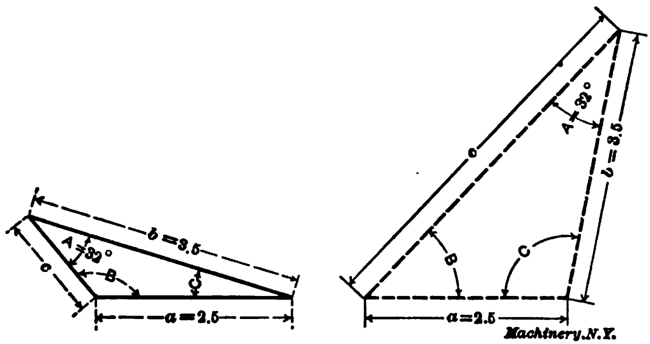


Fig. 37

quired length and the angle opposite one of the sides the required size. In Fig. 37 is shown a triangle in which side a is 2.5 inches, side b, 3.5 inches, and angle A, 32 degrees. Another triangle is shown by dotted lines in the same figure in which sides a and b have the same length as in the triangle drawn by full lines, and angle A opposite side a still remains 32 degrees; but it will be seen that in this triangle the angle B is very much smaller than in the triangle drawn by the full lines. In every case, therefore, when two sides and one of the opposite angles are given, the problem is capable of two solutions, there being two triangles which fill the given requirements. In one of these triangles, the unknown angles opposite a given side is greater than a right angle, and in one it is less than a right angle. When the triangle to be calculated is drawn to the correct shape, it is, therefore, possible to determine from the shape of the triangle which of the two solutions applies. When the triangle is not drawn to the required shape, both solutions must be found and applied to the practical problem requiring the solution of the triangle; it can then usually be determined which of the solutions applies to the practical problem in hand.

Example 1.—Assume that the sides a and b in Fig. 38 are 20 and 17 inches, respectively, as shown, and that angle A opposite the known side a is 61 degrees. Find angles B and C, side c, and the area of the triangle.

The angle B opposite the known side b may be found by the following rule: *The sine of the angle opposite one of the known sides equals the product of the side opposite this angle times the sine of the known angle, divided by the side opposite the known angle.*

From this rule we derive the following formula for the sine of angle B:

$$\sin B = \frac{b \times \sin A}{a} \qquad (11)$$

If we insert the known values for sides b and a and angle A in this formula we have:

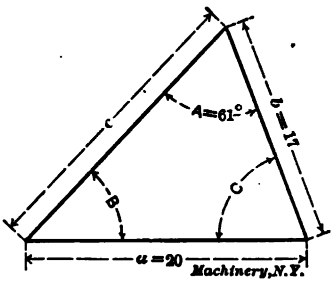


Fig. 38

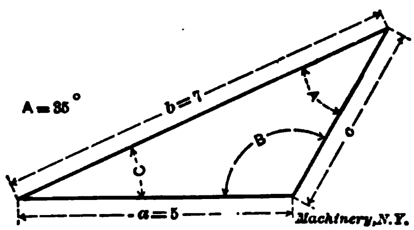


Fig. 39

$$\sin B = \frac{17 \times \sin 61°}{20} = \frac{17 \times 0.87462}{20} = 0.74343.$$

Having $\sin B = 0.74343$, we find from the tables that $B = 48° \ 1'$. As it is shown in Fig. 38 that angle B is less than a right angle, the solution found is the one which applies in this case.

Angle C is now found from Formula (9):

$$C = 180° - (A + B) = 180° - (61° + 48° \ 1') = 70° \ 59'.$$

Side c is found by Formula (5):

$$c = \frac{a \times \sin C}{\sin A} = \frac{20 \times \sin 70° \ 59'}{\sin 61°} = \frac{20 \times 0.94542}{0.87462} = 21.62 \text{ inches.}$$

The area is found by Formula (6):

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{20 \times 17 \times \sin 70° \ 59'}{2} = 160.72 \text{ square inches.}$$

All the required quantities of this triangle have now been found.

Example 2.—In Fig. 39, $a = 5$ inches, $b = 7$ inches, and $A = 35$ degrees. Find B, C, c and the area.

According to the rule and formula in the previous example:

$$\sin B = \frac{b \times \sin A}{a} = \frac{7 \times \sin 35°}{5} = \frac{7 \times 0.57358}{5} = 0.80301$$

Having $\sin B = 0.80301$, we find from the tables that $B = 53° 25'$. However, in the present case we see from the figure that B is greater than 90 degrees. The solution obtained is, therefore, not the solution applying to this case. It is explained in Chapter V that the sine of an angle also equals the sine of 180 degrees minus the angle. Therefore, 0.80301 is the sine not only of $53° 25'$, but also of $180° - 53° 25' = 126° 35'$. The value of angle B applying to the triangle shown in Fig. 39 is therefore $126° 35'$, because of the two values obtained this is the one which is greater than a right angle.

When angle B is found, angle C, side c and the area are found in the same manner as in Example 1.

Example 3.—In Fig. 40, $a = 2$ feet, $b = 3$ feet and $A = 30$ degrees. Find B, C, c and the area.

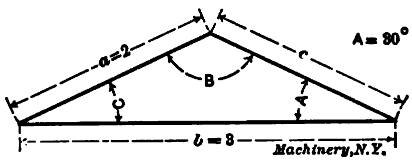


Fig. 40

The sine of angle B is found as in the previous example:

$$\sin B = \frac{b \times \sin A}{a} = \frac{3 \times \sin 30°}{2} = 0.75000.$$

Having $\sin B = 0.75000$, we find from the tables that $B = 48° 35'$. From Fig. 40 it is apparent, however, that B is greater than 90 degrees, and as 0.75000 is the sine not only of $48° 35'$, but also of $180° - 48° 35' = 131° 25'$, angle B in this case equals $131° 25'$.

When the angle B is found, angle C, side c and the area are found in the same manner as in Example 1.

Summary of Formulas

If the angles of a triangle are called A, B and C, and the sides opposite each of the angles a, b and c, respectively, as shown in Fig. 37; then if any two sides and one angle opposite one of the known sides are given, the other angles, the remaining side, and the area may be found. The angle opposite the other known side is first found by any of the formulas below:

$$\sin A = \frac{a \times \sin B}{b} \qquad \sin A = \frac{a \times \sin C}{c}$$

$$\sin B = \frac{b \times \sin A}{a} \qquad \sin B = \frac{b \times \sin C}{c}$$

$$\sin C = \frac{c \times \sin A}{a} \qquad \sin C = \frac{c \times \sin B}{b}$$

The third angle, the remaining side and the area are then found by using Formulas (4) to (9) inclusive.

4. Three Sides Known

Example 1.—In Fig. 41 the three sides a, b and c of the triangle are given; $a = 8$ inches, $b = 9$ inches and $c = 10$ inches. Find the angles A, B and C and the area.

Either of the angles can be found by the formulas given below:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c} \qquad (12)$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2 \times a \times c} \qquad (13)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2 \times a \times b} \qquad (14)$$

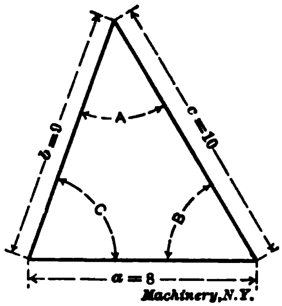


Fig. 41

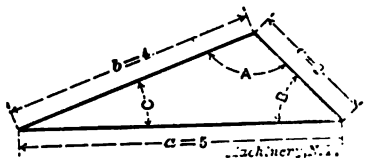


Fig. 42

If we insert the given lengths of the sides in the first of the formulas above we have:

$$\cos A = \frac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10} = \frac{9 \times 9 + 10 \times 10 - 8 \times 8}{2 \times 9 \times 10} = \frac{81 + 100 - 64}{180}$$

$$= \frac{117}{180} = 0.65000$$

Having $\cos A = 0.65000$ we find from the tables that angle $A = 49° 27'$.

Having found angle A, the easiest method for finding angle B is by Formula (11). From this formula we have:

$$\sin B = \frac{b \times \sin A}{a} = \frac{9 \times \sin 49° 27'}{8} = \frac{9 \times 0.75984}{8} = 0.85482$$

Having sin $B = 0.85482$, we find from the tables that $B = 58° 44'$.

Angle C is now found by Formula (9):

$C = 180° - (A + B) = 180° - (49° 27' + 58° 44') = 71° 49'$.

The area is finally found from Formula (6):

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{8 \times 9 \times \sin 71° 49'}{2} = \frac{8 \times 9 \times 0.95006}{2}$$

$= 34.20$ square inches.

Example 2.—In Fig. 42, $a = 5$ inches, $b = 4$ inches and $c = 2$ inches. Find the angles of the triangle.

Using Formula (12), given in Example 1, we have:

$$\cos A = \frac{4^2 + 2^2 - 5^2}{2 \times 4 \times 2} = \frac{16 + 4 - 25}{16} = \frac{20 - 25}{16}$$

It will be seen that in the numerator of the last fraction above, the number to be subtracted from 20 is greater than 20. The numbers are therefore reversed, as explained in Chapter III, 20 being subtracted from 25, the remainder then being negative. Hence:

$$\cos A = \frac{20 - 25}{16} = \frac{-5}{16} = -0.31250.$$

The final result is negative, because a negative number (-5) is divided by a positive number (16). In Chapter V it is stated that the cosines of angles greater than 90 degrees and smaller than 180 degrees are negative. In an example in the same chapter is shown how to find the angle whose tangent is negative; an angle whose cosine is negative is found in a similar manner: Find the value nearest to 0.31250 in the columns of cosines in the tables. It will be seen that the nearest value is 0.31261, which is the cosine of 71° 47'. As the cosine here is negative, angle A, however, is not 71° 47' but $= 180° - 71° 47' = 108° 13'$. Now angle B is found by the formula:

$$\sin B = \frac{b \times \sin A}{a} = \frac{4 \times \sin 108° 13'}{5}$$

As stated in Chapter V, $\sin 108° 13' = \sin (180° - 108° 13') = \sin 71° 47'$. Hence:

$$\sin B = \frac{4 \times \sin 71° 47'}{5} = \frac{4 \times 0.94988}{5} = 0.75990$$

and $B = 49° 27'$.

Finally, angle C is found by the formula:

$C = 180° - (A + B) = 180° - (108° 13' + 49° 27') = 22° 20'$.

CHAPTER X

SUMMARY OF FORMULAS FOR SOLUTION OF TRIANGLES

In the following will be given a summary of all the required formulas, and the methods of procedure for solving both right- and oblique-angled triangles.

Right-angled Triangles

In all the formulas for right-angled triangles reference is made to Fig. 43, in which the sides and angles are given the same names as in the formulas. Use the formulas in the order given.

1. When the hypotenuse and one of the sides forming the right angle are given, call the hypotenuse a and the known side b. Then:

$$c = \sqrt{a^2 - b^2} \qquad \sin B = \frac{b}{a} \qquad C = 90° - B$$

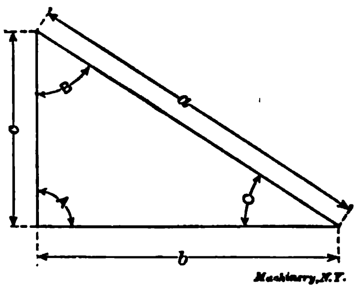


Fig. 43

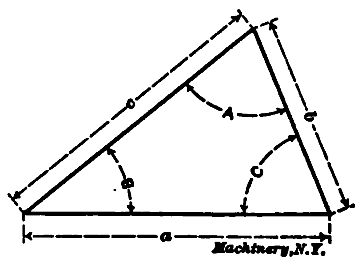


Fig. 44

2. When the two sides forming the right angle are given, call them b and c. Then:

$$a = \sqrt{b^2 + c^2} \qquad \tan B = \frac{b}{c} \qquad C = 90° - B$$

3. When the hypotenuse and one acute angle are given, call the hypotenuse a and the known angle B. Then:

$$c = a \times \cos B \qquad b = a \times \sin B \qquad C = 90° - B$$

4. When one acute angle and its adjacent side are given, call the angle B and the adjacent known side c. Then:

$$a = \frac{c}{\cos B} \qquad b = c \times \tan B \qquad C = 90° - B$$

5. When one acute angle and the side opposite it are given, call the angle B and the known opposite side b. Then:

$$a = \frac{b}{\sin B} \qquad c = b \times \cot B \qquad C = 90° - B$$

The area of all right-angled triangles equals the product of the sides forming the right angle divided by 2; or, referring to Fig. 43:

$$\text{Area} = \frac{b \times c}{2}$$

Oblique-angled Triangles

In all the formulas for oblique-angled triangles reference is made to Fig. 44, in which the sides and angles are given the same names as in the formulas. Use the formulas in the order given.

1. When two angles and one side are given, call the given side a, the angle opposite it A, and the other angle B. Then if A is known:

$$C = 180° - (A + B)$$

$$b = \frac{a \times \sin B}{\sin A}$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being as above.

2. When two sides and the included angle are given, call the given sides a and b and the given angle between them C. Then:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C}$$

$$B = 180° - (A + C)$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

3. When two sides and the angle opposite one of the sides are given, call the given angle A, the side opposite it a and the other given side b. Then:

$$\sin B = \frac{b \times \sin A}{a}$$

$$C = 180° - (A + B)$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

4. When the three sides of a triangle are given, call them a, b and c and the angles opposite them A, B and C, respectively. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$

$$\sin B = \frac{b \times \sin A}{a}$$

$$C = 180° - (A + B)$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

The cases given include all conditions where a solution of the triangle is possible. If all the angles are given, but none of the sides, the triangle may be of any size, but the three sides will be in exact proportion to each other. The formulas below give this relationship:

$$a : b = \sin A : \sin B$$
$$b : c = \sin B : \sin C$$
$$a : c = \sin A : \sin C$$

CHAPTER XI

THE USE OF LOGARITHMS IN SOLVING TRIANGLES

Before undertaking to study the use of logarithms for solving triangles, the student should thoroughly understand the use of logarithms in ordinary numerical examples, as explained in MACHINERY'S Reference Series No. 53, "The Use of Logarithms and Logarithmic Tables." When the use of logarithms in ordinary calculations is well understood, their application to trigonometric problems is very simple. It is merely a question of finding the logarithm for the function of the angle from the tables in Part II of this treatise, and carrying out the calculation in the same manner as with logarithms in general. The heavy-faced figures in the columns headed "Log." in the tables give these logarithms. A few explanatory remarks as to the method in which they are given, will, however, be necessary.

In all cases in these tables, the characteristic is given together with the mantissa. The complete logarithm of the functions, therefore, is found directly from the tables. As however, the values of the natural functions in the three first columns from the left in the tables are always less than 1, the characteristic would always be negative. In order to avoid this negative characteristic, the logarithm as given has had

10 added to its value, so that the actual value of the logarithm for cos 3 deg., for example, is 9.99940 — 10, as is evident if we remember that the logarithm of a number less than 1 must be negative. When using these logarithms in calculations with other logarithms, the calculations can be carried out exactly as explained in Reference Series No 53, if when writing down the logarithm taken from the tables we write $\bar{1}.99940$ for 9.99940, $\bar{2}.71940$ for 8.71940, $\bar{3}.30882$ for 7.30882, and so forth, changing the form to that which was made use of in the previous Reference book. It should be remembered, however, that this change refers only to the three first columns of logarithms. In the fourth column (headed Cot.), the logarithm is given in the exact form in which it is to be used. Of course, if it appears in the divisor of an expression, it must be transformed to its *negative* value, as explained on page 10, Reference Series No. 53.

A few examples will give a better idea of the methods to be followed. The student should carefully study these examples, until all the methods employed are perfectly clear to him. The logarithms of ordinary numbers are found from Reference Series No. 53, and the logarithms for functions of angles from Reference Series No. 55.

Example 1.—Find the area of a triangle where the lengths of two sides are 53 and 82 inches, and the angle between them is 30 degrees.

The area is found by the formula:

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{53 \times 82 \times \sin 30°}{2}$$

Proceed now to find the logarithms:

$$\begin{aligned} \log 53 &= 1.72428 \\ \log 82 &= 1.91381 \\ \log \sin 30° &= \bar{1}.69897 \\ -\log 2 &= \bar{1}.69897 \\ \hline &\ 3.03603 \end{aligned}$$

The logarithm of the area thus is 3.03603, and from the tables in Reference Series No. 53 we find by interpolation that the area then equals 1086.5 square inches.

Example 2.—Angles A and C and side a in a triangle are known. (See Fig. 44.) $A = 37° 42'$; $C = 68° 12'$; $a = 12$ inches. Find side c.

The formula for finding side c is:

$$c = \frac{a \times \sin C}{\sin A} = \frac{12 \times \sin 68° 12'}{\sin 37° 42'}$$

When finding the logarithms, note that as $\log \sin 37° 42' = \bar{1}.78642$, the negative value of the logarithm equals 0.21358.

$$\begin{aligned} \log 12 &= 1.07918 \\ \log \sin 68° 12' &= \bar{1}.96778 \\ -\log \sin 37° 42' &= 0.21358 \\ \hline &\ 1.26054 \end{aligned}$$

Thus $\log c = 1.26054$, and hence $c = 18.22$ inches.

Example 3.—Two sides of a triangle are 9 and 17 inches long. The angle included between them is 32 degrees. Find the angle opposite the side 9 inches long.

The formula by means of which the angle sought can be found is (see Chapter IX):

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{9 \times \sin 32^\circ}{17 - 9 \times \cos 32^\circ}$$

As only multiplications and divisions can be carried out by means of ordinary logarithms, the subtraction in the denominator must be made independently of logarithms; but logarithms can be used for the multiplications and divisions required. The first step will be to find the value of the denominator; we must then first find the product 9 × cos 32°.

$$\begin{aligned} \log 9 &= 0.95424 \\ \log \cos 32^\circ &= \bar{1}.92842 \\ \hline &\ 0.88266 \end{aligned}$$

Hence 9 × cos 32° = 7.6323, and 17 — 7.6323 = 9.3677. Therefore,

$$\tan A = \frac{9 \times \sin 32^\circ}{9.3677}$$

$$\begin{aligned} \log 9 &= 0.95424 \\ \log \sin 32^\circ &= \bar{1}.72421 \\ -\log 9.3677 &= \bar{1}.02837 \\ \hline &\ \bar{1}.70682 \end{aligned}$$

Log tan $A = \bar{1}.70682$, or as given in the tables 9.70682. Hence A = 26° 59′.

The columns "d" (difference) and "c. d." (common differences) in the tables, give the differences between consecutive logarithms for use in interpolation in cases where subdivisions of minutes are required. The method used is the same as that used when interpolating between logarithms of ordinary numbers. It is seldom, however, in ordinary shop calculations or in machine design, that finer divisions of the angle than minutes are required.

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Should, however, the values of these functions be required, they can easily be derived from the tables. The secant is found by dividing 1 by the cosine of the angle, and the cosecant is found by dividing 1 by the sine of the angle. Written as formulas, these rules would be:

$$\sec a = \frac{1}{\cos a}$$

$$\operatorname{cosec} a = \frac{1}{\sin a}$$

Example: Find the secant and cosecant of 15 degrees 42 minutes.

$$\sec 15^\circ\ 42' = \frac{1}{\cos 15^\circ\ 42'} = \frac{1}{0.96269} = 1.0387$$

$$\operatorname{cosec} 15^\circ\ 42' = \frac{1}{\sin 15^\circ\ 42'} = \frac{1}{0.27060} = 3.6955$$

The use of the tables has been fully explained in Part I, and it is, therefore, not necessary to give any further explanations here; if the use of the tables is not thoroughly understood, the explanatory matter found in Chapter V of Part I should be carefully studied.

0°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c. d.	Log. Cot	Nat.	
0	00000	—		1.00000	10.00000	00000	—		—	∞	60
1	029	6.46373	30103	000	0.00000	029	6.46373	30103	3.53627	3437.7	59
2	058	6.76476	17609	000	0.00000	058	6.76476	17609	3.23524	1718.9	58
3	087	6.94085	12494	000	0.00000	087	6.94085	12494	3.05915	1145.9	57
4	116	7.06579	9691	000	0.00000	116	7.06579	9691	2.93421	859.44	56
5	00145	7.16270	7918	1.00000	10.00000	00145	7.16270	7918	2.83730	687.55	55
6	175	7.24188	6694	000	0.00000	175	7.24188	6694	2.75812	572.96	54
7	204	7.30882	5800	000	0.00000	204	7.30882	5800	2.69118	491.11	53
8	233	7.36682	5115	000	0.00000	233	7.36682	5115	2.63318	429.72	52
9	262	7.41797	4576	000	0.00000	262	7.41797	4576	2.58203	381.97	51
10	00291	7.46373	4139	1.00000	10.00000	00291	7.46373	4139	2.53627	343.77	50
11	320	7.50512	3779	99999	0.00000	320	7.50512	3779	2.49488	312.52	49
12	349	7.54291	3476	999	0.00000	349	7.54291	3476	2.45709	286.48	48
13	378	7.57767	3218	999	0.00000	378	7.57767	3219	2.42233	264.44	47
14	407	7.60985	2997	999	0.00000	407	7.60986	2996	2.39014	245.55	46
15	00436	7.63982	2802	99999	10.00000	00436	7.63982	2803	2.36018	229.18	45
16	465	7.66784	2633	999	0.00000	465	7.66785	2633	2.33215	214.86	44
17	495	7.69417	2483	999	9.99999	495	7.69418	2482	2.30582	202.22	43
18	524	7.71900	2348	999	9.99999	524	7.71900	2348	2.28100	190.98	42
19	553	7.74248	2227	998	9.99999	553	7.74248	2228	2.25752	180.93	41
20	00582	7.76475	2119	99998	9.99999	00582	7.76476	2119	2.23524	171.89	40
21	611	7.78594	2021	998	9.99999	611	7.78595	2020	2.21405	163.70	39
22	640	7.80615	1930	998	9.99999	640	7.80615	1931	2.19385	156.26	38
23	669	7.82545	1848	998	9.99999	669	7.82546	1848	2.17454	149.47	37
24	698	7.84393	1773	998	9.99999	698	7.84394	1773	2.15606	143.24	36
25	00727	7.86166	1704	99997	9.99999	00727	7.86167	1704	2.13833	137.51	35
26	756	7.87870	1639	997	9.99999	756	7.87871	1639	2.12129	132.22	34
27	785	7.89509	1579	997	9.99999	785	7.89510	1579	2.10490	127.32	33
28	814	7.91088	1524	997	9.99999	815	7.91089	1524	2.08911	122.77	32
29	844	7.92612	1472	996	9.99998	844	7.92613	1473	2.07387	118.54	31
30	00873	7.94084	1424	99996	9.99998	00873	7.94086	1424	2.05914	114.59	30
31	902	7.95508	1379	996	9.99998	902	7.95510	1379	2.04490	110.89	29
32	931	7.96887	1336	996	9.99998	931	7.96889	1336	2.03111	107.43	28
33	960	7.98223	1297	995	9.99998	960	7.98225	1297	2.01775	104.17	27
34	989	7.99520	1259	995	9.99998	989	7.99522	1259	2.00478	101.11	26
35	01018	8.00779	1223	99995	9.99998	01018	8.00781	1223	1.99219	98.218	25
36	047	8.02002	1190	995	9.99998	047	8.02004	1190	1.97996	95.489	24
37	076	8.03192	1158	994	9.99997	076	8.03194	1159	1.96806	92.908	23
38	105	8.04350	1128	994	9.99997	105	8.04353	1128	1.95647	90.463	22
39	134	8.05478	1100	994	9.99997	135	8.05481	1100	1.94519	88.144	21
40	01164	8.06578	1072	99993	9.99997	01164	8.06581	1072	1.93419	85.940	20
41	193	8.07650	1046	993	9.99997	193	8.07653	1047	1.92347	83.844	19
42	222	8.08696	1022	993	9.99997	222	8.08700	1022	1.91300	81.847	18
43	251	8.09718	999	992	9.99997	251	8.09722	998	1.90278	79.943	17
44	280	8.10717	976	992	9.99996	280	8.10720	976	1.89280	78.126	16
45	01309	8.11693	954	99991	9.99996	01309	8.11696	955	1.88304	76.390	15
46	338	8.12647	934	991	9.99996	338	8.12651	934	1.87349	74.729	14
47	367	8.13581	914	991	9.99996	367	8.13585	915	1.86415	73.139	13
48	396	8.14495	896	990	9.99996	396	8.14500	895	1.85500	71.615	12
49	425	8.15391	877	990	9.99996	425	8.15395	878	1.84605	70.153	11
50	01454	8.16268	860	99989	9.99995	01455	8.16273	860	1.83727	68.750	10
51	483	8.17128	843	989	9.99995	484	8.17133	843	1.82867	67.402	9
52	513	8.17971	827	989	9.99995	513	8.17976	828	1.82024	66.105	8
53	542	8.18798	812	988	9.99995	542	8.18804	812	1.81196	64.858	7
54	571	8.19610	797	988	9.99995	571	8.19616	797	1.80384	63.657	6
55	01600	8.20407	782	99987	9.99994	01600	8.20413	782	1.79587	62.499	5
56	629	8.21189	769	987	9.99994	629	8.21195	769	1.78805	61.383	4
57	658	8.21958	755	986	9.99994	658	8.21964	756	1.78036	60.306	3
58	687	8.22713	743	986	9.99994	687	8.22720	742	1.77280	59.266	2
59	716	8.23456	730	985	9.99994	716	8.23462	730	1.76538	58.261	1
60	745	8.24186		985	9.99993	746	8.24192		1.75808	57.290	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c. d.	Log. Tan	Nat.	′

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1°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	01745	8.24186	717	99985	9.99993	01746	8.24192	718	1.75808	57.290	60
1	774	8.24903	706	984	9.99993	775	8.24910	706	1.75090	56.351	59
2	803	8.25609	695	984	9.99993	804	8.25616	696	1.74384	55.442	58
3	832	8.26304	684	983	9.99993	833	8.26312	684	1.73688	54.561	57
4	862	8.26988	673	983	9.99992	862	8.26996	673	1.73004	53.709	56
5	01891	8.27661	663	99982	9.99992	01891	8.27669	663	1.72331	52.882	55
6	920	8.28324	653	982	9.99992	920	8.28332	654	1.71668	.081	54
7	949	8.28977	644	981	9.99992	949	8.28986	643	1.71014	51.303	53
8	978	8.29621	634	980	9.99992	978	8.29629	634	1.70371	50.549	52
9	02007	8.30255	624	980	9.99991	02007	8.30263	625	1.69737	49.816	51
10	02036	8.30879	616	99979	9.99991	02036	8.30888	617	1.69112	49.104	50
11	065	8.31495	608	979	9.99991	066	8.31505	607	1.68495	48.412	49
12	094	8.32103	599	978	9.99990	095	8.32112	599	1.67888	47.740	48
13	123	8.32702	590	977	9.99990	124	8.32711	591	1.67289	.085	47
14	152	8.33292	583	977	9.99990	153	8.33302	584	1.66698	46.449	46
15	02181	8.33875	575	99976	9.99990	02182	8.33886	575	1.66114	45.829	45
16	211	8.34450	568	976	9.99989	211	8.34461	568	1.65539	.226	44
17	240	8.35018	560	975	9.99989	240	8.35029	561	1.64971	44.639	43
18	269	8.35578	553	974	9.99989	269	8.35590	553	1.64410	.066	42
19	298	8.36131	547	974	9.99989	298	8.36143	546	1.63857	43.508	41
20	02327	8.36678	539	99973	9.99988	02328	8.36689	540	1.63311	42.964	40
21	356	8.37217	533	972	9.99988	357	8.37229	533	1.62771	.433	39
22	385	8.37750	526	972	9.99988	386	8.37762	527	1.62238	41.916	38
23	414	8.38276	520	971	9.99987	415	8.38289	520	1.61711	.411	37
24	443	8.38796	514	970	9.99987	444	8.38809	514	1.61191	40.917	36
25	02472	8.39310	508	99969	9.99987	02473	8.39323	509	1.60677	40.436	35
26	501	8.39818	502	969	9.99986	502	8.39832	502	1.60168	39.965	34
27	530	8.40320	496	968	9.99986	531	8.40334	496	1.59666	.506	33
28	560	8.40816	491	967	9.99986	560	8.40830	491	1.59170	.057	32
29	589	8.41307	485	966	9.99985	589	8.41321	486	1.58679	38.618	31
30	02618	8.41792	480	99966	9.99985	02619	8.41807	480	1.58193	38.188	30
31	647	8.42272	474	965	9.99985	648	8.42287	475	1.57713	37.769	29
32	676	8.42746	470	964	9.99984	677	8.42762	470	1.57238	.358	28
33	705	8.43216	464	963	9.99984	706	8.43232	464	1.56768	36.956	27
34	734	8.43680	459	963	9.99984	735	8.43696	460	1.56304	.563	26
35	02763	8.44139	455	99962	9.99983	02764	8.44156	455	1.55844	36.178	25
36	792	8.44594	450	961	9.99983	793	8.44611	450	1.55389	35.801	24
37	821	8.45044	445	960	9.99983	822	8.45061	446	1.54939	.431	23
38	850	8.45489	441	959	9.99982	851	8.45507	441	1.54493	.070	22
39	879	8.45930	436	959	9.99982	881	8.45948	437	1.54052	34.715	21
40	02908	8.46366	433	99958	9.99982	02910	8.46385	432	1.53615	34.368	20
41	938	8.46799	427	957	9.99981	939	8.46817	428	1.53183	.027	19
42	967	8.47226	424	956	9.99981	968	8.47245	424	1.52755	33.694	18
43	996	8.47650	419	955	9.99981	997	8.47669	420	1.52331	.366	17
44	03025	8.48069	416	954	9.99980	03026	8.48089	416	1.51911	.045	16
45	03054	8.48485	411	99953	9.99980	03055	8.48505	412	1.51495	32.730	15
46	083	8.48896	408	952	9.99979	084	8.48917	408	1.51083	.421	14
47	112	8.49304	404	952	9.99979	114	8.49325	404	1.50675	.118	13
48	141	8.49708	400	951	9.99979	143	8.49729	401	1.50271	31.821	12
49	170	8.50108	396	950	9.99978	172	8.50130	397	1.49870	.528	11
50	03199	8.50504	393	99949	9.99978	03201	8.50527	393	1.49473	31.242	10
51	228	8.50897	390	948	9.99977	230	8.50920	390	1.49080	30.960	9
52	257	8.51287	386	947	9.99977	259	8.51310	386	1.48690	.683	8
53	286	8.51673	382	946	9.99977	288	8.51696	383	1.48304	.412	7
54	316	8.52055	379	945	9.99976	317	8.52079	380	1.47921	.145	6
55	03345	8.52434	376	99944	9.99976	03346	8.52459	376	1.47541	29.882	5
56	374	8.52810	373	943	9.99975	376	8.52835	373	1.47165	.624	4
57	403	8.53183	369	942	9.99975	405	8.53208	370	1.46792	.371	3
58	432	8.53552	367	941	9.99974	434	8.53578	367	1.46422	.122	2
59	461	8.53919	363	940	9.99974	463	8.53945	363	1.46055	28.877	1
60	490	8.54282		939	9.99974	492	8.54308		1.45692	.636	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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2°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	03490	8.54282	360	99939	9.99974	03492	8.54308	361	1.45692	28.636	60
1	519	8.54642	357	938	9.99973	521	8.54669	358	1.45331	.399	59
2	548	8.54999	355	937	9.99973	550	8.55027	355	1.44973	.166	58
3	577	8.55354	351	936	9.99972	579	8.55382	352	1.44618	27.937	57
4	606	8.55705	349	935	9.99972	609	8.55734	349	1.44266	.712	56
5	03635	8.56054	346	99934	9.99971	03638	8.56083	346	1.43917	27.490	55
6	664	8.56400	343	933	9.99971	667	8.56429	344	1.43571	.271	54
7	693	8.56743	341	932	9.99970	696	8.56773	341	1.43227	.057	53
8	723	8.57084	337	931	9.99970	725	8.57114	338	1.42886	26.845	52
9	752	8.57421	336	930	9.99969	754	8.57452	336	1.42548	.637	51
10	03781	8.57757	332	99929	9.99969	03783	8.57788	333	1.42212	26.432	50
11	810	8.58089	330	927	9.99968	812	8.58121	330	1.41879	.230	49
12	839	8.58419	328	926	9.99968	842	8.58451	328	1.41549	.031	48
13	868	8.58747	325	925	9.99967	871	8.58779	326	1.41221	25.835	47
14	897	8.59072	323	924	9.99967	900	8.59105	323	1.40895	.642	46
15	03926	8.59395	320	99923	9.99967	03929	8.59428	321	1.40572	25.452	45
16	955	8.59715	318	922	9.99966	958	8.59749	319	1.40251	.264	44
17	984	8.60033	316	921	9.99966	987	8.60068	316	1.39932	.080	43
18	04013	8.60349	313	919	9.99965	04016	8.60384	314	1.39616	24.898	42
19	042	8.60662	311	918	9.99964	046	8.60698	311	1.39302	.719	41
20	04071	8.60973	309	99917	9.99964	04075	8.61009	310	1.38991	24.542	40
21	100	8.61282	307	916	9.99963	104	8.61319	307	1.38681	.368	39
22	129	8.61589	305	915	9.99963	133	8.61626	305	1.38374	.196	38
23	159	8.61894	302	913	9.99962	162	8.61931	303	1.38069	.026	37
24	188	8.62196	301	912	9.99962	191	8.62234	301	1.37766	23.859	36
25	04217	8.62497	298	99911	9.99961	04220	8.62535	299	1.37465	23.695	35
26	246	8.62795	296	910	9.99961	250	8.62834	297	1.37166	.532	34
27	275	8.63091	294	909	9.99960	279	8.63131	295	1.36869	.372	33
28	304	8.63385	293	907	9.99960	308	8.63426	292	1.36574	.214	32
29	333	8.63678	290	906	9.99959	337	8.63718	291	1.36282	.058	31
30	04362	8.63968	288	99905	9.99959	04366	8.64009	289	1.35991	22.904	30
31	391	8.64256	287	904	9.99958	395	8.64298	287	1.35702	.752	29
32	420	8.64543	284	902	9.99958	424	8.64585	285	1.35415	.602	28
33	449	8.64827	283	901	9.99957	454	8.64870	284	1.35130	.454	27
34	478	8.65110	281	900	9.99956	483	8.65154	281	1.34846	.308	26
35	04507	8.65391	279	99898	9.99956	04512	8.65435	280	1.34565	22.164	25
36	536	8.65670	277	897	9.99955	541	8.65715	278	1.34285	.022	24
37	565	8.65947	276	896	9.99955	570	8.65993	276	1.34007	21.881	23
38	594	8.66223	274	894	9.99954	599	8.66269	274	1.33731	.743	22
39	623	8.66497	272	893	9.99954	628	8.66543	273	1.33457	.606	21
40	04653	8.66769	270	99892	9.99953	04658	8.66816	271	1.33184	21.470	20
41	682	8.67039	269	890	9.99952	687	8.67087	269	1.32913	.337	19
42	711	8.67308	267	889	9.99952	716	8.67356	268	1.32644	.205	18
43	740	8.67575	266	888	9.99951	745	8.67624	266	1.32376	.075	17
44	769	8.67841	263	886	9.99951	774	8.67890	264	1.32110	20.946	16
45	04798	8.68104	263	99885	9.99950	04803	8.68154	263	1.31846	20.819	15
46	827	8.68367	260	883	9.99949	833	8.68417	261	1.31583	.693	14
47	856	8.68627	259	882	9.99949	862	8.68678	260	1.31322	.569	13
48	885	8.68886	258	881	9.99948	891	8.68938	258	1.31062	.446	12
49	914	8.69144	256	879	9.99948	920	8.69196	257	1.30804	.325	11
50	04943	8.69400	254	99878	9.99947	04949	8.69453	255	1.30547	20.206	10
51	972	8.69654	253	876	9.99946	978	8.69708	254	1.30292	.087	9
52	05001	8.69907	252	875	9.99946	05007	8.69962	252	1.30038	19.970	8
53	030	8.70159	250	873	9.99945	037	8.70214	251	1.29786	.855	7
54	059	8.70409	249	872	9.99944	066	8.70465	249	1.29535	.740	6
55	05088	8.70658	247	99870	9.99944	05095	8.70714	248	1.29286	19.627	5
56	117	8.70905	246	869	9.99943	124	8.70962	246	1.29038	.516	4
57	146	8.71151	244	867	9.99942	153	8.71208	245	1.28792	.405	3
58	175	8.71395	243	866	9.99942	182	8.71453	244	1.28547	.296	2
59	205	8.71638	242	864	9.99941	212	8.71697	243	1.28303	.188	1
60	234	8.71880		863	9.99940	241	8.71940		1.28060	.081	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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3°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	05234	8.71880	240	99863	9.99940	05241	8.71940	241	1.28060	19.081	60
1	263	8.72120	239	861	9.99940	270	8.72181	239	1.27819	18.976	59
2	292	8.72359	238	860	9.99939	299	8.72420	239	1.27580	.871	58
3	321	8.72597	237	858	9.99938	328	8.72659	237	1.27341	.768	57
4	350	8.72834	235	857	9.99938	357	8.72896	236	1.27104	.666	56
5	05379	8.73069	234	99855	9.99937	05387	8.73132	234	1.26868	18.564	55
6	408	8.73303	232	854	9.99936	416	8.73366	234	1.26634	.464	54
7	437	8.73535	232	852	9.99936	445	8.73600	232	1.26400	.366	53
8	466	8.73767	230	851	9.99935	474	8.73832	231	1.26168	.268	52
9	495	8.73997	229	849	9.99934	503	8.74063	229	1.25937	.171	51
10	05524	8.74226	228	99847	9.99934	05533	8.74292	229	1.25708	18.075	50
11	553	8.74454	226	846	9.99933	562	8.74521	227	1.25479	17.980	49
12	582	8.74680	226	844	9.99932	591	8.74748	226	1.25252	.886	48
13	611	8.74906	224	842	9.99932	620	8.74974	225	1.25026	.793	47
14	640	8.75130	223	841	9.99931	649	8.75199	224	1.24801	.702	46
15	05669	8.75353	222	99839	9.99930	05678	8.75423	222	1.24577	17.611	45
16	698	8.75575	220	838	9.99929	708	8.75645	222	1.24355	.521	44
17	727	8.75795	220	836	9.99929	737	8.75867	220	1.24133	.431	43
18	756	8.76015	219	834	9.99928	766	8.76087	219	1.23913	.343	42
19	785	8.76234	217	833	9.99927	795	8.76306	219	1.23694	.256	41
20	05814	8.76451	216	99831	9.99926	05824	8.76525	217	1.23475	17.169	40
21	844	8.76667	216	829	9.99926	854	8.76742	216	1.23258	.084	39
22	873	8.76883	214	827	9.99925	883	8.76958	215	1.23042	16.999	38
23	902	8.77097	213	826	9.99924	912	8.77173	214	1.22827	.915	37
24	931	8.77310	212	824	9.99923	941	8.77387	213	1.22613	.832	36
25	05960	8.77522	211	99822	9.99923	05970	8.77600	211	1.22400	16.750	35
26	989	8.77733	210	821	9.99922	999	8.77811	211	1.22189	.668	34
27	06018	8.77943	209	819	9.99921	06029	8.78022	210	1.21978	.587	33
28	047	8.78152	208	817	9.99920	058	8.78232	209	1.21768	.507	32
29	076	8.78360	208	815	9.99920	087	8.78441	208	1.21559	.428	31
30	06105	8.78568	206	99813	9.99919	06116	8.78649	206	1.21351	16.350	30
31	134	8.78774	205	812	9.99918	145	8.78855	206	1.21145	.272	29
32	163	8.78979	204	810	9.99917	175	8.79061	205	1.20939	.195	28
33	192	8.79183	203	808	9.99917	204	8.79266	204	1.20734	.119	27
34	221	8.79386	202	806	9.99916	233	8.79470	203	1.20530	.043	26
35	06250	8.79588	201	99804	9.99915	06262	8.79673	202	1.20327	15.969	25
36	279	8.79789	201	803	9.99914	291	8.79875	201	1.20125	.895	24
37	308	8.79990	199	801	9.99913	321	8.80076	201	1.19924	.821	23
38	337	8.80189	199	799	9.99913	350	8.80277	199	1.19723	.748	22
39	366	8.80388	197	797	9.99912	379	8.80476	198	1.19524	.676	21
40	06395	8.80585	197	99795	9.99911	06408	8.80674	198	1.19326	15.605	20
41	424	8.80782	196	793	9.99910	438	8.80872	196	1.19128	.534	19
42	453	8.80978	195	792	9.99909	467	8.81068	196	1.18932	.464	18
43	482	8.81173	194	790	9.99909	496	8.81264	195	1.18736	.394	17
44	511	8.81367	193	788	9.99908	525	8.81459	194	1.18541	.325	16
45	06540	8.81560	192	99786	9.99907	06554	8.81653	193	1.18347	15.257	15
46	569	8.81752	192	784	9.99906	584	8.81846	192	1.18154	.189	14
47	598	8.81944	190	782	9.99905	613	8.82038	192	1.17962	.122	13
48	627	8.82134	190	780	9.99904	642	8.82230	190	1.17770	.056	12
49	656	8.82324	189	778	9.99904	671	8.82420	190	1.17580	14.990	11
50	06685	8.82513	188	99776	9.99903	06700	8.82610	189	1.17390	14.924	10
51	714	8.82701	187	774	9.99902	730	8.82799	188	1.17201	.860	9
52	743	8.82888	187	772	9.99901	759	8.82987	188	1.17013	.795	8
53	773	8.83075	186	770	9.99900	788	8.83175	186	1.16825	.732	7
54	802	8.83261	185	768	9.99899	817	8.83361	186	1.16639	.669	6
55	06831	8.83446	184	99766	9.99898	06847	8.83547	185	1.16453	14.606	5
56	860	8.83630	183	764	9.99898	876	8.83732	184	1.16268	.544	4
57	889	8.83813	183	762	9.99897	905	8.83916	184	1.16084	.482	3
58	918	8.83996	181	760	9.99896	934	8.84100	182	1.15900	.421	2
59	947	8.84177	181	758	9.99895	963	8.84282	182	1.15718	.361	1
60	976	8.84358		756	9.99894	993	8.84464		1.15536	.301	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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4°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	′
0	06976	8.84358	181	99756	9.99894	06993	8.84464	182	1.15536	14.301	60
1	07005	8.84539	179	754	9.99893	07022	8.84646	180	1.15354	.241	59
2	034	8.84718	179	752	9.99892	051	8.84826	180	1.15174	.182	58
3	063	8.84897	178	750	9.99891	080	8.85006	179	1.14994	.124	57
4	092	8.85075	177	748	9.99891	110	8.85185	178	1.14815	.065	56
5	07121	8.85252	177	99746	9.99890	07139	8.85363	177	1.14637	14.008	55
6	150	8.85429	176	744	9.99889	168	8.85540	177	1.14460	13.951	54
7	179	8.85605	175	742	9.99888	197	8.85717	176	1.14283	.894	53
8	208	8.85780	175	740	9.99887	227	8.85893	176	1.14107	.838	52
9	237	8.85955	173	738	9.99886	256	8.86069	174	1.13931	.782	51
10	07266	8.86128	173	99736	9.99885	07285	8.86243	174	1.13757	13.727	50
11	295	8.86301	173	734	9.99884	314	8.86417	174	1.13583	.672	49
12	324	8.86474	171	731	9.99883	344	8.86591	172	1.13409	.617	48
13	353	8.86645	171	729	9.99882	373	8.86763	172	1.13237	.563	47
14	382	8.86816	171	727	9.99881	402	8.86935	171	1.13065	.510	46
15	07411	8.86987	169	99725	9.99880	07431	8.87106	171	1.12894	13.457	45
16	440	8.87156	169	723	9.99879	461	8.87277	170	1.12723	.404	44
17	469	8.87325	169	721	9.99879	490	8.87447	169	1.12553	.352	43
18	498	8.87494	167	719	9.99878	519	8.87616	169	1.12384	.300	42
19	527	8.87661	168	716	9.99877	548	8.87785	168	1.12215	.248	41
20	07556	8.87829	166	99714	9.99876	07578	8.87953	167	1.12047	13.197	40
21	585	8.87995	166	712	9.99875	607	8.88120	167	1.11880	.146	39
22	614	8.88161	165	710	9.99874	636	8.88287	166	1.11713	.096	38
23	643	8.88326	164	708	9.99873	665	8.88453	165	1.11547	.046	37
24	672	8.88490	164	705	9.99872	695	8.88618	165	1.11382	12.996	36
25	07701	8.88654	163	99703	9.99871	07724	8.88783	165	1.11217	12.947	35
26	730	8.88817	163	701	9.99870	753	8.88948	163	1.11052	.898	34
27	759	8.88980	162	699	9.99869	782	8.89111	163	1.10889	.850	33
28	788	8.89142	162	696	9.99868	812	8.89274	163	1.10726	.801	32
29	817	8.89304	160	694	9.99867	841	8.89437	161	1.10563	.754	31
30	07846	8.89464	161	99692	9.99866	07870	8.89598	162	1.10402	12.706	30
31	875	8.89625	159	689	9.99865	899	8.89760	160	1.10240	.659	29
32	904	8.89784	159	687	9.99864	929	8.89920	160	1.10080	.612	28
33	933	8.89943	159	685	9.99863	958	8.90080	160	1.09920	.566	27
34	962	8.90102	158	683	9.99862	987	8.90240	159	1.09760	.520	26
35	07991	8.90260	157	99680	9.99861	08017	8.90399	158	1.09601	12.474	25
36	08020	8.90417	157	678	9.99860	046	8.90557	158	1.09443	.429	24
37	049	8.90574	156	676	9.99859	075	8.90715	157	1.09285	.384	23
38	078	8.90730	155	673	9.99858	104	8.90872	157	1.09128	.339	22
39	107	8.90885	155	671	9.99857	134	8.91029	156	1.08971	.295	21
40	08136	8.91040	155	99668	9.99856	08163	8.91185	155	1.08815	12.251	20
41	165	8.91195	154	666	9.99855	192	8.91340	155	1.08660	.207	19
42	194	8.91349	153	664	9.99854	221	8.91495	155	1.08505	.163	18
43	223	8.91502	153	661	9.99853	251	8.91650	153	1.08350	.120	17
44	252	8.91655	152	659	9.99852	280	8.91803	154	1.08197	.077	16
45	08281	8.91807	152	99657	9.99851	08309	8.91957	153	1.08043	12.035	15
46	310	8.91959	151	654	9.99850	339	8.92110	152	1.07890	11.992	14
47	339	8.92110	151	652	9.99848	368	8.92262	152	1.07738	.950	13
48	368	8.92261	150	649	9.99847	397	8.92414	151	1.07586	.909	12
49	397	8.92411	150	647	9.99846	427	8.92565	151	1.07435	.867	11
50	08426	8.92561	149	99644	9.99845	08456	8.92716	150	1.07284	11.826	10
51	455	8.92710	149	642	9.99844	485	8.92866	150	1.07134	.785	9
52	484	8.92859	148	639	9.99843	514	8.93016	149	1.06984	.745	8
53	513	8.93007	147	637	9.99842	544	8.93165	148	1.06835	.705	7
54	542	8.93154	147	635	9.99841	573	8.93313	149	1.06687	.664	6
55	08571	8.93301	147	99632	9.99840	08602	8.93462	147	1.06538	11.625	5
56	600	8.93448	146	630	9.99839	632	8.93609	147	1.06391	.585	4
57	629	8.93594	146	627	9.99838	661	8.93756	147	1.06244	.546	3
58	658	8.93740	145	625	9.99837	690	8.93903	146	1.06097	.507	2
59	687	8.93885	145	622	9.99836	720	8.94049	146	1.05951	.468	1
60	716	8.94030		619	9.99834	749	8.94195		1.05805	.430	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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5°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	08716	8.94030	144	99619	9.99834	08749	8.94195	145	1.05805	11.430	60
1	745	8.94174	143	617	9.99833	778	8.94340	145	1.05660	.392	59
2	774	8.94317	144	614	9.99832	807	8.94485	145	1.05515	.354	58
3	803	8.94461	142	612	9.99831	837	8.94630	143	1.05370	.316	57
4	831	8.94603	143	609	9.99830	866	8.94773	144	1.05227	.279	56
5	08860	8.94746	141	99607	9.99829	08895	8.94917	143	1.05083	11.242	55
6	889	8.94887	142	604	9.99828	925	8.95060	142	1.04940	.205	54
7	918	8.95029	141	602	9.99827	954	8.95202	142	1.04798	.168	53
8	947	8.95170	140	599	9.99825	983	8.95344	142	1.04656	.132	52
9	976	8.95310	140	596	9.99824	09013	8.95486	141	1.04514	.095	51
10	09005	8.95450	139	99594	9.99823	09042	8.95627	140	1.04373	11.059	50
11	034	8.95589	139	591	9.99822	071	8.95767	141	1.04233	.024	49
12	063	8.95728	139	588	9.99821	101	8.95908	139	1.04092	10.988	48
13	092	8.95867	138	586	9.99820	130	8.96047	140	1.03953	.953	47
14	121	8.96005	138	583	9.99819	159	8.96187	138	1.03813	.918	46
15	09150	8.96143	137	99580	9.99817	09189	8.96325	139	1.03675	10.883	45
16	179	8.96280	137	578	9.99816	218	8.96464	138	1.03536	.848	44
17	208	8.96417	136	575	9.99815	247	8.96602	137	1.03398	.814	43
18	237	8.96553	136	572	9.99814	277	8.96739	138	1.03261	.780	42
19	266	8.96689	136	570	9.99813	306	8.96877	136	1.03123	.746	41
20	09295	8.96825	135	99567	9.99812	09335	8.97013	137	1.02987	10.712	40
21	324	8.96960	135	564	9.99810	365	8.97150	135	1.02850	.678	39
22	353	8.97095	134	562	9.99809	394	8.97285	136	1.02715	.645	38
23	382	8.97229	134	559	9.99808	423	8.97421	135	1.02579	.612	37
24	411	8.97363	133	556	9.99807	453	8.97556	135	1.02444	.579	36
25	09440	8.97496	133	99553	9.99806	09482	8.97691	134	1.02309	10.546	35
26	469	8.97629	133	551	9.99804	511	8.97825	134	1.02175	.514	34
27	498	8.97762	132	548	9.99803	541	8.97959	133	1.02041	.481	33
28	527	8.97894	132	545	9.99802	570	8.98092	133	1.01908	.449	32
29	556	8.98026	131	542	9.99801	600	8.98225	133	1.01775	.417	31
30	09585	8.98157	131	99540	9.99800	09629	8.98358	132	1.01642	10.385	30
31	614	8.98288	131	537	9.99798	658	8.98490	132	1.01510	.354	29
32	642	8.98419	130	534	9.99797	688	8.98622	131	1.01378	.322	28
33	671	8.98549	130	531	9.99796	717	8.98753	131	1.01247	.291	27
34	700	8.98679	129	528	9.99795	746	8.98884	131	1.01116	.260	26
35	09729	8.98808	129	99526	9.99793	09776	8.99015	130	1.00985	10.229	25
36	758	8.98937	129	523	9.99792	805	8.99145	130	1.00855	.199	24
37	787	8.99066	128	520	9.99791	834	8.99275	130	1.00725	.168	23
38	816	8.99194	128	517	9.99790	864	8.99405	129	1.00595	.138	22
39	845	8.99322	128	514	9.99788	893	8.99534	128	1.00466	.108	21
40	09874	8.99450	127	99511	9.99787	09923	8.99662	129	1.00338	10.078	20
41	903	8.99577	127	508	9.99786	952	8.99791	128	1.00209	.048	19
42	932	8.99704	126	506	9.99785	981	8.99919	127	1.00081	.019	18
43	961	8.99830	126	503	9.99783	10011	9.00046	128	0.99954	9.9893	17
44	990	8.99956	126	500	9.99782	040	9.00174	127	0.99826	601	16
45	10019	9.00082	125	99497	9.99781	10069	9.00301	126	0.99699	9.9310	15
46	048	9.00207	125	494	9.99780	099	9.00427	126	0.99573	021	14
47	077	9.00332	124	491	9.99778	128	9.00553	126	0.99447	9.8734	13
48	106	9.00456	125	488	9.99777	158	9.00679	126	0.99321	448	12
49	135	9.00581	123	485	9.99776	187	9.00805	125	0.99195	164	11
50	10164	9.00704	124	99482	9.99775	10216	9.00930	125	0.99070	9.7882	10
51	192	9.00828	123	479	9.99773	246	9.01055	124	0.98945	601	9
52	221	9.00951	123	476	9.99772	275	9.01179	124	0.98821	322	8
53	250	9.01074	122	473	9.99771	305	9.01303	124	0.98697	044	7
54	279	9.01196	122	470	9.99769	334	9.01427	123	0.98573	9.6768	6
55	10308	9.01318	122	99467	9.99768	10363	9.01550	123	0.98450	9.6493	5
56	337	9.01440	121	464	9.99767	393	9.01673	123	0.98327	220	4
57	366	9.01561	121	461	9.99765	422	9.01796	122	0.98204	9.5949	3
58	395	9.01682	121	458	9.99764	452	9.01918	122	0.98082	679	2
59	424	9.01803	120	455	9.99763	481	9.02040	122	0.97960	411	1
60	453	9.01923		452	9.99761	510	9.02162		0.97838	144	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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6°

′	Nat. Sin	Sin Log.	d.	Nat. Cos	Cos Log.	Nat. Tan	Tan Log.	c.d.	Cot Log.	Cot Nat.	
0	10453	9.01923	120	99452	9.99761	10510	9.02162	121	0.97838	9.5144	60
1	482	9.02043	120	449	9.99760	540	9.02283	121	0.97717	9.4878	59
2	511	9.02163	120	446	9.99759	569	9.02404	121	0.97596	614	58
3	540	9.02283	119	443	9.99757	599	9.02525	120	0.97475	352	57
4	569	9.02402	118	440	9.99756	628	9.02645	121	0.97355	090	56
5	10597	9.02520	119	99437	9.99755	10657	9.02766	119	0.97234	9.3831	55
6	626	9.02639	118	434	9.99753	687	9.02885	120	0.97115	572	54
7	655	9.02757	117	431	9.99752	716	9.03005	119	0.96995	315	53
8	684	9.02874	118	428	9.99751	746	9.03124	118	0.96876	060	52
9	713	9.02992	117	424	9.99749	775	9.03242	119	0.96758	9.2806	51
10	10742	9.03109	117	99421	9.99748	10805	9.03361	118	0.96639	9.2553	50
11	771	9.03226	116	418	9.99747	834	9.03479	118	0.96521	302	49
12	800	9.03342	116	415	9.99745	863	9.03597	117	0.96403	052	48
13	829	9.03458	116	412	9.99744	893	9.03714	118	0.96286	9.1803	47
14	858	9.03574	116	409	9.99742	922	9.03832	116	0.96168	555	46
15	10887	9.03690	115	99406	9.99741	10952	9.03948	117	0.96052	9.1309	45
16	916	9.03805	115	402	9.99740	981	9.04065	116	0.95935	065	44
17	945	9.03920	114	399	9.99738	11011	9.04181	116	0.95819	9.0821	43
18	973	9.04034	115	396	9.99737	040	9.04297	116	0.95703	579	42
19	11002	9.04149	113	393	9.99736	070	9.04413	115	0.95587	338	41
20	11031	9.04262	114	99390	9.99734	11099	9.04528	115	0.95472	9.0098	40
21	060	9.04376	114	386	9.99733	128	9.04643	115	0.95357	8.9860	39
22	089	9.04490	113	383	9.99731	158	9.04758	115	0.95242	623	38
23	118	9.04603	112	380	9.99730	187	9.04873	114	0.95127	387	37
24	147	9.04715	113	377	9.99728	217	9.04987	114	0.95013	152	36
25	11176	9.04828	112	99374	9.99727	11246	9.05101	113	0.94899	8.8919	35
26	205	9.04940	112	370	9.99726	276	9.05214	114	0.94786	686	34
27	234	9.05052	112	367	9.99724	305	9.05328	113	0.94672	455	33
28	263	9.05164	111	364	9.99723	335	9.05441	112	0.94559	225	32
29	291	9.05275	111	360	9.99721	364	9.05553	113	0.94447	8.7996	31
30	11320	9.05386	111	99357	9.99720	11394	9.05666	112	0.94334	8.7769	30
31	349	9.05497	110	354	9.99718	423	9.05778	112	0.94222	542	29
32	378	9.05607	110	351	9.99717	452	9.05890	112	0.94110	317	28
33	407	9.05717	110	347	9.99716	482	9.06002	111	0.93998	093	27
34	436	9.05827	110	344	9.99714	511	9.06113	111	0.93887	8.6870	26
35	11465	9.05937	109	99341	9.99713	11541	9.06224	111	0.93776	8.6648	25
36	494	9.06046	109	337	9.99711	570	9.06335	110	0.93665	427	24
37	523	9.06155	109	334	9.99710	600	9.06445	111	0.93555	208	23
38	552	9.06264	108	331	9.99708	629	9.06556	110	0.93444	8.5989	22
39	580	9.06372	109	327	9.99707	659	9.06666	109	0.93334	772	21
40	11609	9.06481	108	99324	9.99705	11688	9.06775	110	0.93225	8.5555	20
41	638	9.06589	107	320	9.99704	718	9.06885	109	0.93115	340	19
42	667	9.06696	108	317	9.99702	747	9.06994	109	0.93006	126	18
43	696	9.06804	107	314	9.99701	777	9.07103	108	0.92897	8.4913	17
44	725	9.06911	107	310	9.99699	806	9.07211	109	0.92789	701	16
45	11754	9.07018	106	99307	9.99698	11836	9.07320	108	0.92680	8.4490	15
46	783	9.07124	107	303	9.99696	865	9.07428	108	0.92572	280	14
47	812	9.07231	106	300	9.99695	895	9.07536	107	0.92464	071	13
48	840	9.07337	105	297	9.99693	924	9.07643	108	0.92357	8.3863	12
49	869	9.07442	106	293	9.99692	954	9.07751	107	0.92249	656	11
50	11898	9.07548	105	99290	9.99690	11983	9.07858	106	0.92142	8.3450	10
51	927	9.07653	105	286	9.99689	12013	9.07964	107	0.92036	245	9
52	956	9.07758	105	283	9.99687	042	9.08071	106	0.91929	041	8
53	985	9.07863	105	279	9.99686	072	9.08177	106	0.91823	8.2838	7
54	12014	9.07968	104	276	9.99684	101	9.08283	106	0.91717	636	6
55	12043	9.08072	104	99272	9.99683	12131	9.08389	106	0.91611	8.2434	5
56	071	9.08176	104	269	9.99681	160	9.08495	105	0.91505	234	4
57	100	9.08280	103	265	9.99680	190	9.08600	105	0.91400	035	3
58	129	9.08383	103	262	9.99678	219	9.08705	105	0.91295	8.1837	2
59	158	9.08486	103	258	9.99677	249	9.08810	104	0.91190	640	1
60	187	9.08589		255	9.99675	278	9.08914		0.91086	443	0
	Nat. Cos	Cos Log.	d.	Nat. Sin	Sin Log.	Nat. Cot	Cot Log.	c.d.	Tan Log.	Tan Nat.	′

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7°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	12187	9.08589	103	99255	9.99675	12278	9.08914	105	0.91086	8.1443	60
1	216	9.08692	103	251	9.99674	308	9.09019	104	0.90981	248	59
2	245	9.08795	102	248	9.99672	338	9.09123	104	0.90877	054	58
3	274	9.08897	102	244	9.99670	367	9.09227	103	0.90773	8.0860	57
4	302	9.08999	102	240	9.99669	397	9.09330	104	0.90670	667	56
5	12331	9.09101	101	99237	9.99667	12426	9.09434	103	0.90566	8.0476	55
6	360	9.09202	102	233	9.99666	456	9.09537	103	0.90463	285	54
7	389	9.09304	101	230	9.99664	485	9.09640	102	0.90360	095	53
8	418	9.09405	101	226	9.99663	515	9.09742	103	0.90258	7.9906	52
9	447	9.09506	100	222	9.99661	544	9.09845	102	0.90155	718	51
10	12476	9.09606	101	99219	9.99659	12574	9.09947	102	0.90053	7.9530	50
11	504	9.09707	100	215	9.99658	603	9.10049	101	0.89951	344	49
12	533	9.09807	100	211	9.99656	633	9.10150	102	0.89850	158	48
13	562	9.09907	99	208	9.99655	662	9.10252	101	0.89748	7.8973	47
14	591	9.10006	100	204	9.99653	692	9.10353	101	0.89647	789	46
15	12620	9.10106	99	99200	9.99651	12722	9.10454	101	0.89546	7.8606	45
16	649	9.10205	99	197	9.99650	751	9.10555	101	0.89445	424	44
17	678	9.10304	98	193	9.99648	781	9.10656	100	0.89344	243	43
18	706	9.10402	99	189	9.99647	810	9.10756	100	0.89244	062	42
19	735	9.10501	98	186	9.99645	840	9.10856	100	0.89144	7.7882	41
20	12764	9.10599	98	99182	9.99643	12869	9.10956	100	0.89044	7.7704	40
21	793	9.10697	98	178	9.99642	899	9.11056	99	0.88944	525	39
22	822	9.10795	98	175	9.99640	929	9.11155	99	0.88845	348	38
23	851	9.10893	97	171	9.99638	958	9.11254	99	0.88746	171	37
24	880	9.10990	97	167	9.99637	988	9.11353	99	0.88647	7.6996	36
25	12908	9.11087	97	99163	9.99635	13017	9.11452	99	0.88548	7.6821	35
26	937	9.11184	97	160	9.99633	047	9.11551	98	0.88449	647	34
27	966	9.11281	96	156	9.99632	076	9.11649	98	0.88351	473	33
28	995	9.11377	97	152	9.99630	106	9.11747	98	0.88253	301	32
29	13024	9.11474	96	148	9.99629	136	9.11845	98	0.88155	129	31
30	13053	9.11570	96	99144	9.99627	13165	9.11943	97	0.88057	7.5958	30
31	081	9.11666	95	141	9.99625	195	9.12040	98	0.87960	787	29
32	110	9.11761	96	137	9.99624	224	9.12138	97	0.87862	618	28
33	139	9.11857	95	133	9.99622	254	9.12235	97	0.87765	449	27
34	168	9.11952	95	129	9.99620	284	9.12332	96	0.87668	281	26
35	13197	9.12047	95	99125	9.99618	13313	9.12428	97	0.87572	7.5113	25
36	226	9.12142	94	122	9.99617	343	9.12525	96	0.87475	7.4947	24
37	254	9.12236	95	118	9.99615	372	9.12621	96	0.87379	781	23
38	283	9.12331	94	114	9.99613	402	9.12717	96	0.87283	615	22
39	312	9.12425	94	110	9.99612	432	9.12813	96	0.87187	451	21
40	13341	9.12519	93	99106	9.99610	13461	9.12909	95	0.87091	7.4287	20
41	370	9.12612	94	102	9.99608	491	9.13004	95	0.86996	124	19
42	399	9.12706	93	098	9.99607	521	9.13099	95	0.86901	7.3962	18
43	427	9.12799	93	094	9.99605	550	9.13194	95	0.86806	800	17
44	456	9.12892	93	091	9.99603	580	9.13289	95	0.86711	639	16
45	13485	9.12985	93	99087	9.99601	13609	9.13384	94	0.86616	7.3479	15
46	514	9.13078	93	083	9.99600	639	9.13478	95	0.86522	319	14
47	543	9.13171	92	079	9.99598	669	9.13573	94	0.86427	160	13
48	572	9.13263	92	075	9.99596	698	9.13667	94	0.86333	002	12
49	600	9.13355	92	071	9.99595	728	9.13761	93	0.86239	7.2844	11
50	13629	9.13447	92	99067	9.99593	13758	9.13854	94	0.86146	7.2687	10
51	658	9.13539	91	063	9.99591	787	9.13948	93	0.86052	531	9
52	687	9.13630	92	059	9.99589	817	9.14041	93	0.85959	375	8
53	716	9.13722	91	055	9.99588	846	9.14134	93	0.85866	220	7
54	744	9.13813	91	051	9.99586	876	9.14227	93	0.85773	066	6
55	13773	9.13904	90	99047	9.99584	13906	9.14320	92	0.85680	7.1912	5
56	802	9.13994	91	043	9.99582	935	9.14412	92	0.85588	759	4
57	831	9.14085	90	039	9.99581	965	9.14504	93	0.85496	607	3
58	860	9.14175	91	035	9.99579	995	9.14597	91	0.85403	455	2
59	889	9.14266	90	031	9.99577	14024	9.14688	92	0.85312	304	1
60	917	9.14356		027	9.99575	054	9.14780		0.85220	154	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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8°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	13917	9.14356	89	99027	9.99575	14054	9.14780	92	0.85220	7.1154	60
1	946	9.14445	90	023	9.99574	084	9.14872	91	0.85128	004	59
2	975	9.14535	89	019	9.99572	113	9.14963	91	0.85037	7.0855	58
3	14004	9.14624	90	015	9.99570	143	9.15054	91	0.84946	706	57
4	033	9.14714	89	011	9.99568	173	9.15145	91	0.84855	558	56
5	14061	9.14803	88	99006	9.99566	14202	9.15236	91	0.84764	7.0410	55
6	090	9.14891	89	002	9.99565	232	9.15327	90	0.84673	264	54
7	119	9.14980	89	98998	9.99563	262	9.15417	91	0.84583	117	53
8	148	9.15069	88	994	9.99561	291	9.15508	90	0.84492	6.9972	52
9	177	9.15157	88	990	9.99559	321	9.15598	90	0.84402	827	51
10	14205	9.15245	88	98986	9.99557	14351	9.15688	89	0.84312	6.9682	50
11	234	9.15333	88	982	9.99556	381	9.15777	90	0.84223	538	49
12	263	9.15421	87	978	9.99554	410	9.15867	89	0.84133	395	48
13	292	9.15508	88	973	9.99552	440	9.15956	90	0.84044	252	47
14	320	9.15596	87	969	9.99550	470	9.16046	89	0.83954	110	46
15	14349	9.15683	87	98965	9.99548	14499	9.16135	89	0.83865	6.8969	45
16	378	9.15770	87	961	9.99546	529	9.16224	88	0.83776	828	44
17	407	9.15857	87	957	9.99545	559	9.16312	89	0.83688	687	43
18	436	9.15944	86	953	9.99543	588	9.16401	88	0.83599	548	42
19	464	9.16030	86	948	9.99541	618	9.16489	88	0.83511	408	41
20	14493	9.16116	87	98944	9.99539	14648	9.16577	88	0.83423	6.8269	40
21	522	9.16203	86	940	9.99537	678	9.16665	88	0.83335	131	39
22	551	9.16289	85	936	9.99535	707	9.16753	88	0.83247	6.7994	38
23	580	9.16374	86	931	9.99533	737	9.16841	87	0.83159	856	37
24	608	9.16460	85	927	9.99532	767	9.16928	88	0.83072	720	36
25	14637	9.16545	86	98923	9.99530	14796	9.17016	87	0.82984	6.7584	35
26	666	9.16631	85	919	9.99528	826	9.17103	87	0.82897	448	34
27	695	9.16716	85	914	9.99526	856	9.17190	87	0.82810	313	33
28	723	9.16801	85	910	9.99524	886	9.17277	86	0.82723	179	32
29	752	9.16886	84	906	9.99522	915	9.17363	87	0.82637	045	31
30	14781	9.16970	85	98902	9.99520	14945	9.17450	86	0.82550	6.6912	30
31	810	9.17055	84	897	9.99518	975	9.17536	86	0.82464	779	29
32	838	9.17139	84	893	9.99517	15005	9.17622	86	0.82378	646	28
33	867	9.17223	84	889	9.99515	034	9.17708	86	0.82292	514	27
34	896	9.17307	84	884	9.99513	064	9.17794	86	0.82206	383	26
35	14925	9.17391	83	98880	9.99511	15094	9.17880	85	0.82120	6.6252	25
36	954	9.17474	84	876	9.99509	124	9.17965	86	0.82035	122	24
37	982	9.17558	83	871	9.99507	153	9.18051	85	0.81949	6.5992	23
38	15011	9.17641	83	867	9.99505	183	9.18136	85	0.81864	863	22
39	040	9.17724	83	863	9.99503	213	9.18221	85	0.81779	734	21
40	15069	9.17807	83	98858	9.99501	15243	9.18306	85	0.81694	6.5606	20
41	097	9.17890	83	854	9.99499	272	9.18391	84	0.81609	478	19
42	126	9.17973	82	849	9.99497	302	9.18475	85	0.81525	350	18
43	155	9.18055	82	845	9.99495	332	9.18560	84	0.81440	223	17
44	184	9.18137	83	841	9.99494	362	9.18644	84	0.81356	097	16
45	15212	9.18220	82	98836	9.99492	15391	9.18728	84	0.81272	6.4971	15
46	241	9.18302	81	832	9.99490	421	9.18812	84	0.81188	846	14
47	270	9.18383	82	827	9.99488	451	9.18896	83	0.81104	721	13
48	299	9.18465	82	823	9.99486	481	9.18979	84	0.81021	596	12
49	327	9.18547	81	818	9.99484	511	9.19063	83	0.80937	472	11
50	15356	9.18628	81	98814	9.99482	15540	9.19146	83	0.80854	6.4348	10
51	385	9.18709	81	809	9.99480	570	9.19229	83	0.80771	225	9
52	414	9.18790	81	805	9.99478	600	9.19312	83	0.80688	103	8
53	442	9.18871	81	800	9.99476	630	9.19395	83	0.80605	6.3980	7
54	471	9.18952	81	796	9.99474	660	9.19478	83	0.80522	859	6
55	15500	9.19033	80	98791	9.99472	15689	9.19561	82	0.80439	6.3737	5
56	529	9.19113	80	787	9.99470	719	9.19643	82	0.80357	617	4
57	557	9.19193	80	782	9.99468	749	9.19725	82	0.80275	496	3
58	586	9.19273	80	778	9.99466	779	9.19807	82	0.80193	376	2
59	615	9.19353	80	773	9.99464	809	9.19889	82	0.80111	257	1
60	643	9.19433		769	9.99462	838	9.19971		0.80029	138	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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9°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	15643	9.19433	80	98769	9.99462	15838	9.19971	82	0.80029	6.3138	60
1	672	9.19513	79	764	9.99460	868	9.20053	81	0.79947	019	59
2	701	9.19592	80	760	9.99458	898	9.20134	82	0.79866	6.2901	58
3	730	9.19672	79	755	9.99456	928	9.20216	81	0.79784	783	57
4	758	9.19751	79	751	9.99454	958	9.20297	81	0.79703	666	56
5	15787	9.19830	79	98746	9.99452	15988	9.20378	81	0.79622	6.2549	55
6	816	9.19909	79	741	9.99450	16017	9.20459	81	0.79541	432	54
7	845	9.19988	79	737	9.99448	047	9.20540	81	0.79460	316	53
8	873	9.20067	78	732	9.99446	077	9.20621	80	0.79379	200	52
9	902	9.20145	78	728	9.99444	107	9.20701	81	0.79299	085	51
10	15931	9.20223	79	98723	9.99442	16137	9.20782	80	0.79218	6.1970	50
11	959	9.20302	78	718	9.99440	167	9.20862	80	0.79138	856	49
12	988	9.20380	78	714	9.99438	196	9.20942	80	0.79058	742	48
13	16017	9.20458	77	709	9.99436	226	9.21022	80	0.78978	628	47
14	046	9.20535	78	704	9.99434	256	9.21102	80	0.78898	515	46
15	16074	9.20613	78	98700	9.99432	16286	9.21182	79	0.78818	6.1402	45
16	103	9.20691	77	695	9.99429	316	9.21261	80	0.78739	290	44
17	132	9.20768	77	690	9.99427	346	9.21341	79	0.78659	178	43
18	160	9.20845	77	686	9.99425	376	9.21420	79	0.78580	066	42
19	189	9.20922	77	681	9.99423	405	9.21499	79	0.78501	6.0955	41
20	16218	9.20999	77	98676	9.99421	16435	9.21578	79	0.78422	6.0844	40
21	246	9.21076	77	671	9.99419	465	9.21657	79	0.78343	734	39
22	275	9.21153	76	667	9.99417	495	9.21736	78	0.78264	624	38
23	304	9.21229	77	662	9.99415	525	9.21814	79	0.78186	514	37
24	333	9.21306	76	657	9.99413	555	9.21893	78	0.78107	405	36
25	16361	9.21382	76	98652	9.99411	16585	9.21971	78	0.78029	6.0296	35
26	390	9.21458	76	648	9.99409	615	9.22049	78	0.77951	188	34
27	419	9.21534	76	643	9.99407	645	9.22127	78	0.77873	080	33
28	447	9.21610	75	638	9.99404	674	9.22205	78	0.77795	5.9972	32
29	476	9.21685	76	633	9.99402	704	9.22283	78	0.77717	865	31
30	16505	9.21761	75	98629	9.99400	16734	9.22361	77	0.77639	5.9758	30
31	533	9.21836	76	624	9.99398	764	9.22438	78	0.77562	651	29
32	562	9.21912	75	619	9.99396	794	9.22516	77	0.77484	545	28
33	591	9.21987	75	614	9.99394	824	9.22593	77	0.77407	439	27
34	620	9.22062	75	609	9.99392	854	9.22670	77	0.77330	333	26
35	16648	9.22137	74	98604	9.99390	16884	9.22747	77	0.77253	5.9228	25
36	677	9.22211	75	600	9.99388	914	9.22824	77	0.77176	124	24
37	706	9.22286	75	595	9.99385	944	9.22901	76	0.77099	019	23
38	734	9.22361	74	590	9.99383	974	9.22977	77	0.77023	5.8915	22
39	763	9.22435	74	585	9.99381	17004	9.23054	76	0.76946	811	21
40	16792	9.22509	74	98580	9.99379	17033	9.23130	76	0.76870	5.8708	20
41	820	9.22583	74	575	9.99377	063	9.23206	77	0.76794	605	19
42	849	9.22657	74	570	9.99375	093	9.23283	76	0.76717	502	18
43	878	9.22731	74	565	9.99372	123	9.23359	76	0.76641	400	17
44	906	9.22805	73	561	9.99370	153	9.23435	75	0.76565	298	16
45	16935	9.22878	74	98556	9.99368	17183	9.23510	76	0.76490	5.8197	15
46	964	9.22952	73	551	9.99366	213	9.23586	75	0.76414	095	14
47	992	9.23025	73	546	9.99364	243	9.23661	76	0.76339	5.7994	13
48	17021	9.23098	73	541	9.99362	273	9.23737	75	0.76263	894	12
49	050	9.23171	73	536	9.99359	303	9.23812	75	0.76188	794	11
50	17078	9.23244	73	98531	9.99357	17333	9.23887	75	0.76113	5.7694	10
51	107	9.23317	73	526	9.99355	363	9.23962	75	0.76038	594	9
52	136	9.23390	72	521	9.99353	393	9.24037	75	0.75963	495	8
53	164	9.23462	73	516	9.99351	423	9.24112	74	0.75888	396	7
54	193	9.23535	72	511	9.99348	453	9.24186	75	0.75814	297	6
55	17222	9.23607	72	98506	9.99346	17483	9.24261	74	0.75739	5.7199	5
56	250	9.23679	73	501	9.99344	513	9.24335	75	0.75665	101	4
57	279	9.23752	71	496	9.99342	543	9.24410	74	0.75590	004	3
58	308	9.23823	72	491	9.99340	573	9.24484	74	0.75516	5.6906	2
59	336	9.23895	72	486	9.99337	603	9.24558	74	0.75442	809	1
60	365	9.23967		481	9.99335	633	9.24632		0.75368	713	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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10°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	17365	9.23967	72	98481	9.99335	2	17633	9.24632	74	0.75368	5.6713	60
1	393	9.24039	71	476	9.99333	2	663	9.24706	73	0.75294	617	59
2	422	9.24110	71	471	9.99331	3	693	9.24779	74	0.75221	521	58
3	451	9.24181	72	466	9.99328	2	723	9.24853	73	0.75147	425	57
4	479	9.24253	71	461	9.99326	2	753	9.24926	74	0.75074	329	56
5	17508	9.24324	71	98455	9.99324	2	17783	9.25000	73	0.75000	5.6234	55
6	537	9.24395	71	450	9.99322	3	813	9.25073	73	0.74927	140	54
7	565	9.24466	70	445	9.99319	2	843	9.25146	73	0.74854	045	53
8	594	9.24536	71	440	9.99317	2	873	9.25219	73	0.74781	5.5951	52
9	623	9.24607	70	435	9.99315	2	903	9.25292	73	0.74708	857	51
10	17651	9.24677	71	98430	9.99313	3	17933	9.25365	72	0.74635	5.5764	50
11	680	9.24748	70	425	9.99310	2	963	9.25437	73	0.74563	671	49
12	708	9.24818	70	420	9.99308	2	993	9.25510	72	0.74490	578	48
13	737	9.24888	70	414	9.99306	2	18023	9.25582	73	0.74418	485	47
14	766	9.24958	70	409	9.99304	3	053	9.25655	72	0.74345	393	46
15	17794	9.25028	70	98404	9.99301	2	18083	9.25727	72	0.74273	5.5301	45
16	823	9.25098	70	399	9.99299	2	113	9.25799	72	0.74201	209	44
17	852	9.25168	69	394	9.99297	3	143	9.25871	72	0.74129	118	43
18	880	9.25237	70	389	9.99294	2	173	9.25943	72	0.74057	026	42
19	909	9.25307	69	383	9.99292	2	203	9.26015	71	0.73985	5.4936	41
20	17937	9.25376	69	98378	9.99290	2	18233	9.26086	72	0.73914	5.4845	40
21	966	9.25445	69	373	9.99288	3	263	9.26158	71	0.73842	755	39
22	995	9.25514	69	368	9.99285	2	293	9.26229	72	0.73771	665	38
23	18023	9.25583	69	362	9.99283	2	323	9.26301	71	0.73699	575	37
24	052	9.25652	69	357	9.99281	3	353	9.26372	71	0.73628	486	36
25	18081	9.25721	69	98352	9.99278	2	18384	9.26443	71	0.73557	5.4397	35
26	109	9.25790	68	347	9.99276	2	414	9.26514	71	0.73486	308	34
27	138	9.25858	69	341	9.99274	3	444	9.26585	70	0.73415	219	33
28	166	9.25927	68	336	9.99271	2	474	9.26655	71	0.73345	131	32
29	195	9.25995	68	331	9.99269	2	504	9.26726	71	0.73274	043	31
30	18224	9.26063	68	98325	9.99267	3	18534	9.26797	70	0.73203	5.3955	30
31	252	9.26131	68	320	9.99264	2	564	9.26867	70	0.73133	868	29
32	281	9.26199	68	315	9.99262	2	594	9.26937	71	0.73063	781	28
33	309	9.26267	68	310	9.99260	3	624	9.27008	70	0.72992	694	27
34	338	9.26335	68	304	9.99257	2	654	9.27078	70	0.72922	607	26
35	18367	9.26403	67	98299	9.99255	3	18684	9.27148	70	0.72852	5.3521	25
36	395	9.26470	68	294	9.99252	2	714	9.27218	70	0.72782	435	24
37	424	9.26538	67	288	9.99250	2	745	9.27288	69	0.72712	349	23
38	452	9.26605	67	283	9.99248	3	775	9.27357	70	0.72643	263	22
39	481	9.26672	67	277	9.99245	2	805	9.27427	69	0.72573	178	21
40	18509	9.26739	67	98272	9.99243	2	18835	9.27496	70	0.72504	5.3093	20
41	538	9.26806	67	267	9.99241	3	865	9.27566	69	0.72434	008	19
42	567	9.26873	67	261	9.99238	2	895	9.27635	69	0.72365	5.2924	18
43	595	9.26940	67	256	9.99236	3	925	9.27704	69	0.72296	839	17
44	624	9.27007	66	250	9.99233	2	955	9.27773	69	0.72227	755	16
45	18652	9.27073	67	98245	9.99231	2	18986	9.27842	69	0.72158	5.2672	15
46	681	9.27140	66	240	9.99229	3	19016	9.27911	69	0.72089	588	14
47	710	9.27206	67	234	9.99226	2	046	9.27980	69	0.72020	505	13
48	738	9.27273	66	229	9.99224	3	076	9.28049	68	0.71951	422	12
49	767	9.27339	66	223	9.99221	2	106	9.28117	69	0.71883	339	11
50	18795	9.27405	66	98218	9.99219	2	19136	9.28186	68	0.71814	5.2257	10
51	824	9.27471	66	212	9.99217	3	166	9.28254	69	0.71746	174	9
52	852	9.27537	65	207	9.99214	2	197	9.28323	68	0.71677	092	8
53	881	9.27602	66	201	9.99212	3	227	9.28391	68	0.71609	011	7
54	910	9.27668	66	196	9.99209	2	257	9.28459	68	0.71541	5.1929	6
55	18938	9.27734	65	98190	9.99207	3	19287	9.28527	68	9.71473	5.1848	5
56	967	9.27799	65	185	9.99204	2	317	9.28595	67	0.71405	767	4
57	995	9.27864	66	179	9.99202	2	347	9.28662	68	0.71338	686	3
58	19024	9.27930	65	174	9.99200	3	378	9.28730	68	0.71270	606	2
59	052	9.27995	65	168	9.99197	2	408	9.28798	67	0.71202	526	1
60	081	9.28060		163	9.99195		438	9.28865		0.71135	446	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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'	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	'
0	19081	9.28060	[illegible]	98163	9.99195	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60
1	109	9.28125	[illegible]	157	9.99192	2	[illegible]	9.28933	[illegible]	[illegible]	[illegible]	59
2	138	9.28190	64	152	9.99190	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58
3	167	9.28254	[illegible]	146	9.99187	2	[illegible]	9.29067	[illegible]	[illegible]	[illegible]	57
4	195	9.28319	[illegible]	140	9.99185	3	[illegible]	9.29134	[illegible]	[illegible]	[illegible]	56
5	19224	9.28384	64	98135	9.99182	2	[illegible]	9.29201	[illegible]	[illegible]	[illegible]	55
6	252	9.28448	64	129	9.99180	3	[illegible]	9.29268	[illegible]	0.70732	[illegible]	54
7	281	9.28512	65	124	9.99177	2	[illegible]	9.29335	[illegible]	[illegible]	[illegible]	53
8	309	9.28577	64	118	9.99175	3	680	9.29402	[illegible]	[illegible]	[illegible]	52
9	338	9.28641	64	112	9.99172	2	710	9.29468	[illegible]	0.70532	[illegible]	51
10	19366	9.28705	64	98107	9.99170	3	19740	9.29535	[illegible]	[illegible]	[illegible]	50
11	395	9.28769	64	101	9.99167	2	770	9.29601	[illegible]	[illegible]	[illegible]	49
12	423	9.28833	63	096	9.99165	3	801	9.29668	66	0.70332	[illegible]	48
13	452	9.28896	64	090	9.99162	2	831	9.29734	66	[illegible]	[illegible]	47
14	481	9.28960	64	084	9.99160	3	801	9.29800	66	0.70200	[illegible]	46
15	19509	9.29024	63	98079	9.99157	2	19891	9.29866	66	0.70134	[illegible]	45
16	538	9.29087	63	073	9.99155	3	921	9.29932	66	0.70068	[illegible]	44
17	566	9.29150	64	067	9.99152	2	952	9.29998	66	0.70002	[illegible]	43
18	595	9.29214	63	061	9.99150	3	982	9.30064	66	0.69936	[illegible]	42
19	623	9.29277	63	056	9.99147	2	20012	9.30130	65	0.69870	[illegible]	41
20	19652	9.29340	63	98050	9.99145	3	20042	9.30195	66	0.69805	[illegible]	40
21	680	9.29403	63	044	9.99142	2	073	9.30261	65	0.69739	[illegible]	39
22	709	9.29466	63	039	9.99140	3	103	9.30326	65	0.69674	[illegible]	38
23	737	9.29529	62	033	9.99137	2	133	9.30391	66	0.69609	[illegible]	37
24	766	9.29591	63	027	9.99135	3	164	9.30457	65	0.69543	[illegible]	36
25	19794	9.29654	62	98021	9.99132	2	20194	9.30522	65	0.69478	[illegible]	35
26	823	9.29716	63	016	9.99130	3	224	9.30587	65	0.69413	[illegible]	34
27	851	9.29779	62	010	9.99127	3	254	9.30652	65	0.69348	[illegible]	33
28	880	9.29841	62	004	9.99124	2	285	9.30717	65	0.69283	[illegible]	32
29	908	9.29903	63	97998	9.99122	3	315	9.30782	64	0.69218	[illegible]	31
30	19937	9.29966	62	97992	9.99119	2	20345	9.30846	65	0.69154	[illegible]	30
31	965	9.30028	62	987	9.99117	3	376	9.30911	64	0.69089	[illegible]	29
32	994	9.30090	61	981	9.99114	2	406	9.30975	65	0.69025	[illegible]	28
33	20022	9.30151	62	975	9.99112	3	436	9.31040	64	0.68960	[illegible]	27
34	051	9.30213	62	969	9.99109	3	466	9.31104	64	0.68896	[illegible]	26
35	20079	9.30275	61	97963	9.99106	2	20497	9.31168	65	0.68832	4.8788	25
36	108	9.30336	62	958	9.99104	3	527	9.31233	64	0.68767	710	24
37	136	9.30398	61	952	9.99101	2	557	9.31297	64	0.68703	644	23
38	165	9.30459	62	946	9.99099	3	588	9.31361	64	0.68639	573	22
39	193	9.30521	61	940	9.99096	3	618	9.31425	64	0.68575	501	21
40	20222	9.30582	61	97934	9.99093	2	20648	9.31489	63	0.68511	4.8430	20
41	250	9.30643	61	928	9.99091	3	679	9.31552	64	0.68448	359	19
42	279	9.30704	61	922	9.99088	2	709	9.31616	63	0.68384	288	18
43	307	9.30765	61	916	9.99086	3	739	9.31679	64	0.68321	218	17
44	336	9.30826	61	910	9.99083	3	770	9.31743	63	0.68257	147	16
45	20364	9.30887	60	97905	9.99080	2	20800	9.31806	64	0.68194	4.8077	15
46	393	9.30947	61	899	9.99078	3	830	9.31870	63	0.68130	007	14
47	421	9.31008	60	893	9.99075	3	861	9.31933	63	0.68067	4.7937	13
48	450	9.31068	61	887	9.99072	2	891	9.31996	63	0.68004	867	12
49	478	9.31129	60	881	9.99070	3	921	9.32059	63	0.67941	798	11
50	20507	9.31189	61	97875	9.99067	3	20952	9.32122	63	0.67878	4.7729	10
51	535	9.31250	60	869	9.99064	2	982	9.32185	63	0.67815	659	9
52	563	9.31310	60	863	9.99062	3	21013	9.32248	63	0.67752	591	8
53	592	9.31370	60	857	9.99059	3	043	9.32311	62	0.67689	522	7
54	620	9.31430	60	851	9.99056	2	073	9.32373	63	0.67627	453	6
55	20649	9.31490	59	97845	9.99054	3	21104	9.32436	62	0.67564	4.7385	5
56	677	9.31549	60	839	9.99051	3	134	9.32498	63	0.67502	317	4
57	706	9.31609	60	833	9.99048	2	164	9.32561	62	0.67439	249	3
58	734	9.31669	59	827	9.99046	3	195	9.32623	62	0.67377	181	2
59	763	9.31728	60	821	9.99043	3	225	9.32685	62	0.67315	114	1
60	791	9.31788		815	9.99040		256	9.32747		0.67253	046	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	'

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	20791	9.31788		97815	9.99040		21256	9.32747		0.67253	4.7046	60
1	820	9.31847	59	809	9.99038	2	286	9.32810	63	0.67190	4.6979	59
2	848	9.31907	60	803	9.99035	3	316	9.32872	62	0.67128	912	58
3	877	9.31966	59	797	9.99032	3	347	9.32933	61	0.67067	845	57
4	905	9.32025	59	791	9.99030	2	377	9.32995	62	0.67005	779	56
5	20933	9.32084	59	97784	9.99027	3	21408	9.33057	62	0.66943	4.6712	55
6	962	9.32143	59	778	9.99024	3	438	9.33119	62	0.66881	646	54
7	990	9.32202	59	772	9.99022	2	469	9.33180	61	0.66820	580	53
8	21019	9.32261	59	766	9.99019	3	499	9.33242	62	0.66758	514	52
9	047	9.32319	58	760	9.99016	3	529	9.33303	61	0.66697	448	51
10	21076	9.32378	59	97754	9.99013	3	21560	9.33365	62	0.66635	4.6382	50
11	104	9.32437	59	748	9.99011	2	590	9.33426	61	0.66574	317	49
12	132	9.32495	58	742	9.99008	3	621	9.33487	61	0.66513	252	48
13	161	9.32553	58	735	9.99005	3	651	9.33548	61	0.66452	187	47
14	189	9.32612	59	729	9.99002	3	682	9.33609	61	0.66391	122	46
15	21218	9.32670	58	97723	9.99000	2	21712	9.33670	61	0.66330	4.6057	45
16	246	9.32728	58	717	9.98997	3	743	9.33731	61	0.66269	4.5993	44
17	275	9.32786	58	711	9.98994	3	773	9.33792	61	0.66208	928	43
18	303	9.32844	58	705	9.98991	3	804	9.33853	61	0.66147	864	42
19	331	9.32902	58	698	9.98989	2	834	9.33913	60	0.66087	800	41
20	21360	9.32960	58	97692	9.98986	3	21864	9.33974	61	0.66026	4.5736	40
21	388	9.33018	58	686	9.98983	3	895	9.34034	60	0.65966	673	39
22	417	9.33075	57	680	9.98980	3	925	9.34095	61	0.65905	609	38
23	445	9.33133	58	673	9.98978	2	956	9.34155	60	0.65845	546	37
24	474	9.33190	57	667	9.98975	3	986	9.34215	60	0.65785	483	36
25	21502	9.33248	58	97661	9.98972	3	22017	9.34276	61	0.65724	4.5420	35
26	530	9.33305	57	655	9.98969	3	047	9.34336	60	0.65664	357	34
27	559	9.33362	57	648	9.98967	2	078	9.34396	60	0.65604	294	33
28	587	9.33420	58	642	9.98964	3	108	9.34456	60	0.65544	232	32
29	616	9.33477	57	636	9.98961	3	139	9.34516	60	0.65484	169	31
30	21644	9.33534	57	97630	9.98958	3	22169	9.34576	60	0.65424	4.5107	30
31	672	9.33591	57	623	9.98955	3	200	9.34635	59	0.65365	045	29
32	701	9.33647	56	617	9.98953	2	231	9.34695	60	0.65305	4.4983	28
33	729	9.33704	57	611	9.98950	3	261	9.34755	60	0.65245	922	27
34	758	9.33761	57	604	9.98947	3	292	9.34814	59	0.65186	860	26
35	21786	9.33818	57	97598	9.98944	3	22322	9.34874	60	0.65126	4.4799	25
36	814	9.33874	56	592	9.98941	3	353	9.34933	59	0.65067	737	24
37	843	9.33931	57	585	9.98938	3	383	9.34992	59	0.65008	676	23
38	871	9.33987	56	579	9.98936	2	414	9.35051	59	0.64949	615	22
39	899	9.34043	56	573	9.98933	3	444	9.35111	60	0.64889	555	21
40	21928	9.34100	57	97566	9.98930	3	22475	9.35170	59	0.64830	4.4494	20
41	956	9.34156	56	560	9.98927	3	505	9.35229	59	0.64771	434	19
42	985	9.34212	56	553	9.98924	3	536	9.35288	59	0.64712	373	18
43	22013	9.34268	56	547	9.98921	3	567	9.35347	59	0.64653	313	17
44	041	9.34324	56	541	9.98919	2	597	9.35405	58	0.64595	253	16
45	22070	9.34380	56	97534	9.98916	3	22628	9.35464	59	0.64536	4.4194	15
46	098	9.34436	56	528	9.98913	3	658	9.35523	59	0.64477	134	14
47	126	9.34491	55	521	9.98910	3	689	9.35581	58	0.64419	075	13
48	155	9.34547	56	515	9.98907	3	719	9.35640	59	0.64360	015	12
49	183	9.34602	55	508	9.98904	3	750	9.35698	58	0.64302	4.3956	11
50	22212	9.34658	56	97502	9.98901	3	22781	9.35757	59	0.64243	4.3897	10
51	240	9.34713	55	496	9.98898	3	811	9.35815	58	0.64185	838	9
52	268	9.34769	56	489	9.98896	2	842	9.35873	58	0.64127	779	8
53	297	9.34824	55	483	9.98893	3	872	9.35931	58	0.64069	721	7
54	325	9.34879	55	476	9.98890	3	903	9.35989	58	0.64011	662	6
55	22353	9.34934	55	97470	9.98887	3	22934	9.36047	58	0.63953	4.3604	5
56	382	9.34989	55	463	9.98884	3	964	9.36105	58	0.63895	546	4
57	410	9.35044	55	457	9.98881	3	995	9.36163	58	0.63837	488	3
58	438	9.35099	55	450	9.98878	3	23026	9.36221	58	0.63779	430	2
59	467	9.35154	55	444	9.98875	3	056	9.36279	58	0.63721	372	1
60	495	9.35209	55	437	9.98872	3	087	9.36336	57	0.63664	315	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	22495	9.35209		97437	9.98872		23087	9.36336		0.63664	4.3315	60
1	523	9.35263	54	430	9.98869	3	117	9.36394	58	0.63606	257	59
2	552	9.35318	55	424	9.98867	2	148	9.36452	58	0.63548	200	58
3	580	9.35373	55	417	9.98864	3	179	9.36509	57	0.63491	143	57
4	608	9.35427	54	411	9.98861	3	209	9.36566	57	0.63434	086	56
5	22637	9.35481	54	97404	9.98858	3	23240	9.36624	58	0.63376	4.3029	55
6	665	9.35536	55	398	9.98855	3	271	9.36681	57	0.63319	4.2972	54
7	693	9.35590	54	391	9.98852	3	301	9.36738	57	0.63262	916	53
8	722	9.35644	54	384	9.98849	3	332	9.36795	57	0.63205	859	52
9	750	9.35698	54	378	9.98846	3	363	9.36852	57	0.63148	803	51
10	22778	9.35752	54	97371	9.98843	3	23393	9.36909	57	0.63091	4.2747	50
11	807	9.35806	54	365	9.98840	3	424	9.36966	57	0.63034	691	49
12	835	9.35860	54	358	9.98837	3	455	9.37023	57	0.62977	635	48
13	863	9.35914	54	351	9.98834	3	485	9.37080	57	0.62920	580	47
14	892	9.35968	54	345	9.98831	3	516	9.37137	57	0.62863	524	46
15	22920	9.36022	54	97338	9.98828	3	23547	9.37193	56	0.62807	4.2468	45
16	948	9.36075	53	331	9.98825	3	578	9.37250	57	0.62750	413	44
17	977	9.36129	54	325	9.98822	3	608	9.37306	56	0.62694	358	43
18	23005	9.36182	53	318	9.98819	3	639	9.37363	57	0.62637	303	42
19	033	9.36236	54	311	9.98816	3	670	9.37419	56	0.62581	248	41
20	23062	9.36289	53	97304	9.98813	3	23700	9.37476	57	0.62524	4.2193	40
21	090	9.36342	53	298	9.98810	3	731	9.37532	56	0.62468	139	39
22	118	9.36395	53	291	9.98807	3	762	9.37588	56	0.62412	084	38
23	146	9.36449	54	284	9.98804	3	793	9.37644	56	0.62356	030	37
24	175	9.36502	53	278	9.98801	3	823	9.37700	56	0.62300	4.1976	36
25	23203	9.36555	53	97271	9.98798	3	23854	9.37756	56	0.62244	4.1922	35
26	231	9.36608	53	264	9.98795	3	885	9.37812	56	0.62188	868	34
27	260	9.36660	52	257	9.98792	3	916	9.37868	56	0.62132	814	33
28	288	9.36713	53	251	9.98789	3	946	9.37924	56	0.62076	760	32
29	316	9.36766	53	244	9.98786	3	977	9.37980	56	0.62020	706	31
30	23345	9.36819	53	97237	9.98783	3	24008	9.38035	55	0.61965	4.1653	30
31	373	9.36871	52	230	9.98780	3	039	9.38091	56	0.61909	600	29
32	401	9.36924	53	223	9.98777	3	069	9.38147	56	0.61853	547	28
33	429	9.36976	52	217	9.98774	3	100	9.38202	55	0.61798	493	27
34	458	9.37028	52	210	9.98771	3	131	9.38257	55	0.61743	441	26
35	23486	9.37081	53	97203	9.98768	3	24162	9.38313	56	0.61687	4.1388	25
36	514	9.37133	52	196	9.98765	3	193	9.38368	55	0.61632	335	24
37	542	9.37185	52	189	9.98762	3	223	9.38423	55	0.61577	282	23
38	571	9.37237	52	182	9.98759	3	254	9.38479	56	0.61521	230	22
39	599	9.37289	52	176	9.98756	3	285	9.38534	55	0.61466	178	21
40	23627	9.37341	52	97169	9.98753	3	24316	9.38589	55	0.61411	4.1126	20
41	656	9.37393	52	162	9.98750	3	347	9.38644	55	0.61356	074	19
42	684	9.37445	52	155	9.98746	4	377	9.38699	55	0.61301	022	18
43	712	9.37497	52	148	9.98743	3	408	9.38754	55	0.61246	4.0970	17
44	740	9.37549	52	141	9.98740	3	439	9.38808	54	0.61192	918	16
45	23769	9.37600	51	97134	9.98737	3	24470	9.38863	55	0.61137	4.0867	15
46	797	9.37652	52	127	9.98734	3	501	9.38918	55	0.61082	815	14
47	825	9.37703	51	120	9.98731	3	532	9.38972	54	0.61028	764	13
48	853	9.37755	52	113	9.98728	3	562	9.39027	55	0.60973	713	12
49	882	9.37806	51	106	9.98725	3	593	9.39082	55	0.60918	662	11
50	23910	9.37858	52	97100	9.98722	3	24624	9.39136	54	0.60864	4.0611	10
51	938	9.37909	51	093	9.98719	3	655	9.39190	54	0.60810	560	9
52	966	9.37960	51	086	9.98715	4	686	9.39245	55	0.60755	509	8
53	995	9.38011	51	079	9.98712	3	717	9.39299	54	0.60701	459	7
54	24023	9.38062	51	072	9.98709	3	747	9.39353	54	0.60647	408	6
55	24051	9.38113	51	97065	9.98706	3	24778	9.39407	54	0.60593	4.0358	5
56	079	9.38164	51	058	9.98703	3	809	9.39461	54	0.60539	308	4
57	108	9.38215	51	051	9.98700	3	840	9.39515	54	0.60485	257	3
58	136	9.38266	51	044	9.98697	3	871	9.39569	54	0.60431	207	2
59	164	9.38317	51	037	9.98694	3	902	9.39623	54	0.60377	158	1
60	192	9.38368	51	030	9.98690	4	933	9.39677	54	0.60323	108	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

14°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	24192	9.38368		97030	9.98690		24933	9.39677		0.60323	4.0108	60
1	220	9.38418	50	023	9.98687	3	964	9.39731	54	0.60269	058	59
2	249	9.38469	51	015	9.98684	3	995	9.39785	54	0.60215	009	58
3	277	9.38519	50	008	9.98681	3	25026	9.39838	53	0.60162	3.9959	57
4	305	9.38570	51	001	9.98678	3	056	9.39892	54	0.60108	910	56
5	24333	9.38620	50	96994	9.98675	3	25087	9.39945	53	0.60055	3.9861	55
6	362	9.38670	50	987	9.98671	4	118	9.39999	54	0.60001	812	54
7	390	9.38721	51	980	9.98668	3	149	9.40052	53	0.59948	763	53
8	418	9.38771	50	973	9.98665	3	180	9.40106	54	0.59894	714	52
9	446	9.38821	50	966	9.98662	3	211	9.40159	53	0.59841	665	51
10	24474	9.38871	50	96959	9.98659	3	25242	9.40212	53	0.59788	3.9617	50
11	503	9.38921	50	952	9.98656	3	273	9.40266	54	0.59734	568	49
12	531	9.38971	50	945	9.98652	4	304	9.40319	53	0.59681	520	48
13	559	9.39021	50	937	9.98649	3	335	9.40372	53	0.59628	471	47
14	587	9.39071	50	930	9.98646	3	366	9.40425	53	0.59575	423	46
15	24615	9.39121	50	96923	9.98643	3	25397	9.40478	53	0.59522	3.9375	45
16	644	9.39170	49	916	9.98640	3	428	9.40531	53	0.59469	327	44
17	672	9.39220	50	909	9.98636	4	459	9.40584	53	0.59416	279	43
18	700	9.39270	50	902	9.98633	3	490	9.40636	52	0.59364	232	42
19	728	9.39319	49	894	9.98630	3	521	9.40689	53	0.59311	184	41
20	24756	9.39369	50	96887	9.98627	3	25552	9.40742	53	0.59258	3.9136	40
21	784	9.39418	49	880	9.98623	4	583	9.40795	53	0.59205	089	39
22	813	9.39467	49	873	9.98620	3	614	9.40847	52	0.59153	042	38
23	841	9.39517	50	866	9.98617	3	645	9.40900	53	0.59100	3.8995	37
24	869	9.39566	49	858	9.98614	3	676	9.40952	52	0.59048	947	36
25	24897	9.39615	49	96851	9.98610	4	25707	9.41005	53	0.58995	3.8900	35
26	925	9.39664	49	844	9.98607	3	738	9.41057	52	0.58943	854	34
27	954	9.39713	49	837	9.98604	3	769	9.41109	52	0.58891	807	33
28	982	9.39762	49	829	9.98601	3	800	9.41161	52	0.58839	760	32
29	25010	9.39811	49	822	9.98597	4	831	9.41214	53	0.58786	714	31
30	25038	9.39860	49	96815	9.98594	3	25862	9.41266	52	0.58734	3.8667	30
31	066	9.39909	49	807	9.98591	3	893	9.41318	52	0.58682	621	29
32	094	9.39958	49	800	9.98588	3	924	9.41370	52	0.58630	575	28
33	122	9.40006	48	793	9.98584	4	955	9.41422	52	0.58578	528	27
34	151	9.40055	49	786	9.98581	3	986	9.41474	52	0.58526	482	26
35	25179	9.40103	48	96778	9.98578	3	26017	9.41526	52	0.58474	3.8436	25
36	207	9.40152	49	771	9.98574	4	048	9.41578	52	0.58422	391	24
37	235	9.40200	48	764	9.98571	3	079	9.41629	51	0.58371	345	23
38	263	9.40249	49	756	9.98568	3	110	9.41681	52	0.58319	299	22
39	291	9.40297	48	749	9.98565	3	141	9.41733	52	0.58267	254	21
40	25320	9.40346	49	96742	9.98561	4	26172	9.41784	51	0.58216	3.8208	20
41	348	9.40394	48	734	9.98558	3	203	9.41836	52	0.58164	163	19
42	376	9.40442	48	727	9.98555	3	235	9.41887	51	0.58113	118	18
43	404	9.40490	48	719	9.98551	4	266	9.41939	52	0.58061	073	17
44	432	9.40538	48	712	9.98548	3	297	9.41990	51	0.58010	028	16
45	25460	9.40586	48	96705	9.98545	3	26328	9.42041	51	0.57959	3.7983	15
46	488	9.40634	48	697	9.98541	4	359	9.42093	52	0.57907	938	14
47	516	9.40682	48	690	9.98538	3	390	9.42144	51	0.57856	893	13
48	545	9.40730	48	682	9.98535	3	421	9.42195	51	0.57805	848	12
49	573	9.40778	48	675	9.98531	4	452	9.42246	51	0.57754	804	11
50	25601	9.40825	47	96667	9.98528	3	26483	9.42297	51	0.57703	3.7760	10
51	629	9.40873	48	660	9.98525	3	515	9.42348	51	0.57652	715	9
52	657	9.40921	48	653	9.98521	4	546	9.42399	51	0.57601	671	8
53	685	9.40968	47	645	9.98518	3	577	9.42450	51	0.57550	627	7
54	713	9.41016	48	638	9.98515	3	608	9.42501	51	0.57499	583	6
55	25741	9.41063	47	96630	9.98511	4	26639	9.42552	51	0.57448	3.7539	5
56	769	9.41111	48	623	9.98508	3	670	9.42603	51	0.57397	495	4
57	798	9.41158	47	615	9.98505	3	701	9.42653	50	0.57347	451	3
58	826	9.41205	47	608	9.98501	4	733	9.42704	51	0.57296	408	2
59	854	9.41252	47	600	9.98498	3	764	9.42755	51	0.57245	364	1
60	882	9.41300	48	593	9.98494	4	795	9.42805	50	0.57195	321	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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15°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	25882	9.41300	47	96593	9.98494	3	26795	9.42805	51	0.57195	3.7321	60
1	910	9.41347	47	585	9.98491	3	826	9.42856	50	0.57144	277	59
2	938	9.41394	47	578	9.98488	4	857	9.42906	51	0.57094	234	58
3	966	9.41441	47	570	9.98484	3	888	9.42957	50	0.57043	191	57
4	994	9.41488	47	562	9.98481	4	920	9.43007	50	0.56993	148	56
5	26022	9.41535	47	96555	9.98477	3	26951	9.43057	51	0.56943	3.7105	55
6	050	9.41582	46	547	9.98474	3	982	9.43108	50	0.56892	062	54
7	079	9.41628	47	540	9.98471	4	27013	9.43158	50	0.56842	019	53
8	107	9.41675	47	532	9.98467	3	044	9.43208	50	0.56792	3.6976	52
9	135	9.41722	46	524	9.98464	4	076	9.43258	50	0.56742	933	51
10	26163	9.41768	47	96517	9.98460	3	27107	9.43308	50	0.56692	3.6891	50
11	191	9.41815	46	509	9.98457	4	138	9.43358	50	0.56642	848	49
12	219	9.41861	47	502	9.98453	3	169	9.43408	50	0.56592	806	48
13	247	9.41908	46	494	9.98450	3	201	9.43458	50	0.56542	764	47
14	275	9.41954	47	486	9.98447	4	232	9.43508	50	0.56492	722	46
15	26303	9.42001	46	96479	9.98443	3	27263	9.43558	49	0.56442	3.6680	45
16	331	9.42047	46	471	9.98440	4	294	9.43607	50	0.56393	638	44
17	359	9.42093	47	463	9.98436	3	326	9.43657	50	0.56343	596	43
18	387	9.42140	46	456	9.98433	4	357	9.43707	49	0.56293	554	42
19	415	9.42186	46	448	9.98429	3	388	9.43756	50	0.56244	512	41
20	26443	9.42232	46	96440	9.98426	4	27419	9.43806	49	0.56194	3.6470	40
21	471	9.42278	46	433	9.98422	3	451	9.43855	50	0.56145	429	39
22	500	9.42324	46	425	9.98419	4	482	9.43905	49	0.56095	387	38
23	528	9.42370	46	417	9.98415	3	513	9.43954	50	0.56046	346	37
24	556	9.42416	45	410	9.98412	3	545	9.44004	49	0.55996	305	36
25	26584	9.42461	46	96402	9.98409	4	27576	9.44053	49	0.55947	3.6264	35
26	612	9.42507	46	394	9.98405	3	607	9.44102	49	0.55898	222	34
27	640	9.42553	46	386	9.98402	4	638	9.44151	50	0.55849	181	33
28	668	9.42599	45	379	9.98398	3	670	9.44201	49	0.55799	140	32
29	696	9.42644	46	371	9.98395	4	701	9.44250	49	0.55750	100	31
30	26724	9.42690	45	96363	9.98391	3	27732	9.44299	49	0.55701	3.6059	30
31	752	9.42735	46	355	9.98388	4	764	9.44348	49	0.55652	018	29
32	780	9.42781	45	347	9.98384	3	795	9.44397	49	0.55603	3.5978	28
33	808	9.42826	46	340	9.98381	4	826	9.44446	49	0.55554	937	27
34	836	9.42872	45	332	9.98377	4	858	9.44495	49	0.55505	897	26
35	26864	9.42917	45	96324	9.98373	3	27889	9.44544	48	0.55456	3.5856	25
36	892	9.42962	46	316	9.98370	4	921	9.44592	49	0.55408	816	24
37	920	9.43008	45	308	9.98366	3	952	9.44641	49	0.55359	776	23
38	948	9.43053	45	301	9.98363	4	983	9.44690	48	0.55310	736	22
39	976	9.43098	45	293	9.98359	3	28015	9.44738	49	0.55262	696	21
40	27004	9.43143	45	96285	9.98356	4	28046	9.44787	49	0.55213	3.5656	20
41	032	9.43188	45	277	9.98352	3	077	9.44836	48	0.55164	616	19
42	060	9.43233	45	269	9.98349	4	109	9.44884	49	0.55116	576	18
43	088	9.43278	45	261	9.98345	3	140	9.44933	48	0.55067	536	17
44	116	9.43323	44	253	9.98342	4	172	9.44981	48	0.55019	497	16
45	27144	9.43367	45	96246	9.98338	4	28203	9.45029	49	0.54971	3.5457	15
46	172	9.43412	45	238	9.98334	3	234	9.45078	48	0.54922	418	14
47	200	9.43457	45	230	9.98331	4	266	9.45126	48	0.54874	379	13
48	228	9.43502	44	222	9.98327	3	297	9.45174	48	0.54826	339	12
49	256	9.43546	45	214	9.98324	4	329	9.45222	49	0.54778	300	11
50	27284	9.43591	44	96206	9.98320	3	28360	9.45271	48	0.54729	3.5261	10
51	312	9.43635	45	198	9.98317	4	391	9.45319	48	0.54681	222	9
52	340	9.43680	44	190	9.98313	4	423	9.45367	48	0.54633	183	8
53	368	9.43724	45	182	9.98309	3	454	9.45415	48	0.54585	144	7
54	396	9.43769	44	174	9.98306	4	486	9.45463	48	0.54537	105	6
55	27424	9.43813	44	96166	9.98302	3	28517	9.45511	48	0.54489	3.5067	5
56	452	9.43857	44	158	9.98299	4	549	9.45559	47	0.54441	028	4
57	480	9.43901	45	150	9.98295	4	580	9.45606	48	0.54394	3.4989	3
58	508	9.43946	44	142	9.98291	3	612	9.45654	48	0.54346	951	2
59	536	9.43990	44	134	9.98288	4	643	9.45702	48	0.54298	912	1
60	564	9.44034		126	9.98284		675	9.45750		0.54250	874	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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16°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	27564	9.44034	44	96126	9.98284	3	28675	9.45750	47	0.54250	3.4874	60
1	592	9.44078	44	118	9.98281	4	706	9.45797	48	0.54203	836	59
2	620	9.44122	44	110	9.98277	4	738	9.45845	47	0.54155	798	58
3	648	9.44166	44	102	9.98273	3	769	9.45892	48	0.54108	760	57
4	676	9.44210	43	094	9.98270	4	801	9.45940	47	0.54060	722	56
5	27704	9.44253	44	96086	9.98266	4	28832	9.45987	48	0.54013	3.4684	55
6	731	9.44297	44	078	9.98262	3	864	9.46035	47	0.53965	646	54
7	759	9.44341	44	070	9.98259	4	895	9.46082	48	0.53918	608	53
8	787	9.44385	43	062	9.98255	4	927	9.46130	47	0.53870	570	52
9	815	9.44428	44	054	9.98251	3	958	9.46177	47	0.53823	533	51
10	27843	9.44472	44	96046	9.98248	4	28990	9.46224	47	0.53776	3.4495	50
11	871	9.44516	43	037	9.98244	4	29021	9.46271	48	0.53729	458	49
12	899	9.44559	43	029	9.98240	3	053	9.46319	47	0.53681	420	48
13	927	9.44602	44	021	9.98237	4	084	9.46366	47	0.53634	383	47
14	955	9.44646	43	013	9.98233	4	116	9.46413	47	0.53587	346	46
15	27983	9.44689	44	96005	9.98229	3	29147	9.46460	47	0.53540	3.4308	45
16	28011	9.44733	43	95997	9.98226	4	179	9.46507	47	0.53493	271	44
17	039	9.44776	43	989	9.98222	4	210	9.46554	47	0.53446	234	43
18	067	9.44819	43	981	9.98218	3	242	9.46601	47	0.53399	197	42
19	095	9.44862	43	972	9.98215	4	274	9.46648	46	0.53352	160	41
20	28123	9.44905	43	95964	9.98211	4	29305	9.46694	47	0.53306	3.4124	40
21	150	9.44948	44	956	9.98207	3	337	9.46741	47	0.53259	087	39
22	178	9.44992	43	948	9.98204	4	368	9.46788	47	0.53212	050	38
23	206	9.45035	42	940	9.98200	4	400	9.46835	46	0.53165	014	37
24	234	9.45077	43	931	9.98196	4	432	9.46881	47	0.53119	3.3977	36
25	28262	9.45120	43	95923	9.98192	3	29463	9.46928	47	0.53072	3.3941	35
26	290	9.45163	43	915	9.98189	4	495	9.46975	46	0.53025	904	34
27	318	9.45206	43	907	9.98185	4	526	9.47021	47	0.52979	868	33
28	346	9.45249	43	898	9.98181	4	558	9.47068	46	0.52932	832	32
29	374	9.45292	42	890	9.98177	3	590	9.47114	46	0.52886	796	31
30	28402	9.45334	43	95882	9.98174	4	29621	9.47160	47	0.52840	3.3759	30
31	429	9.45377	42	874	9.98170	4	653	9.47207	46	0.52793	723	29
32	457	9.45419	43	865	9.98166	4	685	9.47253	46	0.52747	687	28
33	485	9.45462	42	857	9.98162	3	716	9.47299	47	0.52701	652	27
34	513	9.45504	43	849	9.98159	4	748	9.47346	46	0.52654	616	26
35	28541	9.45547	42	95841	9.98155	4	29780	9.47392	46	0.52608	3.3580	25
36	569	9.45589	43	832	9.98151	4	811	9.47438	46	0.52562	544	24
37	597	9.45632	42	824	9.98147	3	843	9.47484	46	0.52516	509	23
38	625	9.45674	42	816	9.98144	4	875	9.47530	46	0.52470	473	22
39	652	9.45716	42	807	9.98140	4	906	9.47576	46	0.52424	438	21
40	28680	9.45758	43	95799	9.98136	4	29938	9.47622	46	0.52378	3.3402	20
41	708	9.45801	42	791	9.98132	3	970	9.47668	46	0.52332	367	19
42	736	9.45843	42	782	9.98129	4	30001	9.47714	46	0.52286	332	18
43	764	9.45885	42	774	9.98125	4	033	9.47760	46	0.52240	297	17
44	792	9.45927	42	766	9.98121	4	065	9.47806	46	0.52194	261	16
45	28820	9.45969	42	95757	9.98117	4	30097	9.47852	45	0.52148	3.3226	15
46	847	9.46011	42	749	9.98113	3	128	9.47897	46	0.52103	191	14
47	875	9.46053	42	740	9.98110	4	160	9.47943	46	0.52057	156	13
48	903	9.46095	41	732	9.98106	4	192	9.47989	46	0.52011	122	12
49	931	9.46136	42	724	9.98102	4	224	9.48035	45	0.51965	087	11
50	28959	9.46178	42	95715	9.98098	4	30255	9.48080	46	0.51920	3.3052	10
51	987	9.46220	42	707	9.98094	4	287	9.48126	45	0.51874	017	9
52	29015	9.46262	41	698	9.98090	3	319	9.48171	46	0.51829	3.2983	8
53	042	9.46303	42	690	9.98087	4	351	9.48217	45	0.51783	948	7
54	070	9.46345	41	681	9.98083	4	382	9.48262	45	0.51738	914	6
55	29098	9.46386	42	95673	9.98079	4	30414	9.48307	46	0.51693	3.2879	5
56	126	9.46428	41	664	9.98075	4	446	9.48353	45	0.51647	845	4
57	154	9.46469	42	656	9.98071	4	478	9.48398	45	0.51602	811	3
58	182	9.46511	41	647	9.98067	4	509	9.48443	46	0.51557	777	2
59	209	9.46552	42	639	9.98063	3	541	9.48489	45	0.51511	743	1
60	237	9.46594		630	9.98060		573	9.48534		0.51466	709	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	29237	9.46594	41	95630	9.98060	4	30573	9.48534	45	0.51466	3.2709	60
1	265	9.46635	41	622	9.98056	4	605	9.48579	45	0.51421	675	59
2	293	9.46676	41	613	9.98052	4	637	9.48624	45	0.51376	641	58
3	321	9.46717	41	605	9.98048	4	669	9.48669	45	0.51331	607	57
4	348	9.46758	42	596	9.98044	4	700	9.48714	45	0.51286	573	56
5	29376	9.46800	41	95588	9.98040	4	30732	9.48759	45	0.51241	3.2539	55
6	404	9.46841	41	579	9.98036	4	764	9.48804	45	0.51196	506	54
7	432	9.46882	41	571	9.98032	3	796	9.48849	45	0.51151	472	53
8	460	9.46923	41	562	9.98029	4	828	9.48894	45	0.51106	438	52
9	487	9.46964	41	554	9.98025	4	860	9.48939	45	0.51061	405	51
10	29515	9.47005	40	95545	9.98021	4	30891	9.48984	45	0.51016	3.2371	50
11	543	9.47045	41	536	9.98017	4	923	9.49029	44	0.50971	338	49
12	571	9.47086	41	528	9.98013	4	955	9.49073	45	0.50927	305	48
13	599	9.47127	41	519	9.98009	4	987	9.49118	45	0.50882	272	47
14	626	9.47168	41	511	9.98005	4	31019	9.49163	44	0.50837	238	46
15	29654	9.47209	40	95502	9.98001	4	31051	9.49207	45	0.50793	3.2205	45
16	682	9.47249	41	493	9.97997	4	083	9.49252	44	0.50748	172	44
17	710	9.47290	40	485	9.97993	4	115	9.49296	45	0.50704	139	43
18	737	9.47330	41	476	9.97989	3	147	9.49341	44	0.50659	106	42
19	765	9.47371	40	467	9.97986	4	178	9.49385	45	0.50615	073	41
20	29793	9.47411	41	95459	9.97982	4	31210	9.49430	44	0.50570	3.2041	40
21	821	9.47452	40	450	9.97978	4	242	9.49474	45	0.50526	008	39
22	849	9.47492	41	441	9.97974	4	274	9.49519	44	0.50481	3.1975	38
23	876	9.47533	40	433	9.97970	4	306	9.49563	44	0.50437	943	37
24	904	9.47573	40	424	9.97966	4	338	9.49607	45	0.50393	910	36
25	29932	9.47613	41	95415	9.97962	4	31370	9.49652	44	0.50348	3.1878	35
26	960	9.47654	40	407	9.97958	4	402	9.49696	44	0.50304	845	34
27	987	9.47694	40	398	9.97954	4	434	9.49740	44	0.50260	813	33
28	30015	9.47734	40	389	9.97950	4	466	9.49784	44	0.50216	780	32
29	043	9.47774	40	380	9.97946	4	498	9.49828	44	0.50172	748	31
30	30071	9.47814	40	95372	9.97942	4	31530	9.49872	44	0.50128	3.1716	30
31	098	9.47854	40	363	9.97938	4	562	9.49916	44	0.50084	684	29
32	126	9.47894	40	354	9.97934	4	594	9.49960	44	0.50040	652	28
33	154	9.47934	40	345	9.97930	4	626	9.50004	44	0.49996	620	27
34	182	9.47974	40	337	9.97926	4	658	9.50048	44	0.49952	588	26
35	30209	9.48014	40	95328	9.97922	4	31690	9.50092	44	0.49908	3.1556	25
36	237	9.48054	40	319	9.97918	4	722	9.50136	44	0.49864	524	24
37	265	9.48094	39	310	9.97914	4	754	9.50180	43	0.49820	492	23
38	292	9.48133	40	301	9.97910	4	786	9.50223	44	0.49777	460	22
39	320	9.48173	40	293	9.97906	4	818	9.50267	44	0.49733	429	21
40	30348	9.48213	39	95284	9.97902	4	31850	9.50311	44	0.49689	3.1397	20
41	376	9.48252	40	275	9.97898	4	882	9.50355	43	0.49645	366	19
42	403	9.48292	40	266	9.97894	4	914	9.50398	44	0.49602	334	18
43	431	9.48332	39	257	9.97890	4	946	9.50442	43	0.49558	303	17
44	459	9.48371	40	248	9.97886	4	978	9.50485	44	0.49515	271	16
45	30486	9.48411	39	95240	9.97882	4	32010	9.50529	43	0.49471	3.1240	15
46	514	9.48450	40	231	9.97878	4	042	9.50572	44	0.49428	209	14
47	542	9.48490	39	222	9.97874	4	074	9.50616	43	0.49384	178	13
48	570	9.48529	39	213	9.97870	4	106	9.50659	44	0.49341	146	12
49	597	9.48568	39	204	9.97866	5	139	9.50703	43	0.49297	115	11
50	30625	9.48607	40	95195	9.97861	4	32171	9.50746	43	0.49254	3.1084	10
51	653	9.48647	39	186	9.97857	4	203	9.50789	44	0.49211	053	9
52	680	9.48686	39	177	9.97853	4	235	9.50833	43	0.49167	022	8
53	708	9.48725	39	168	9.97849	4	267	9.50876	43	0.49124	3.0991	7
54	736	9.48764	39	159	9.97845	4	299	9.50919	43	0.49081	961	6
55	30763	9.48803	39	95150	9.97841	4	32331	9.50962	43	0.49038	3.0930	5
56	791	9.48842	39	142	9.97837	4	363	9.51005	43	0.48995	899	4
57	819	9.48881	39	133	9.97833	4	396	9.51048	44	0.48952	868	3
58	846	9.48920	39	124	9.97829	4	428	9.51092	43	0.48908	838	2
59	874	9.48959	39	115	9.97825	4	460	9.51135	43	0.48865	807	1
60	902	9.48998		106	9.97821		492	9.51178		0.48822	777	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	30902	9.48998	39	95106	9.97821	4	32492	9.51178	43	0.48822	3.0777	60
1	929	9.49037	39	097	9.97817	5	524	9.51221	43	0.48779	746	59
2	957	9.49076	39	088	9.97812	4	556	9.51264	42	0.48736	716	58
3	985	9.49115	38	079	9.97808	4	588	9.51306	43	0.48694	686	57
4	31012	9.49153	39	070	9.97804	4	621	9.51349	43	0.48651	655	56
5	31040	9.49192	39	95061	9.97800	4	32653	9.51392	43	0.48608	3.0625	55
6	068	9.49231	38	052	9.97796	4	685	9.51435	43	0.48565	595	54
7	095	9.49269	39	043	9.97792	4	717	9.51478	42	0.48522	565	53
8	123	9.49308	39	033	9.97788	4	749	9.51520	43	0.48480	535	52
9	151	9.49347	38	024	9.97784	5	782	9.51563	43	0.48437	505	51
10	31178	9.49385	39	95015	9.97779	4	32814	9.51606	42	0.48394	3.0475	50
11	206	9.49424	38	006	9.97775	4	846	9.51648	43	0.48352	445	49
12	233	9.49462	38	94997	9.97771	4	878	9.51691	43	0.48309	415	48
13	261	9.49500	39	988	9.97767	4	911	9.51734	42	0.48266	385	47
14	289	9.49539	38	979	9.97763	4	943	9.51776	43	0.48224	356	46
15	31316	9.49577	38	94970	9.97759	5	32975	9.51819	42	0.48181	3.0326	45
16	344	9.49615	39	961	9.97754	4	33007	9.51861	42	0.48139	296	44
17	372	9.49654	38	952	9.97750	4	040	9.51903	43	0.48097	267	43
18	399	9.49692	38	943	9.97746	4	072	9.51946	42	0.48054	237	42
19	427	9.49730	38	933	9.97742	4	104	9.51988	43	0.48012	208	41
20	31454	9.49768	38	94924	9.97738	4	33136	9.52031	42	0.47969	3.0178	40
21	482	9.49806	38	915	9.97734	5	169	9.52073	42	0.47927	149	39
22	510	9.49844	38	906	9.97729	4	201	9.52115	42	0.47885	120	38
23	537	9.49882	38	897	9.97725	4	233	9.52157	43	0.47843	090	37
24	565	9.49920	38	888	9.97721	4	266	9.52200	42	0.47800	061	36
25	31593	9.49958	38	94878	9.97717	4	33298	9.52242	42	0.47758	3.0032	35
26	620	9.49996	38	869	9.97713	5	330	9.52284	42	0.47716	003	34
27	648	9.50034	38	860	9.97708	4	363	9.52326	42	0.47674	2.9974	33
28	675	9.50072	38	851	9.97704	4	395	9.52368	42	0.47632	945	32
29	703	9.50110	38	842	9.97700	4	427	9.52410	42	0.47590	916	31
30	31730	9.50148	37	94832	9.97696	5	33460	9.52452	42	0.47548	2.9887	30
31	758	9.50185	38	823	9.97691	4	492	9.52494	42	0.47506	858	29
32	786	9.50223	38	814	9.97687	4	524	9.52536	42	0.47464	829	28
33	813	9.50261	37	805	9.97683	4	557	9.52578	42	0.47422	800	27
34	841	9.50298	38	795	9.97679	5	589	9.52620	41	0.47380	772	26
35	31868	9.50336	38	94786	9.97674	4	33621	9.52661	42	0.47339	2.9743	25
36	896	9.50374	37	777	9.97670	4	654	9.52703	42	0.47297	714	24
37	923	9.50411	38	768	9.97666	4	686	9.52745	42	0.47255	686	23
38	951	9.50449	37	758	9.97662	5	718	9.52787	42	0.47213	657	22
39	979	9.50486	37	749	9.97657	4	751	9.52829	41	0.47171	629	21
40	32006	9.50523	38	94740	9.97653	4	33783	9.52870	42	0.47130	2.9600	20
41	034	9.50561	37	730	9.97649	4	816	9.52912	41	0.47088	572	19
42	061	9.50598	37	721	9.97645	5	848	9.52953	42	0.47047	544	18
43	089	9.50635	38	712	9.97640	4	881	9.52995	42	0.47005	515	17
44	116	9.50673	37	702	9.97636	4	913	9.53037	41	0.46963	487	16
45	32144	9.50710	37	94693	9.97632	4	33945	9.53078	42	0.46922	2.9459	15
46	171	9.50747	37	684	9.97628	5	978	9.53120	41	0.46880	431	14
47	199	9.50784	37	674	9.97623	4	34010	9.53161	41	0.46839	403	13
48	227	9.50821	37	665	9.97619	4	043	9.53202	42	0.46798	375	12
49	254	9.50858	38	656	9.97615	5	075	9.53244	41	0.46756	347	11
50	32282	9.50896	37	94646	9.97610	4	34108	9.53285	42	0.46715	2.9319	10
51	309	9.50933	37	637	9.97606	4	140	9.53327	41	0.46673	291	9
52	337	9.50970	37	627	9.97602	5	173	9.53368	41	0.46632	263	8
53	364	9.51007	36	618	9.97597	4	205	9.53409	41	0.46591	235	7
54	392	9.51043	37	609	9.97593	4	238	9.53450	42	0.46550	208	6
55	32419	9.51080	37	94599	9.97589	5	34270	9.53492	41	0.46508	2.9180	5
56	447	9.51117	37	590	9.97584	4	303	9.53533	41	0.46467	152	4
57	474	9.51154	37	580	9.97580	4	335	9.53574	41	0.46426	125	3
58	502	9.51191	36	571	9.97576	5	368	9.53615	41	0.46385	097	2
59	529	9.51227	37	561	9.97571	4	400	9.53656	41	0.46344	070	1
60	557	9.51264		552	9.97567		433	9.53697		0.46303	042	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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19°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	32557	9.51264	37	94552	9.97567	4	34433	9.53697	41	0.46303	2.9042	60
1	584	9.51301	37	542	9.97563	5	465	9.53738	41	0.46262	015	59
2	612	9.51338	36	533	9.97558	4	498	9.53779	41	0.46221	2.8987	58
3	639	9.51374	37	523	9.97554	4	530	9.53820	41	0.46180	960	57
4	667	9.51411	36	514	9.97550	5	563	9.53861	41	0.46139	933	56
5	32694	9.51447	37	94504	9.97545	4	34596	9.53902	41	0.46098	2.8905	55
6	722	9.51484	36	495	9.97541	5	628	9.53943	41	0.46057	878	54
7	749	9.51520	37	485	9.97536	4	661	9.53984	41	0.46016	851	53
8	777	9.51557	36	476	9.97532	4	693	9.54025	40	0.45975	824	52
9	804	9.51593	36	466	9.97528	5	726	9.54065	41	0.45935	797	51
10	32832	9.51629	37	94457	9.97523	4	34758	9.54106	41	0.45894	2.8770	50
11	859	9.51666	36	447	9.97519	4	791	9.54147	40	0.45853	743	49
12	887	9.51702	36	438	9.97515	5	824	9.54187	41	0.45813	716	48
13	914	9.51738	36	428	9.97510	4	856	9.54228	41	0.45772	689	47
14	942	9.51774	37	418	9.97506	5	889	9.54269	40	0.45731	662	46
15	32969	9.51811	36	94409	9.97501	4	34922	9.54309	41	0.45691	2.8636	45
16	997	9.51847	36	399	9.97497	5	954	9.54350	40	0.45650	609	44
17	33024	9.51883	36	390	9.97492	4	987	9.54390	41	0.45610	582	43
18	051	9.51919	36	380	9.97488	4	35020	9.54431	40	0.45569	556	42
19	079	9.51955	36	370	9.97484	5	052	9.54471	41	0.45529	529	41
20	33106	9.51991	36	94361	9.97479	4	35085	9.54512	40	0.45488	2.8502	40
21	134	9.52027	36	351	9.97475	5	118	9.54552	41	0.45448	476	39
22	161	9.52063	36	342	9.97470	4	150	9.54593	40	0.45407	449	38
23	189	9.52099	36	332	9.97466	5	183	9.54633	40	0.45367	423	37
24	216	9.52135	36	322	9.97461	4	216	9.54673	41	0.45327	397	36
25	33244	9.52171	36	94313	9.97457	4	35248	9.54714	40	0.45286	2.8370	35
26	271	9.52207	35	303	9.97453	5	281	9.54754	40	0.45246	344	34
27	298	9.52242	36	293	9.97448	4	314	9.54794	41	0.45206	318	33
28	326	9.52278	36	284	9.97444	5	346	9.54835	40	0.45165	291	32
29	353	9.52314	36	274	9.97439	4	379	9.54875	40	0.45125	265	31
30	33381	9.52350	35	94264	9.97435	5	35412	9.54915	40	0.45085	2.8239	30
31	408	9.52385	36	254	9.97430	4	445	9.54955	40	0.45045	213	29
32	436	9.52421	35	245	9.97426	5	477	9.54995	40	0.45005	187	28
33	463	9.52456	36	235	9.97421	4	510	9.55035	40	0.44965	161	27
34	490	9.52492	35	225	9.97417	5	543	9.55075	40	0.44925	135	26
35	33518	9.52527	36	94215	9.97412	4	35576	9.55115	40	0.44885	2.8109	25
36	545	9.52563	35	206	9.97408	5	608	9.55155	40	0.44845	083	24
37	573	9.52598	36	196	9.97403	4	641	9.55195	40	0.44805	057	23
38	600	9.52634	35	186	9.97399	5	674	9.55235	40	0.44765	032	22
39	627	9.52669	36	176	9.97394	4	707	9.55275	40	0.44725	006	21
40	33655	9.52705	35	94167	9.97390	5	35740	9.55315	40	0.44685	2.7980	20
41	682	9.52740	35	157	9.97385	4	772	9.55355	40	0.44645	955	19
42	710	9.52775	36	147	9.97381	5	805	9.55395	39	0.44605	929	18
43	737	9.52811	35	137	9.97376	4	838	9.55434	40	0.44566	903	17
44	764	9.52846	35	127	9.97372	5	871	9.55474	40	0.44526	878	16
45	33792	9.52881	35	94118	9.97367	4	35904	9.55514	40	0.44486	2.7852	15
46	819	9.52916	35	108	9.97363	5	937	9.55554	39	0.44446	827	14
47	846	9.52951	35	098	9.97358	5	969	9.55593	40	0.44407	801	13
48	874	9.52986	35	088	9.97353	4	36002	9.55633	40	0.44367	776	12
49	901	9.53021	35	078	9.97349	5	035	9.55673	39	0.44327	751	11
50	33929	9.53056	36	94068	9.97344	4	36068	9.55712	40	0.44288	2.7725	10
51	956	9.53092	34	058	9.97340	5	101	9.55752	39	0.44248	700	9
52	983	9.53126	35	049	9.97335	4	134	9.55791	40	0.44209	675	8
53	34011	9.53161	35	039	9.97331	5	167	9.55831	39	0.44169	650	7
54	038	9.53196	35	029	9.97326	4	199	9.55870	40	0.44130	625	6
55	34065	9.53231	35	94019	9.97322	5	36232	9.55910	39	0.44090	2.7600	5
56	093	9.53266	35	009	9.97317	5	265	9.55949	40	0.44051	575	4
57	120	9.53301	35	93999	9.97312	4	298	9.55989	39	0.44011	550	3
58	147	9.53336	34	989	9.97308	5	331	9.56028	39	0.43972	525	2
59	175	9.53370	35	979	9.97303	4	364	9.56067	40	0.43933	500	1
60	202	9.53405		969	9.97299		397	9.56107		0.43893	475	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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20°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	′
0	34202	9.53405		93969	9.97299		36397	9.56107		0.43893	2.7475	60
1	229	9.53440	35	959	9.97294	5	430	9.56146	39	0.43854	450	59
2	257	9.53475	35	949	9.97289	5	463	9.56185	39	0.43815	425	58
3	284	9.53509	34	939	9.97285	4	496	9.56224	39	0.43776	400	57
4	311	9.53544	35	929	9.97280	5	529	9.56264	40	0.43736	376	56
5	34339	9.53578	34	93919	9.97276	4	36562	9.56303	39	0.43697	2.7351	55
6	366	9.53613	35	909	9.97271	5	595	9.56342	39	0.43658	326	54
7	393	9.53647	34	899	9.97266	5	628	9.56381	39	0.43619	302	53
8	421	9.53682	35	889	9.97262	4	661	9.56420	39	0.43580	277	52
9	448	9.53716	34	879	9.97257	5	694	9.56459	39	0.43541	253	51
10	34475	9.53751	35	93869	9.97252	5	36727	9.56498	39	0.43502	2.7228	50
11	503	9.53785	34	859	9.97248	4	760	9.56537	39	0.43463	204	49
12	530	9.53819	34	849	9.97243	5	793	9.56576	39	0.43424	179	48
13	557	9.53854	35	839	9.97238	5	826	9.56615	39	0.43385	155	47
14	584	9.53888	34	829	9.97234	4	859	9.56654	39	0.43346	130	46
15	34612	9.53922	34	93819	9.97229	5	36892	9.56693	39	0.43307	2.7106	45
16	639	9.53957	35	809	9.97224	5	925	9.56732	39	0.43268	082	44
17	666	9.53991	34	799	9.97220	4	958	9.56771	39	0.43229	058	43
18	694	9.54025	34	789	9.97215	5	991	9.56810	39	0.43190	034	42
19	721	9.54059	34	779	9.97210	5	37024	9.56849	39	0.43151	009	41
20	34748	9.54093	34	93769	9.97206	4	37057	9.56887	38	0.43113	2.6985	40
21	775	9.54127	34	759	9.97201	5	090	9.56926	39	0.43074	961	39
22	803	9.54161	34	748	9.97196	5	123	9.56965	39	0.43035	937	38
23	830	9.54195	34	738	9.97192	4	157	9.57004	39	0.42996	913	37
24	857	9.54229	34	728	9.97187	5	190	9.57042	38	0.42958	889	36
25	34884	9.54263	34	93718	9.97182	5	37223	9.57081	39	0.42919	2.6865	35
26	912	9.54297	34	708	9.97178	4	256	9.57120	39	0.42880	841	34
27	939	9.54331	34	698	9.97173	5	289	9.57158	38	0.42842	818	33
28	966	9.54365	34	688	9.97168	5	322	9.57197	39	0.42803	794	32
29	993	9.54399	34	677	9.97163	5	355	9.57235	38	0.42765	770	31
30	35021	9.54433	34	93667	9.97159	4	37388	9.57274	39	0.42726	2.6746	30
31	048	9.54466	33	657	9.97154	5	422	9.57312	38	0.42688	723	29
32	075	9.54500	34	647	9.97149	5	455	9.57351	39	0.42649	699	28
33	102	9.54534	34	637	9.97145	4	488	9.57389	38	0.42611	675	27
34	130	9.54567	33	626	9.97140	5	521	9.57428	39	0.42572	652	26
35	35157	9.54601	34	93616	9.97135	5	37554	9.57466	38	0.42534	2.6628	25
36	184	9.54635	34	606	9.97130	5	588	9.57504	38	0.42496	605	24
37	211	9.54668	33	596	9.97126	4	621	9.57543	39	0.42457	581	23
38	239	9.54702	34	585	9.97121	5	654	9.57581	38	0.42419	558	22
39	266	9.54735	33	575	9.97116	5	687	9.57619	38	0.42381	534	21
40	35293	9.54769	34	93565	9.97111	5	37720	9.57658	39	0.42342	2.6511	20
41	320	9.54802	33	555	9.97107	4	754	9.57696	38	0.42304	488	19
42	347	9.54836	34	544	9.97102	5	787	9.57734	38	0.42266	464	18
43	375	9.54869	33	534	9.97097	5	820	9.57772	38	0.42228	441	17
44	402	9.54903	34	524	9.97092	5	853	9.57810	38	0.42190	418	16
45	35429	9.54936	33	93514	9.97087	5	37887	9.57849	39	0.42151	2.6395	15
46	456	9.54969	33	503	9.97083	4	920	9.57887	38	0.42113	371	14
47	484	9.55003	34	493	9.97078	5	953	9.57925	38	0.42075	348	13
48	511	9.55036	33	483	9.97073	5	986	9.57963	38	0.42037	325	12
49	538	9.55069	33	472	9.97068	5	38020	9.58001	38	0.41999	302	11
50	35565	9.55102	33	93462	9.97063	5	38053	9.58039	38	0.41961	2.6279	10
51	592	9.55136	34	452	9.97059	4	086	9.58077	38	0.41923	256	9
52	619	9.55169	33	441	9.97054	5	120	9.58115	38	0.41885	233	8
53	647	9.55202	33	431	9.97049	5	153	9.58153	38	0.41847	210	7
54	674	9.55235	33	420	9.97044	5	186	9.58191	38	0.41809	187	6
55	36701	9.55268	33	93410	9.97039	5	38220	9.58229	38	0.41771	2.6165	5
56	728	9.55301	33	400	9.97035	4	253	9.58267	38	0.41733	142	4
57	755	9.55334	33	389	9.97030	5	286	9.58304	37	0.41696	119	3
58	782	9.55367	33	379	9.97025	5	320	9.58342	38	0.41658	096	2
59	810	9.55400	33	368	9.97020	5	353	9.58380	38	0.41620	074	1
60	837	9.55433	33	358	9.97015	5	386	9.58418	38	0.41582	051	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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21°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	35837	9.55433	33	93358	9.97015	5	38386	9.58418	37	0.41582	2.6051	60
1	864	9.55466	33	348	9.97010	5	420	9.58455	38	0.41545	028	59
2	891	9.55499	33	337	9.97005	4	453	9.58493	38	0.41507	006	58
3	918	9.55532	32	327	9.97001	5	487	9.58531	38	0.41469	2.5983	57
4	945	9.55564	33	316	9.96996	5	520	9.58569	37	0.41431	961	56
5	35973	9.55597	33	93306	9.96991	5	38553	9.58606	38	0.41394	2.5938	55
6	36000	9.55630	33	295	9.96986	5	587	9.58644	37	0.41356	916	54
7	027	9.55663	32	285	9.96981	5	620	9.58681	38	0.41319	893	53
8	054	9.55695	33	274	9.96976	5	654	9.58719	38	0.41281	871	52
9	081	9.55728	33	264	9.96971	5	687	9.58757	37	0.41243	848	51
10	36108	9.55761	32	93253	9.96966	4	38721	9.58794	38	0.41206	2.5826	50
11	135	9.55793	33	243	9.96962	5	754	9.58832	37	0.41168	804	49
12	162	9.55826	32	232	9.96957	5	787	9.58869	38	0.41131	782	48
13	190	9.55858	33	222	9.96952	5	821	9.58907	37	0.41093	759	47
14	217	9.55891	32	211	9.96947	5	854	9.58944	37	0.41056	737	46
15	36244	9.55923	33	93201	9.96942	5	38888	9.58981	38	0.41019	2.5715	45
16	271	9.55956	32	190	9.96937	5	921	9.59019	37	0.40981	693	44
17	298	9.55988	33	180	9.96932	5	955	9.59056	38	0.40944	671	43
18	325	9.56021	32	169	9.96927	5	988	9.59094	37	0.40906	649	42
19	352	9.56053	32	159	9.96922	5	39022	9.59131	37	0.40869	627	41
20	36379	9.56085	33	93148	9.96917	5	39055	9.59168	37	0.40832	2.5605	40
21	406	9.56118	32	137	9.96912	5	089	9.59205	38	0.40795	583	39
22	434	9.56150	32	127	9.96907	4	122	9.59243	37	0.40757	561	38
23	461	9.56182	33	116	9.96903	5	156	9.59280	37	0.40720	539	37
24	488	9.56215	32	106	9.96898	5	190	9.59317	37	0.40683	517	36
25	36515	9.56247	32	93095	9.96893	5	39223	9.59354	37	0.40646	2.5495	35
26	542	9.56279	32	084	9.96888	5	257	9.59391	38	0.40609	473	34
27	569	9.56311	32	074	9.96883	5	290	9.59429	37	0.40571	452	33
28	596	9.56343	32	063	9.96878	5	324	9.59466	37	0.40534	430	32
29	623	9.56375	33	052	9.96873	5	357	9.59503	37	0.40497	408	31
30	36650	9.56408	32	93042	9.96868	5	39391	9.59540	37	0.40460	2.5386	30
31	677	9.56440	32	031	9.96863	5	425	9.59577	37	0.40423	365	29
32	704	9.56472	32	020	9.96858	5	458	9.59614	37	0.40386	343	28
33	731	9.56504	32	010	9.96853	5	492	9.59651	37	0.40349	322	27
34	758	9.56536	32	92999	9.96848	5	526	9.59688	37	0.40312	300	26
35	36785	9.56568	31	92988	9.96843	5	39559	9.59725	37	0.40275	2.5279	25
36	812	9.56599	32	978	9.96838	5	593	9.59762	37	0.40238	257	24
37	839	9.56631	32	967	9.96833	5	626	9.59799	36	0.40201	236	23
38	867	9.56663	32	956	9.96828	5	660	9.59835	37	0.40165	214	22
39	894	9.56695	32	945	9.96823	5	694	9.59872	37	0.40128	193	21
40	36921	9.56727	32	92935	9.96818	5	39727	9.59909	37	0.40091	2.5172	20
41	948	9.56759	31	924	9.96813	5	761	9.59946	37	0.40054	150	19
42	975	9.56790	32	913	9.96808	5	795	9.59983	36	0.40017	129	18
43	37002	9.56822	32	902	9.96803	5	829	9.60019	37	0.39981	108	17
44	029	9.56854	32	892	9.96798	5	862	9.60056	37	0.39944	086	16
45	37056	9.56886	31	92881	9.96793	5	39896	9.60093	37	0.39907	2.5065	15
46	083	9.56917	32	870	9.96788	5	930	9.60130	36	0.39870	044	14
47	110	9.56949	31	859	9.96783	5	963	9.60166	37	0.39834	023	13
48	137	9.56980	32	849	9.96778	6	997	9.60203	37	0.39797	002	12
49	164	9.57012	32	838	9.96772	5	40031	9.60240	36	0.39760	2.4981	11
50	37191	9.57044	31	92827	9.96767	5	40065	9.60276	37	0.39724	2.4960	10
51	218	9.57075	32	816	9.96762	5	098	9.60313	36	0.39687	939	9
52	245	9.57107	31	805	9.96757	5	132	9.60349	37	0.39651	918	8
53	272	9.57138	31	794	9.96752	5	166	9.60386	36	0.39614	897	7
54	299	9.57169	32	784	9.96747	5	200	9.60422	37	0.39578	876	6
55	37326	9.57201	31	92773	9.96742	5	40234	9.60459	36	0.39541	2.4855	5
56	353	9.57232	32	762	9.96737	5	267	9.60495	37	0.39505	834	4
57	380	9.57264	31	751	9.96732	5	301	9.60532	36	0.39468	813	3
58	407	9.57295	31	740	9.96727	5	335	9.60568	37	0.39432	792	2
59	434	9.57326	32	729	9.96722	5	369	9.60605	36	0.39395	772	1
60	461	9.57358		718	9.96717		403	9.60641		0.39359	751	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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22°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	37461	9.57358	31	92718	9.96717	6	40403	9.60641	36	0.39359	2.4751	60
1	488	9.57389	31	707	9.96711	5	436	9.60677	37	0.39323	730	59
2	515	9.57420	31	697	9.96706	5	470	9.60714	36	0.39286	709	58
3	542	9.57451	31	686	9.96701	5	504	9.60750	36	0.39250	689	57
4	569	9.57482	32	675	9.96696	5	538	9.60786	37	0.39214	668	56
5	37595	9.57514	31	92664	9.96691	5	40572	9.60823	36	0.39177	2.4648	55
6	622	9.57545	31	653	9.96686	5	606	9.60859	36	0.39141	627	54
7	649	9.57576	31	642	9.96681	5	640	9.60895	36	0.39105	606	53
8	676	9.57607	31	631	9.96676	6	674	9.60931	36	0.39069	586	52
9	703	9.57638	31	620	9.96670	5	707	9.60967	37	0.39033	566	51
10	37730	9.57669	31	92609	9.96665	5	40741	9.61004	36	0.38996	2.4545	50
11	757	9.57700	31	598	9.96660	5	775	9.61040	36	0.38960	525	49
12	784	9.57731	31	587	9.96655	5	809	9.61076	36	0.38924	504	48
13	811	9.57762	31	576	9.96650	5	843	9.61112	36	0.38888	484	47
14	838	9.57793	31	565	9.96645	5	877	9.61148	36	0.38852	464	46
15	37865	9.57824	31	92554	9.96640	6	40911	9.61184	36	0.38816	2.4443	45
16	892	9.57855	30	543	9.96634	5	945	9.61220	36	0.38780	423	44
17	919	9.57885	31	532	9.96629	5	979	9.61256	36	0.38744	403	43
18	946	9.57916	31	521	9.96624	5	41013	9.61292	36	0.38708	383	42
19	973	9.57947	31	510	9.96619	5	047	9.61328	36	0.38672	362	41
20	37999	9.57978	30	92499	9.96614	6	41081	9.61364	36	0.38636	2.4342	40
21	38026	9.58008	31	488	9.96608	5	115	9.61400	36	0.38600	322	39
22	053	9.58039	31	477	9.96603	5	149	9.61436	36	0.38564	302	38
23	080	9.58070	31	466	9.96598	5	183	9.61472	36	0.38528	282	37
24	107	9.58101	30	455	9.96593	5	217	9.61508	36	0.38492	262	36
25	38134	9.58131	31	92444	9.96588	6	41251	9.61544	35	0.38456	2.4242	35
26	161	9.58162	30	432	9.96582	5	285	9.61579	36	0.38421	222	34
27	188	9.58192	31	421	9.96577	5	319	9.61615	36	0.38385	202	33
28	215	9.58223	30	410	9.96572	5	353	9.61651	36	0.38349	182	32
29	241	9.58253	31	399	9.96567	5	387	9.61687	35	0.38313	162	31
30	38268	9.58284	30	92388	9.96562	6	41421	9.61722	36	0.38278	2.4142	30
31	295	9.58314	31	377	9.96556	5	455	9.61758	36	0.38242	122	29
32	322	9.58345	30	366	9.96551	5	490	9.61794	36	0.38206	102	28
33	349	9.58375	31	355	9.96546	5	524	9.61830	35	0.38170	083	27
34	376	9.58406	30	343	9.96541	6	558	9.61865	36	0.38135	063	26
35	38403	9.58436	31	92332	9.96535	5	41592	9.61901	35	0.38099	2.4043	25
36	430	9.58467	30	321	9.96530	5	626	9.61936	36	0.38064	023	24
37	456	9.58497	30	310	9.96525	5	660	9.61972	36	0.38028	004	23
38	483	9.58527	30	299	9.96520	6	694	9.62008	35	0.37992	2.3984	22
39	510	9.58557	31	287	9.96514	5	728	9.62043	36	0.37957	964	21
40	38537	9.58588	30	92276	9.96509	5	41763	9.62079	35	0.37921	2.3945	20
41	564	9.58618	30	265	9.96504	6	797	9.62114	36	0.37886	925	19
42	591	9.58648	30	254	9.96498	5	831	9.62150	35	0.37850	906	18
43	617	9.58678	31	243	9.96493	5	865	9.62185	36	0.37815	886	17
44	644	9.58709	30	231	9.96488	5	899	9.62221	35	0.37779	867	16
45	38671	9.58739	30	92220	9.96483	6	41933	9.62256	36	0.37744	2.3847	15
46	698	9.58769	30	209	9.96477	5	968	9.62292	35	0.37708	828	14
47	725	9.58799	30	198	9.96472	5	42002	9.62327	35	0.37673	808	13
48	752	9.58829	30	186	9.96467	6	036	9.62362	36	0.37638	789	12
49	778	9.58859	30	175	9.96461	5	070	9.62398	35	0.37602	770	11
50	38805	9.58889	30	92164	9.96456	5	42105	9.62433	35	0.37567	2.3750	10
51	832	9.58919	30	152	9.96451	6	139	9.62468	36	0.37532	731	9
52	859	9.58949	30	141	9.96445	5	173	9.62504	35	0.37496	712	8
53	886	9.58979	30	130	9.96440	5	207	9.62539	35	0.37461	693	7
54	912	9.59009	30	119	9.96435	6	242	9.62574	35	0.37426	673	6
55	38939	9.59039	30	92107	9.96429	5	42276	9.62609	36	0.37391	2.3654	5
56	966	9.59069	29	096	9.96424	5	310	9.62645	35	0.37355	635	4
57	993	9.59098	30	085	9.96419	6	345	9.62680	35	0.37320	616	3
58	39020	9.59128	30	073	9.96413	5	379	9.62715	35	0.37285	597	2
59	046	9.59158	30	062	9.96408	5	413	9.62750	35	0.37250	578	1
60	073	9.59188		050	9.96403		447	9.62785		0.37215	559	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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23°

′	Nat. Sin	Sin Log.	d.	Nat. Cos	Cos Log.	d.	Nat Tan	Tan Log.	c.d.	Log. Cot	Cot Nat.	
0	39073	9.59188	30	92050	9.96403	6	42447	9.62785	35	0.37215	2.3559	60
1	100	9.59218	29	039	9.96397	5	482	9.62820	35	0.37180	539	59
2	127	9.59247	30	028	9.96392	5	516	9.62855	35	0.37145	520	58
3	153	9.59277	30	016	9.96387	6	551	9.62890	36	0.37110	501	57
4	180	9.59307	29	005	9.96381	5	585	9.62926	35	0.37074	483	56
5	39207	9.59336	30	91994	9.96376	6	42619	9.62961	35	0.37039	2.3464	55
6	234	9.59366	30	982	9.96370	5	654	9.62996	35	0.37004	445	54
7	260	9.59396	29	971	9.96365	5	688	9.63031	35	0.36969	426	53
8	287	9.59425	30	959	9.96360	6	722	9.63066	35	0.36934	407	52
9	314	9.59455	29	948	9.96354	5	757	9.63101	34	0.36899	388	51
10	39341	9.59484	30	91936	9.96349	6	42791	9.63135	35	0.36865	2.3369	50
11	367	9.59514	29	925	9.96343	5	826	9.63170	35	0.36830	351	49
12	394	9.59543	30	914	9.96338	5	860	9.63205	35	0.36795	332	48
13	421	9.59573	29	902	9.96333	6	894	9.63240	35	0.36760	313	47
14	448	9.59602	30	891	9.96327	5	929	9.63275	35	0.36725	294	46
15	39474	9.59632	29	91879	9.96322	6	42963	9.63310	35	0.36690	2.3276	45
16	501	9.59661	29	868	9.96316	5	998	9.63345	34	0.36655	257	44
17	528	9.59690	30	856	9.96311	6	43032	9.63379	35	0.36621	238	43
18	555	9.59720	29	845	9.96305	5	067	9.63414	35	0.36586	220	42
19	581	9.59749	29	833	9.96300	6	101	9.63449	35	0.36551	201	41
20	39608	9.59778	30	91822	9.96294	5	43136	9.63484	35	0.36516	2.3183	40
21	635	9.59808	29	810	9.96289	5	170	9.63519	34	0.36481	164	39
22	661	9.59837	29	799	9.96284	6	205	9.63553	35	0.36447	146	38
23	688	9.59866	29	787	9.96278	5	239	9.63588	35	0.36412	127	37
24	715	9.59895	29	775	9.96273	6	274	9.63623	34	0.36377	109	36
25	39741	9.59924	30	91764	9.96267	5	43308	9.63657	35	0.36343	2.3090	35
26	768	9.59954	29	752	9.96262	6	343	9.63692	34	0.36308	072	34
27	795	9.59983	29	741	9.96256	5	378	9.63726	35	0.36274	053	33
28	822	9.60012	29	729	9.96251	6	412	9.63761	35	0.36239	035	32
29	848	9.60041	29	718	9.96245	5	447	9.63796	34	0.36204	017	31
30	39875	9.60070	29	91706	9.96240	6	43481	9.63830	35	0.36170	2.2998	30
31	902	9.60099	29	694	9.96234	5	516	9.63865	34	0.36135	980	29
32	928	9.60128	29	683	9.96229	6	550	9.63899	35	0.36101	962	28
33	955	9.60157	29	671	9.96223	5	585	9.63934	34	0.36066	944	27
34	982	9.60186	29	660	9.96218	6	620	9.63968	35	0.36032	925	26
35	40008	9.60215	29	91648	9.96212	5	43654	9.64003	34	0.35997	2.2907	25
36	035	9.60244	29	636	9.96207	6	689	9.64037	35	0.35963	889	24
37	062	9.60273	29	625	9.96201	5	724	9.64072	34	0.35928	871	23
38	088	9.60302	29	613	9.96196	6	758	9.64106	34	0.35894	853	22
39	115	9.60331	28	601	9.96190	5	793	9.64140	35	0.35860	835	21
40	40141	9.60359	29	91590	9.96185	6	43828	9.64175	34	0.35825	2.2817	20
41	168	9.60388	29	578	9.96179	5	862	9.64209	34	0.35791	799	19
42	195	9.60417	29	566	9.96174	6	897	9.64243	35	0.35757	781	18
43	221	9.60446	28	555	9.96168	6	932	9.64278	34	0.35722	763	17
44	248	9.60474	29	543	9.96162	5	966	9.64312	34	0.35688	745	16
45	40275	9.60503	29	91531	9.96157	6	44001	9.64346	35	0.35654	2.2727	15
46	301	9.60532	29	519	9.96151	5	036	9.64381	34	0.35619	709	14
47	328	9.60561	28	508	9.96146	6	071	9.64415	34	0.35585	691	13
48	355	9.60589	29	496	9.96140	5	105	9.64449	34	0.35551	673	12
49	381	9.60618	28	484	9.96135	6	140	9.64483	34	0.35517	655	11
50	40408	9.60646	29	91472	9.96129	6	44175	9.64517	35	0.35483	2.2637	10
51	434	9.60675	29	461	9.96123	5	210	9.64552	34	0.35448	620	9
52	461	9.60704	28	449	9.96118	6	244	9.64586	34	0.35414	602	8
53	488	9.60732	29	437	9.96112	5	279	9.64620	34	0.35380	584	7
54	514	9.60761	28	425	9.96107	6	314	9.64654	34	0.35346	566	6
55	40541	9.60789	29	91414	9.96101	6	44349	9.64688	34	0.35312	2.2549	5
56	567	9.60818	28	402	9.96095	5	384	9.64722	34	0.35278	531	4
57	594	9.60846	29	390	9.96090	6	418	9.64756	34	0.35244	513	3
58	621	9.60875	28	378	9.96084	5	453	9.64790	34	0.35210	496	2
59	647	9.60903	28	366	9.96079	6	488	9.64824	34	0.35176	478	1
60	674	9.60931		355	9.96073		523	9.64858		0.35142	460	0
	Nat. Cos	Cos Log.	d.	Nat. Sin	Sin Log.	d.	Nat. Cot	Cot Log.	c.d.	Log. Tan	Tan Nat.	′

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24°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	40674	9.60931	29	91355	9.96073	6	44523	9.64858	34	0.35142	2.2460	60
1	700	9.60960	28	343	9.96067	5	558	9.64892	34	0.35108	443	59
2	727	9.60988	28	331	9.96062	6	593	9.64926	34	0.35074	425	58
3	753	9.61016	29	319	9.96056	6	627	9.64960	34	0.35040	408	57
4	780	9.61045	28	307	9.96050	5	662	9.64994	34	0.35006	390	56
5	40806	9.61073	28	91295	9.96045	6	44697	9.65028	34	0.34972	2.2373	55
6	833	9.61101	28	283	9.96039	5	732	9.65062	34	0.34938	355	54
7	860	9.61129	29	272	9.96034	6	767	9.65096	34	0.34904	338	53
8	886	9.61158	28	260	9.96028	6	802	9.65130	34	0.34870	320	52
9	913	9.61186	28	248	9.96022	5	837	9.65164	33	0.34836	303	51
10	40939	9.61214	28	91236	9.96017	6	44872	9.65197	34	0.34803	2.2286	50
11	966	9.61242	28	224	9.96011	6	907	9.65231	34	0.34769	268	49
12	992	9.61270	28	212	9.96005	5	942	9.65265	34	0.34735	251	48
13	41019	9.61298	28	200	9.96000	6	977	9.65299	34	0.34701	234	47
14	045	9.61326	28	188	9.95994	6	45012	9.65333	33	0.34667	216	46
15	41072	9.61354	28	91176	9.95988	6	45047	9.65366	34	0.34634	2.2199	45
16	098	9.61382	29	164	9.95982	5	082	9.65400	34	0.34600	182	44
17	125	9.61411	27	152	9.95977	6	117	9.65434	33	0.34566	165	43
18	151	9.61438	28	140	9.95971	6	152	9.65467	34	0.34533	148	42
19	178	9.61466	28	128	9.95965	5	187	9.65501	34	0.34499	130	41
20	41204	9.61494	28	91116	9.95960	6	45222	9.65535	33	0.34465	2.2113	40
21	231	9.61522	28	104	9.95954	6	257	9.65568	34	0.34432	096	39
22	257	9.61550	28	092	9.95948	6	292	9.65602	34	0.34398	079	38
23	284	9.61578	28	080	9.95942	5	327	9.65636	33	0.34364	062	37
24	310	9.61606	28	068	9.95937	6	362	9.65669	34	0.34331	045	36
25	41337	9.61634	28	91056	9.95931	6	45397	9.65703	33	0.34297	2.2028	35
26	363	9.61662	27	044	9.95925	5	432	9.65736	34	0.34264	011	34
27	390	9.61689	28	032	9.95920	6	467	9.65770	33	0.34230	2.1994	33
28	416	9.61717	28	020	9.95914	6	502	9.65803	34	0.34197	977	32
29	443	9.61745	28	008	9.95908	6	538	9.65837	33	0.34163	960	31
30	41469	9.61773	27	90996	9.95902	5	45573	9.65870	34	0.34130	2.1943	30
31	496	9.61800	28	984	9.95897	6	608	9.65904	33	0.34096	926	29
32	522	9.61828	28	972	9.95891	6	643	9.65937	34	0.34063	909	28
33	549	9.61856	27	960	9.95885	6	678	9.65971	33	0.34029	892	27
34	575	9.61883	28	948	9.95879	6	713	9.66904	34	0.33996	876	26
35	41602	9.61911	28	90936	9.95873	5	45748	9.66038	33	0.33962	2.1859	25
36	628	9.61939	27	924	9.95868	6	784	9.66071	33	0.33929	842	24
37	655	9.61966	28	911	9.95862	6	819	9.66104	34	0.33896	825	23
38	681	9.61994	27	899	9.95856	6	854	9.66138	33	0.33862	808	22
39	707	9.62021	28	887	9.95850	6	889	9.66171	33	0.33829	792	21
40	41734	9.62049	27	90875	9.95844	5	45924	9.66204	34	0.33796	2.1775	20
41	760	9.62076	28	863	9.95839	6	960	9.66238	33	0.33762	758	19
42	787	9.62104	27	851	9.95833	6	995	9.66271	33	0.33729	742	18
43	813	9.62131	28	839	9.95827	6	46030	9.66304	33	0.33696	725	17
44	840	9.62159	27	826	9.95821	6	065	9.66337	34	0.33663	708	16
45	41866	9.62186	28	90814	9.95815	5	46101	9.66371	33	0.33629	2.1692	15
46	892	9.62214	27	802	9.95810	6	136	9.66404	33	0.33596	675	14
47	919	9.62241	27	790	9.95804	6	171	9.66437	33	0.33563	659	13
48	945	9.62268	28	778	9.95798	6	206	9.66470	33	0.33530	642	12
49	972	9.62296	27	766	9.95792	6	242	9.66503	34	0.33497	625	11
50	41998	9.62323	27	90753	9.95786	6	46277	9.66537	33	0.33463	2.1609	10
51	42024	9.62350	27	741	9.95780	5	312	9.66570	33	0.33430	592	9
52	051	9.62377	28	729	9.95775	6	348	9.66603	33	0.33397	576	8
53	077	9.62405	27	717	9.95769	6	383	9.66636	33	0.33364	560	7
54	104	9.62432	27	704	9.95763	6	418	9.66669	33	0.33331	543	6
55	42130	9.62459	27	90692	9.95757	6	46454	9.66702	33	0.33298	2.1527	5
56	156	9.62486	27	680	9.95751	6	489	9.66735	33	0.33265	510	4
57	183	9.62513	28	668	9.95745	6	525	9.66768	33	0.33232	494	3
58	209	9.62541	27	655	9.95739	6	560	9.66801	33	0.33199	478	2
59	235	9.62568	27	643	9.95733	5	595	9.66834	33	0.33166	461	1
60	262	9.62595		631	9.95728		631	9.66867		0.33133	445	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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25°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	42262	9.62595	27	90631	9.95728	6	46631	9.66867	33	0.33133	2.1445	60
1	288	9.62622	27	618	9.95722	6	666	9.66900	33	0.33100	429	59
2	315	9.62649	27	606	9.95716	6	702	9.66933	33	0.33067	413	58
3	341	9.62676	27	594	9.95710	6	737	9.66966	33	0.33034	396	57
4	367	9.62703	27	582	9.95704	6	772	9.66999	33	0.33001	380	56
5	42394	9.62730	27	90569	9.95698	6	46808	9.67032	33	0.32968	2.1364	55
6	420	9.62757	27	557	9.95692	6	843	9.67065	33	0.32935	348	54
7	446	9.62784	27	545	9.95686	6	879	9.67098	33	0.32902	332	53
8	473	9.62811	27	532	9.95680	6	914	9.67131	32	0.32869	315	52
9	499	9.62838	27	520	9.95674	6	950	9.67163	33	0.32837	299	51
10	42525	9.62865	27	90507	9.95668	5	46985	9.67196	33	0.32804	2.1283	50
11	552	9.62892	26	495	9.95663	6	47021	9.67229	33	0.32771	267	49
12	578	9.62918	27	483	9.95657	6	056	9.67262	33	0.32738	251	48
13	604	9.62945	27	470	9.95651	6	092	9.67295	32	0.32705	235	47
14	631	9.62972	27	458	9.95645	6	128	9.67327	33	0.32673	219	46
15	42657	9.62999	27	90446	9.95639	6	47163	9.67360	33	0.32640	2.1203	45
16	683	9.63026	26	433	9.95633	6	199	9.67393	33	0.32607	187	44
17	709	9.63052	27	421	9.95627	6	234	9.67426	32	0.32574	171	43
18	736	9.63079	27	408	9.95621	6	270	9.67458	33	0.32542	155	42
19	762	9.63106	27	396	9.95615	6	305	9.67491	33	0.32509	139	41
20	42788	9.63133	26	90383	9.95609	6	47341	9.67524	32	0.32476	2.1123	40
21	815	9.63159	27	371	9.95603	6	377	9.67556	33	0.32444	107	39
22	841	9.63186	27	358	9.95597	6	412	9.67589	33	0.32411	092	38
23	867	9.63213	26	346	9.95591	6	448	9.67622	32	0.32378	076	37
24	894	9.63239	27	334	9.95585	6	483	9.67654	33	0.32346	060	36
25	42920	9.63266	26	90321	9.95579	6	47519	9.67687	32	0.32313	2.1044	35
26	946	9.63292	27	309	9.95573	6	555	9.67719	33	0.32281	028	34
27	972	9.63319	26	296	9.95567	6	590	9.67752	33	0.32248	013	33
28	999	9.63345	27	284	9.95561	6	626	9.67785	32	0.32215	2.0997	32
29	43025	9.63372	26	271	9.95555	6	662	9.67817	33	0.32183	981	31
30	43051	9.63398	27	90259	9.95549	6	47698	9.67850	32	0.32150	2.0965	30
31	077	9.63425	26	246	9.95543	6	733	9.67882	33	0.32118	950	29
32	104	9.63451	27	233	9.95537	6	769	9.67915	32	0.32085	934	28
33	130	9.63478	26	221	9.95531	6	805	9.67947	33	0.32053	918	27
34	156	9.63504	27	208	9.95525	6	840	9.67980	32	0.32020	903	26
35	43182	9.63531	26	90196	9.95519	6	47876	9.68012	32	0.31988	2.0887	25
36	209	9.63557	26	183	9.95513	6	912	9.68044	33	0.31956	872	24
37	235	9.63583	27	171	9.95507	7	948	9.68077	32	0.31923	856	23
38	261	9.63610	26	158	9.95500	6	984	9.68109	33	0.31891	840	22
39	287	9.63636	26	146	9.95494	6	48019	9.68142	32	0.31858	825	21
40	43313	9.63662	27	90133	9.95488	6	48055	9.68174	32	0.31826	2.0809	20
41	340	9.63689	26	120	9.95482	6	091	9.68206	33	0.31794	794	19
42	366	9.63715	26	108	9.95476	6	127	9.68239	32	0.31761	778	18
43	392	9.63741	26	095	9.95470	6	163	9.68271	32	0.31729	763	17
44	418	9.63767	27	082	9.95464	6	198	9.68303	33	0.31697	748	16
45	43445	9.63794	26	90070	9.95458	6	48234	9.68336	32	0.31664	2.0732	15
46	471	9.63820	26	057	9.95452	6	270	9.68368	32	0.31632	717	14
47	497	9.63846	26	045	9.95446	6	306	9.68400	32	0.31600	701	13
48	523	9.63872	26	032	9.95440	6	342	9.68432	33	0.31568	686	12
49	549	9.63898	26	019	9.95434	7	378	9.68465	32	0.31535	671	11
50	43575	9.63924	26	90007	9.95427	6	48414	9.68497	32	0.31503	2.0655	10
51	602	9.63950	26	89994	9.95421	6	450	9.68529	32	0.31471	640	9
52	628	9.63976	26	981	9.95415	6	486	9.68561	32	0.31439	625	8
53	654	9.64002	26	968	9.95409	6	521	9.68593	33	0.31407	609	7
54	680	9.64028	26	956	9.95403	6	557	9.68626	32	0.31374	594	6
55	43706	9.64054	26	89943	9.95397	6	48593	9.68658	32	0.31342	2.0579	5
56	733	9.64080	26	930	9.95391	7	629	9.68690	32	0.31310	564	4
57	759	9.64106	26	918	9.95384	6	665	9.68722	32	0.31278	549	3
58	785	9.64132	26	905	9.95378	6	701	9.68754	32	0.31246	533	2
59	811	9.64158	26	892	9.95372	6	737	9.68786	32	0.31214	518	1
60	837	9.64184		879	9.95366		773	9.68818		0.31182	503	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	43837	9.64184	26	89879	9.95366	6	48773	9.68818	32	0.31182	2.0503	60
1	863	9.64210	26	867	9.95360	6	809	9.68850	32	0.31150	488	59
2	889	9.64236	26	854	9.95354	6	845	9.68882	32	0.31118	473	58
3	916	9.64262	26	841	9.95348	7	881	9.68914	32	0.31086	458	57
4	942	9.64288	25	828	9.95341	6	917	9.68946	32	0.31054	443	56
5	43968	9.64313	26	89816	9.95335	6	48953	9.68978	32	0.31022	2.0428	55
6	994	9.64339	26	803	9.95329	6	989	9.69010	32	0.30990	413	54
7	44020	9.64365	26	790	9.95323	6	49026	9.69042	32	0.30958	398	53
8	046	9.64391	26	777	9.95317	7	062	9.69074	32	0.30926	383	52
9	072	9.64417	25	764	9.95310	6	098	9.69106	32	0.30894	368	51
10	44098	9.64442	26	89752	9.95304	6	49134	9.69138	32	0.30862	2.0353	50
11	124	9.64468	26	739	9.95298	6	170	9.69170	32	0.30830	338	49
12	151	9.64494	25	726	9.95292	6	206	9.69202	32	0.30798	323	48
13	177	9.64519	26	713	9.95286	7	242	9.69234	32	0.30766	308	47
14	203	9.64545	26	700	9.95279	6	278	9.69266	32	0.30734	293	46
15	44229	9.64571	25	89687	9.95273	6	49315	9.69298	31	0.30702	2.0278	45
16	255	9.64596	26	674	9.95267	6	351	9.69329	32	0.30671	263	44
17	281	9.64622	25	662	9.95261	7	387	9.69361	32	0.30639	248	43
18	307	9.64647	26	649	9.95254	6	423	9.69393	32	0.30607	233	42
19	333	9.64673	25	636	9.95248	6	459	9.69425	32	0.30575	219	41
20	44359	9.64698	26	89623	9.95242	6	49495	9.69457	31	0.30543	2.0204	40
21	385	9.64724	25	610	9.95236	7	532	9.69488	32	0.30512	189	39
22	411	9.64749	26	597	9.95229	6	568	9.69520	32	0.30480	174	38
23	437	9.64775	25	584	9.95223	6	604	9.69552	32	0.30448	160	37
24	464	9.64800	26	571	9.95217	6	640	9.69584	31	0.30416	145	36
25	44490	9.64826	25	89558	9.95211	7	49677	9.69615	32	0.30385	2.0130	35
26	516	9.64851	26	545	9.95204	6	713	9.69647	32	0.30353	115	34
27	542	9.64877	25	532	9.95198	6	749	9.69679	31	0.30321	101	33
28	568	9.64902	25	519	9.95192	7	786	9.69710	32	0.30290	086	32
29	594	9.64927	26	506	9.95185	6	822	9.69742	32	0.30258	072	31
30	44620	9.64953	25	89493	9.95179	6	49858	9.69774	31	0.30226	2.0057	30
31	646	9.64978	25	480	9.95173	6	894	9.69805	32	0.30195	042	29
32	672	9.65003	26	467	9.95167	7	931	9.69837	31	0.30163	028	28
33	698	9.65029	25	454	9.95160	6	967	9.69868	32	0.30132	013	27
34	724	9.65054	25	441	9.95154	6	50004	9.69900	32	0.30100	1.9999	26
35	44750	9.65079	25	89428	9.95148	7	50040	9.69932	31	0.30068	1.9984	25
36	776	9.65104	26	415	9.95141	6	076	9.69963	32	0.30037	970	24
37	802	9.65130	25	402	9.95135	6	113	9.69995	31	0.30005	955	23
38	828	9.65155	25	389	9.95129	7	149	9.70026	32	0.29974	941	22
39	854	9.65180	25	376	9.95122	6	185	9.70058	31	0.29942	926	21
40	44880	9.65205	25	89363	9.95116	6	50222	9.70089	32	0.29911	1.9912	20
41	906	9.65230	25	350	9.95110	7	258	9.70121	31	0.29879	897	19
42	932	9.65255	26	337	9.95103	6	295	9.70152	32	0.29848	883	18
43	958	9.65281	25	324	9.95097	7	331	9.70184	31	0.29816	868	17
44	984	9.65306	25	311	9.95090	6	368	9.70215	32	0.29785	854	16
45	45010	9.65331	25	89298	9.95084	6	50404	9.70247	31	0.29753	1.9840	15
46	036	9.65356	25	285	9.95078	7	441	9.70278	31	0.29722	825	14
47	062	9.65381	25	272	9.95071	6	477	9.70309	32	0.29691	811	13
48	088	9.65406	25	259	9.95065	6	514	9.70341	31	0.29659	797	12
49	114	9.65431	25	245	9.95059	7	550	9.70372	32	0.29628	782	11
50	45140	9.65456	25	89232	9.95052	6	50587	9.70404	31	0.29596	1.9768	10
51	166	9.65481	25	219	9.95046	7	623	9.70435	31	0.29565	754	9
52	192	9.65506	25	206	9.95039	6	660	9.70466	32	0.29534	740	8
53	218	9.65531	25	193	9.95033	6	696	9.70498	31	0.29502	725	7
54	243	9.65556	24	180	9.95027	7	733	9.70529	31	0.29471	711	6
55	45269	9.65580	25	89167	9.95020	6	50769	9.70560	32	0.29440	1.9697	5
56	295	9.65605	25	153	9.95014	7	806	9.70592	31	0.29408	683	4
57	321	9.65630	25	140	9.95007	6	843	9.70623	31	0.29377	669	3
58	347	9.65655	25	127	9.95001	6	879	9.70654	31	0.29346	654	2
59	373	9.65680	25	114	9.94995	7	916	9.70685	32	0.29315	640	1
60	399	9.65705		101	9.94988		953	9.70717		0.29283	626	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat Tan	Log.	c.d.	Log. Cot	Nat.	
0	45399	9.65705	24	89101	9.94988	6	50953	9.70717	31	0.29283	1.9626	60
1	425	9.65729	25	087	9.94982	7	989	9.70748	31	0.29252	612	59
2	451	9.65754	25	074	9.94975	6	51026	9.70779	31	0.29221	598	58
3	477	9.65779	25	061	9.94969	7	063	9.70810	31	0.29190	584	57
4	503	9.65804	24	048	9.94962	6	099	9.70841	32	0.29159	570	56
5	45529	9.65828	25	89035	9.94956	7	51136	9.70873	31	0.29127	1.9556	55
6	554	9.65853	25	021	9.94949	6	173	9.70904	31	0.29096	542	54
7	580	9.65878	24	008	9.94943	7	209	9.70935	31	0.29065	528	53
8	606	9.65902	25	88995	9.94936	6	246	9.70966	31	0.29034	514	52
9	632	9.65927	25	981	9.94930	7	283	9.70997	31	0.29003	500	51
10	45658	9.65952	24	88968	9.94923	6	51319	9.71028	31	0.28972	1.9486	50
11	684	9.65976	25	955	9.94917	6	356	9.71059	31	0.28941	472	49
12	710	9.66001	24	942	9.94911	7	393	9.71090	31	0.28910	458	48
13	736	9.66025	25	928	9.94904	6	430	9.71121	32	0.28879	444	47
14	762	9.66050	25	915	9.94898	7	467	9.71153	31	0.28847	430	46
15	45787	9.66075	24	88902	9.94891	6	51503	9.71184	31	0.28816	1.9416	45
16	813	9.66099	25	888	9.94885	7	540	9.71215	31	0.28785	402	44
17	839	9.66124	24	875	9.94878	7	577	9.71246	31	0.28754	388	43
18	865	9.66148	25	862	9.94871	6	614	9.71277	31	0.28723	375	42
19	891	9.66173	24	848	9.94865	7	651	9.71308	31	0.28692	361	41
20	45917	9.66197	24	88835	9.94858	6	51688	9.71339	31	0.28661	1.9347	40
21	942	9.66221	25	822	9.94852	7	724	9.71370	31	0.28630	333	39
22	968	9.66246	24	808	9.94845	6	761	9.71401	30	0.28599	319	38
23	994	9.66270	25	795	9.94839	7	798	9.71431	31	0.28569	306	37
24	46020	9.66295	24	782	9.94832	6	835	9.71462	31	0.28538	292	36
25	46046	9.66319	24	88768	9.94826	7	51872	9.71493	31	0.28507	1.9278	35
26	072	9.66343	25	755	9.94819	6	909	9.71524	31	0.28476	265	34
27	097	9.66368	24	741	9.94813	7	946	9.71555	31	0.28445	251	33
28	123	9.66392	24	728	9.94806	7	983	9.71586	31	0.28414	237	32
29	149	9.66416	25	715	9.94799	6	52020	9.71617	31	0.28383	223	31
30	46175	9.66441	24	88701	9.94793	7	52057	9.71648	31	0.28352	1.9210	30
31	201	9.66465	24	688	9.94786	6	094	9.71679	30	0.28321	196	29
32	226	9.66489	24	674	9.94780	7	131	9.71709	31	0.28291	183	28
33	252	9.66513	24	661	9.94773	6	168	9.71740	31	0.28260	169	27
34	278	9.66537	25	647	9.94767	7	205	9.71771	31	0.28229	155	26
35	46304	9.66562	24	88634	9.94760	7	52242	9.71802	31	0.28198	1.9142	25
36	330	9.66586	24	620	9.94753	6	279	9.71833	30	0.28167	128	24
37	355	9.66610	24	607	9.94747	7	316	9.71863	31	0.28137	115	23
38	381	9.66634	24	593	9.94740	6	353	9.71894	31	0.28106	101	22
39	407	9.66658	24	580	9.94734	7	390	9.71925	30	0.28075	088	21
40	46433	9.66682	24	88566	9.94727	7	52427	9.71955	31	0.28045	1.9074	20
41	458	9.66706	25	553	9.94720	6	464	9.71986	31	0.28014	061	19
42	484	9.66731	24	539	9.94714	7	501	9.72017	31	0.27983	047	18
43	510	9.66755	24	526	9.94707	7	538	9.72048	30	0.27952	034	17
44	536	9.66779	24	512	9.94700	6	575	9.72078	31	0.27922	020	16
45	46561	9.66803	24	88499	9.94694	7	52613	9.72109	31	0.27891	1.9007	15
46	587	9.66827	24	485	9.94687	7	650	9.72140	30	0.27860	1.8993	14
47	613	9.66851	24	472	9.94680	6	687	9.72170	31	0.27830	980	13
48	639	9.66875	24	458	9.94674	7	724	9.72201	30	0.27799	967	12
49	664	9.66899	23	445	9.94667	7	761	9.72231	31	0.27769	953	11
50	46690	9.66922	24	88431	9.94660	6	52798	9.72262	31	0.27738	1.8940	10
51	716	9.66946	24	417	9.94654	7	836	9.72293	30	0.27707	927	9
52	742	9.66970	24	404	9.94647	7	873	9.72323	31	0.27677	913	8
53	767	9.66994	24	390	9.94640	6	910	9.72354	30	0.27646	900	7
54	793	9.67018	24	377	9.94634	7	947	9.72384	31	0.27616	887	6
55	46819	9.67042	24	88363	9.94627	7	52985	9.72415	30	0.27585	1.8873	5
56	844	9.67066	24	349	9.94620	6	53022	9.72445	31	0.27555	860	4
57	870	9.67090	23	336	9.94614	7	059	9.72476	30	0.27524	847	3
58	896	9.67113	24	322	9.94607	7	096	9.72506	31	0.27494	834	2
59	921	9.67137	24	308	9.94600	7	134	9.72537	30	0.27463	820	1
60	947	9.67161		295	9.94593		171	9.72567		0.27433	807	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	46947	9.67161	24	88295	9.94593	6	53171	9.72567	31	0.27433	1.8807	60
1	973	9.67185	23	281	9.94587	7	208	9.72598	30	0.27402	794	59
2	999	9.67208	24	267	9.94580	7	246	9.72628	31	0.27372	781	58
3	47024	9.67232	24	254	9.94573	6	283	9.72659	30	0.27341	768	57
4	050	9.67256	24	240	9.94567	7	320	9.72689	31	0.27311	755	56
5	47076	9.67280	23	88226	9.94560	7	53358	9.72720	30	0.27280	1.8741	55
6	101	9.67303	24	213	9.94553	7	395	9.72750	30	0.27250	728	54
7	127	9.67327	23	199	9.94546	6	432	9.72780	31	0.27220	715	53
8	153	9.67350	24	185	9.94540	7	470	9.72811	30	0.27189	702	52
9	178	9.67374	24	172	9.94533	7	507	9.72841	31	0.27159	689	51
10	47204	9.67398	23	88158	9.94526	7	53545	9.72872	30	0.27128	1.8676	50
11	229	9.67421	24	144	9.94519	6	582	9.72902	30	0.27098	663	49
12	255	9.67445	23	130	9.94513	7	620	9.72932	31	0.27068	650	48
13	281	9.67468	24	117	9.94506	7	657	9.72963	30	0.27037	637	47
14	306	9.67492	23	103	9.94499	7	694	9.72993	30	0.27007	624	46
15	47332	9.67515	24	88089	9.94492	7	53732	9.73023	31	0.26977	1.8611	45
16	358	9.67539	23	075	9.94485	6	769	9.73054	30	0.26946	598	44
17	383	9.67562	24	062	9.94479	7	807	9.83084	30	0.26916	585	43
18	409	9.67586	23	048	9.94472	7	844	9.73114	30	0.26886	572	42
19	434	9.67609	24	034	9.94465	7	882	9.73144	31	0.26856	559	41
20	47460	9.67633	23	88020	9.94458	7	53920	9.73175	30	0.26825	1.8546	40
21	486	9.67656	24	006	9.94451	6	957	9.73205	30	0.26795	533	39
22	511	9.67680	23	87993	9.94445	7	995	9.73235	30	0.26765	520	38
23	537	9.67703	23	979	9.94438	7	54032	9.73265	30	0.26735	507	37
24	562	9.67726	24	965	9.94431	7	070	9.73295	31	0.26705	495	36
25	47588	9.67750	23	87951	9.94424	7	54107	9.73326	30	0.26674	1.8482	35
26	614	9.67773	23	937	9.94417	7	145	9.73356	30	0.26644	469	34
27	639	9.67796	24	923	9.94410	6	183	9.73386	30	0.26614	456	33
28	665	9.67820	23	909	9.94404	7	220	9.73416	30	0.26584	443	32
29	690	9.67843	23	896	9.94397	7	258	9.73446	30	0.26554	430	31
30	47716	9.67866	24	87882	9.94390	7	54296	9.73476	31	0.26524	1.8418	30
31	741	9.67890	23	868	9.94383	7	333	9.73507	30	0.26493	405	29
32	767	9.67913	23	854	9.94376	7	371	9.73537	30	0.26463	392	28
33	793	9.67936	23	840	9.94369	7	409	9.73567	30	0.26433	379	27
34	818	9.67959	23	826	9.94362	7	446	9.73597	30	0.26403	367	26
35	47844	9.67982	24	87812	9.94355	6	54484	9.73627	30	0.26373	1.8354	25
36	869	9.68006	23	798	9.94349	7	522	9.73657	30	0.26343	341	24
37	895	9.68029	23	784	9.94342	7	560	9.73687	30	0.26313	329	23
38	920	9.68052	23	770	9.94335	7	597	9.73717	30	0.26283	316	22
39	946	9.68075	23	756	9.94328	7	635	9.73747	30	0.26253	303	21
40	47971	9.68098	23	87743	9.94321	7	54673	9.73777	30	0.26223	1.8291	20
41	997	9.68121	23	729	9.94314	7	711	9.73807	30	0.26193	278	19
42	48022	9.68144	23	715	9.94307	7	748	9.73837	30	0.26163	265	18
43	048	9.68167	23	701	9.94300	7	786	9.73867	30	0.26133	253	17
44	073	9.68190	23	687	9.94293	7	824	9.73897	30	0.26103	240	16
45	48099	9.68213	24	87673	9.94286	7	54862	9.73927	30	0.26073	1.8228	15
46	124	9.68237	23	659	9.94279	6	900	9.73957	30	0.26043	215	14
47	150	9.68260	23	645	9.94273	7	938	9.73987	30	0.26013	202	13
48	175	9.68283	22	631	9.94266	7	975	9.74017	30	0.25983	190	12
49	201	9.68305	23	617	9.94259	7	55013	9.74047	30	0.25953	177	11
50	48226	9.68328	23	87603	9.94252	7	55051	9.74077	30	0.25923	1.8165	10
51	252	9.68351	23	589	9.94245	7	089	9.74107	30	0.25893	152	9
52	277	9.68374	23	575	9.94238	7	127	9.74137	29	0.25863	140	8
53	303	9.68397	23	561	9.94231	7	165	9.74166	30	0.25834	127	7
54	328	9.68420	23	546	9.94224	7	203	9.74196	30	0.25804	115	6
55	48354	9.68443	23	87532	9.94217	7	55241	9.74226	30	0.25774	1.8103	5
56	379	9.68466	23	518	9.94210	7	279	9.74256	30	0.25744	090	4
57	405	9.68489	23	504	9.94203	7	317	9.74286	30	0.25714	078	3
58	430	9.68512	22	490	9.94196	7	355	9.74316	29	0.25684	065	2
59	456	9.68534	23	476	9.94189	7	393	9.74345	30	0.25655	053	1
60	481	9.68557		462	9.94182		431	9.74375		0.25625	040	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	′
0	48481	9.68557	23	87462	9.94182	7	55431	9.74375	30	0.25625	1.8040	60
1	506	9.68580	23	448	9.94175	7	469	9.74405	30	0.25595	028	59
2	532	9.68603	22	434	9.94168	7	507	9.74435	30	0.25565	016	58
3	557	9.68625	23	420	9.94161	7	545	9.74465	29	0.25535	003	57
4	583	9.68648	23	406	9.94154	7	583	9.74494	30	0.25506	1.7991	56
5	48608	9.68671	23	87391	9.94147	7	55621	9.74524	30	0.25476	1.7979	55
6	634	9.68694	22	377	9.94140	7	659	9.74554	29	0.25446	966	54
7	659	9.68716	23	363	9.94133	7	697	9.74583	30	0.25417	954	53
8	684	9.68739	23	349	9.94126	7	736	9.74613	30	0.25387	942	52
9	710	9.68762	22	335	9.94119	7	774	9.74643	30	0.25357	930	51
10	48735	9.68784	23	87321	9.94112	7	55812	9.74673	29	0.25327	1.7917	50
11	761	9.68807	22	306	9.94105	7	850	9.74702	30	0.25298	905	49
12	786	9.68829	23	292	9.94098	8	888	9.74732	30	0.25268	893	48
13	811	9.68852	23	278	9.94090	7	926	9.74762	29	0.25238	881	47
14	837	9.68875	22	264	9.94083	7	964	9.74791	30	0.25209	868	46
15	48862	9.68897	23	87250	9.94076	7	56003	9.74821	30	0.25179	1.7856	45
16	888	9.68920	22	235	9.94069	7	041	9.74851	29	0.25149	844	44
17	913	9.68942	23	221	9.94062	7	079	9.74880	30	0.25120	832	43
18	938	9.68965	22	207	9.94055	7	117	9.74910	29	0.25090	820	42
19	964	9.68987	23	193	9.94048	7	156	9.74939	30	0.25061	808	41
20	48989	9.69010	22	87178	9.94041	7	56194	9.74969	29	0.25031	1.7796	40
21	49014	9.69032	23	164	9.94034	7	232	9.74998	30	0.25002	783	39
22	040	9.69055	22	150	9.94027	7	270	9.75028	30	0.24972	771	38
23	065	9.69077	23	136	9.94020	8	309	9.75058	29	0.24942	759	37
24	090	9.69100	22	121	9.94012	7	347	9.75087	30	0.24913	747	36
25	49116	9.69122	22	87107	9.94005	7	56385	9.75117	29	0.24883	1.7735	35
26	141	9.69144	23	093	9.93998	7	424	9.75146	30	0.24854	723	34
27	166	9.69167	22	079	9.93991	7	462	9.75176	29	0.24824	711	33
28	192	9.69189	23	064	9.93984	7	501	9.75205	30	0.24795	699	32
29	217	9.69212	22	050	9.93977	7	539	9.75235	29	0.24765	687	31
30	49242	9.69234	22	87036	9.93970	7	56577	9.75264	30	0.24736	1.7675	30
31	268	9.69256	23	021	9.93963	8	616	9.75294	29	0.24706	663	29
32	293	9.69279	22	007	9.93955	7	654	9.75323	30	0.24677	651	28
33	318	9.69301	22	86993	9.93948	7	693	9.75353	29	0.24647	639	27
34	344	9.69323	22	978	9.93941	7	731	9.75382	29	0.24618	627	26
35	49369	9.69345	23	86964	9.93934	7	56769	9.75411	30	0.24589	1.7615	25
36	394	9.69368	22	949	9.93927	7	808	9.75441	29	0.24559	603	24
37	419	9.69390	22	935	9.93920	8	846	9.75470	30	0.24530	591	23
38	445	9.69412	22	921	9.93912	7	885	9.75500	29	0.24500	579	22
39	470	9.69434	22	906	9.93905	7	923	9.75529	29	0.24471	567	21
40	49495	9.69456	23	86892	9.93898	7	56962	9.75558	30	0.24442	1.7556	20
41	521	9.69479	22	878	9.93891	7	57000	9.75588	29	0.24412	544	19
42	546	9.69501	22	863	9.93884	8	039	9.75617	30	0.24383	532	18
43	571	9.69523	22	849	9.93876	7	078	9.75647	29	0.24353	520	17
44	596	9.69545	22	834	9.93869	7	116	9.75676	29	0.24324	508	16
45	49622	9.69567	22	86820	9.93862	7	57155	9.75705	30	0.24295	1.7496	15
46	647	9.69589	22	805	9.93855	8	193	9.75735	29	0.24265	485	14
47	672	9.69611	22	791	9.93847	7	232	9.75764	29	0.24236	473	13
48	697	9.69633	22	777	9.93840	7	271	9.75793	29	0.24207	461	12
49	723	9.69655	22	762	9.93833	7	309	9.75822	30	0.24178	449	11
50	49748	9.69677	22	86748	9.93826	7	57348	9.75852	29	0.24148	1.7437	10
51	773	9.69699	22	733	9.93819	8	386	9.75881	29	0.24119	426	9
52	798	9.69721	22	719	9.93811	7	425	9.75910	29	0.24090	414	8
53	824	9.69743	22	704	9.93804	7	464	9.75939	30	0.24061	402	7
54	849	9.69765	22	690	9.93797	8	503	9.75969	29	0.24031	391	6
55	49874	9.69787	22	86675	9.93789	7	57541	9.75998	29	0.24002	1.7379	5
56	899	9.69809	22	661	9.93782	7	580	9.76027	29	0.23973	367	4
57	924	9.69831	22	646	9.93775	7	619	9.76056	30	0.23944	355	3
58	950	9.69853	22	632	9.93768	8	657	9.76086	29	0.23914	344	2
59	975	9.69875	22	617	9.93760	7	696	9.76115	29	0.23885	332	1
60	50000	9.69897		603	9.93753		735	9.76144		0.23856	321	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	50000	9.69897	22	86603	9.93753	7	57735	9.76144	29	0.23856	1.7321	60
1	025	9.69919	22	588	9.93746	8	774	9.76173	29	0.23827	309	59
2	050	9.69941	22	573	9.93738	7	813	9.76202	29	0.23798	297	58
3	076	9.69963	21	559	9.93731	7	851	9.76231	30	0.23769	286	57
4	101	9.69984	22	544	9.93724	7	890	9.76261	29	0.23739	274	56
5	50126	9.70006	22	86530	9.93717	8	57929	9.76290	29	0.23710	1.7262	55
6	151	9.70028	22	515	9.93709	7	968	9.76319	29	0.23681	251	54
7	176	9.70050	22	501	9.93702	7	58007	9.76348	29	0.23652	239	53
8	201	9.70072	21	486	9.93695	8	046	9.76377	29	0.23623	228	52
9	227	9.70093	22	471	9.93687	7	085	9.76406	29	0.23594	216	51
10	50252	9.70115	22	86457	9.93680	7	58124	9.76435	29	0.23565	1.7205	50
11	277	9.70137	22	442	9.93673	8	162	9.76464	29	0.23536	193	49
12	302	9.70159	21	427	9.93665	7	201	9.76493	29	0.23507	182	48
13	327	9.70180	22	413	9.93658	8	240	9.76522	29	0.23478	170	47
14	352	9.70202	22	398	9.93650	7	279	9.76551	29	0.23449	159	46
15	50377	9.70224	21	86384	9.93643	7	58318	9.76580	29	0.23420	1.7147	45
16	403	9.70245	22	369	9.93636	8	357	9.76609	30	0.23391	136	44
17	428	9.70267	21	354	9.93628	7	396	9.76639	29	0.23361	124	43
18	453	9.70288	22	340	9.93621	7	435	9.76668	29	0.23332	113	42
19	478	9.70310	22	325	9.93614	8	474	9.76697	28	0.23303	102	41
20	50503	9.70332	21	86310	9.93606	7	58513	9.76725	29	0.23275	1.7090	40
21	528	9.70353	22	295	9.93599	8	552	9.76754	29	0.23246	079	39
22	553	9.70375	21	281	9.93591	7	591	9.76783	29	0.23217	067	38
23	578	9.70396	22	266	9.93584	7	631	9.76812	29	0.23188	056	37
24	603	9.70418	21	251	9.93577	8	670	9.76841	29	0.23159	045	36
25	50628	9.70439	22	86237	9.93569	7	58709	9.76870	29	0.23130	1.7033	35
26	654	9.70461	21	222	9.93562	8	748	9.76899	29	0.23101	022	34
27	679	9.70482	22	207	9.93554	7	787	9.76928	29	0.23072	011	33
28	704	9.70504	21	192	9.93547	8	826	9.76957	29	0.23043	1.6999	32
29	729	9.70525	22	178	9.93539	7	865	9.76986	29	0.23014	988	31
30	50754	9.70547	21	86163	9.93532	7	58905	9.77015	29	0.22985	1.6977	30
31	779	9.70568	22	148	9.93525	8	944	9.77044	29	0.22956	965	29
32	804	9.70590	21	133	9.93517	7	983	9.77073	28	0.22927	954	28
33	829	9.70611	22	119	9.93510	8	59022	9.77101	29	0.22899	943	27
34	854	9.70633	21	104	9.93502	7	061	9.77130	29	0.22870	932	26
35	50879	9.70654	21	86089	9.93495	8	59101	9.77159	29	0.22841	1.6920	25
36	904	9.70675	22	074	9.93487	7	140	9.77188	29	0.22812	909	24
37	929	9.70697	21	059	9.93480	8	179	9.77217	29	0.22783	898	23
38	954	9.70718	21	045	9.93472	7	218	9.77246	28	0.22754	887	22
39	979	9.70739	22	030	9.93465	8	258	9.77274	29	0.22726	875	21
40	51004	9.70761	21	86015	9.93457	7	59297	9.77303	29	0.22697	1.6864	20
41	029	9.70782	21	000	9.93450	8	336	9.77332	29	0.22668	853	19
42	054	9.70803	21	85985	9.93442	7	376	9.77361	29	0.22639	842	18
43	079	9.70824	22	970	9.93435	8	415	9.77390	28	0.22610	831	17
44	104	9.70846	21	956	9.93427	7	454	9.77418	29	0.22582	820	16
45	51129	9.70867	21	85941	9.93420	8	59494	9.77447	29	0.22553	1.6808	15
46	154	9.70888	21	926	9.93412	7	533	9.77476	29	0.22524	797	14
47	179	9.70909	22	911	9.93405	8	573	9.77505	28	0.22495	786	13
48	204	9.70931	21	896	9.93397	7	612	9.77533	29	0.22467	775	12
49	229	9.70952	21	881	9.93390	8	651	9.77562	29	0.22438	764	11
50	51254	9.70973	21	85866	9.93382	7	59691	9.77591	28	0.22409	1.6753	10
51	279	9.70994	21	851	9.93375	8	730	9.77619	29	0.22381	742	9
52	304	9.71015	21	836	9.93367	7	770	9.77648	29	0.22352	731	8
53	329	9.71036	22	821	9.93360	8	809	9.77677	29	0.22323	720	7
54	354	9.71058	21	806	9.93352	8	849	9.77706	28	0.22294	709	6
55	51379	9.71079	21	85792	9.93344	7	59888	9.77734	29	0.22266	1.6698	5
56	404	9.71100	21	777	9.93337	8	928	9.77763	28	0.22237	687	4
57	429	9.71121	21	762	9.93329	7	967	9.77791	29	0.22209	676	3
58	454	9.71142	21	747	9.93322	8	60007	9.77820	29	0.22180	665	2
59	479	9.71163	21	732	9.93314	7	046	9.77849	28	0.22151	654	1
60	504	9.71184		717	9.93307		086	9.77877		0.22123	643	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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31°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	′
0	51504	9.71184	21	85717	9.93307	8	60086	9.77877	29	0.22123	1.6643	60
1	529	9.71205	21	702	9.93299	8	126	9.77906	29	0.22094	632	59
2	554	9.71226	21	687	9.93291	7	165	9.77935	28	0.22065	621	58
3	579	9.71247	21	672	9.93284	8	205	9.77963	29	0.22037	610	57
4	604	9.71268	21	657	9.93276	7	245	9.77992	28	0.22008	599	56
5	51628	9.71289	21	85642	9.93269	8	60284	9.78020	29	0.21980	1.6588	55
6	653	9.71310	21	627	9.93261	8	324	9.78049	28	0.21951	577	54
7	678	9.71331	21	612	9.93253	7	364	9.78077	29	0.21923	566	53
8	703	9.71352	21	597	9.93246	8	403	9.78106	29	0.21894	555	52
9	728	9.71373	20	582	9.93238	8	443	9.78135	28	0.21865	545	51
10	51753	9.71393	21	85567	9.93230	7	60483	9.78163	29	0.21837	1.6534	50
11	778	9.71414	21	551	9.93223	8	522	9.78192	28	0.21808	523	49
12	803	9.71435	21	536	9.93215	8	562	9.78220	29	0.21780	512	48
13	828	9.71456	21	521	9.93207	7	602	9.78249	28	0.21751	501	47
14	852	9.71477	21	506	9.93200	8	642	9.78277	29	0.21723	490	46
15	51877	9.71498	21	85491	9.93192	8	60681	9.78306	28	0.21694	1.6479	45
16	902	9.71519	20	476	9.93184	7	721	9.78334	29	0.21666	469	44
17	927	9.71539	21	461	9.93177	8	761	9.78363	28	0.21637	458	43
18	952	9.71560	21	446	9.93169	8	801	9.78391	28	0.21609	447	42
19	977	9.71581	21	431	9.93161	7	841	9.78419	29	0.21581	436	41
20	52002	9.71602	20	85416	9.93154	8	60881	9.78448	28	0.21552	1.6426	40
21	026	9.71622	21	401	9.93146	8	921	9.78476	29	0.21524	415	39
22	051	9.71643	21	385	9.93138	7	960	9.78505	28	0.21495	404	38
23	076	9.71664	21	370	9.93131	8	61000	9.78533	29	0.21467	393	37
24	101	9.71685	20	355	9.93123	8	040	9.78562	28	0.21438	383	36
25	52126	9.71705	21	85340	9.93115	7	61080	9.78590	28	0.21410	1.6372	35
26	151	9.71726	21	325	9.93108	8	120	9.78618	29	0.21382	361	34
27	175	9.71747	20	310	9.93100	8	160	9.78647	28	0.21353	351	33
28	200	9.71767	21	294	9.93092	8	200	9.78675	29	0.21325	340	32
29	225	9.71788	21	279	9.93084	7	240	9.78704	28	0.21296	329	31
30	52250	9.71809	20	85264	9.93077	8	61280	9.78732	28	0.21268	1.6319	30
31	275	9.71829	21	249	9.93069	8	320	9.78760	29	0.21240	308	29
32	299	9.71850	20	234	9.93061	8	360	9.78789	28	0.21211	297	28
33	324	9.71870	21	218	9.93053	7	400	9.78817	28	0.21183	287	27
34	349	9.71891	20	203	9.93046	8	440	9.78845	29	0.21155	276	26
35	52374	9.71911	21	85188	9.93038	8	61480	9.78874	28	0.21126	1.6265	25
36	399	9.71932	20	173	9.93030	8	520	9.78902	28	0.21098	255	24
37	423	9.71952	21	157	9.93022	8	561	9.78930	29	0.21070	244	23
38	448	9.71973	21	142	9.93014	7	601	9.78959	28	0.21041	234	22
39	473	9.71994	20	127	9.93007	8	641	9.78987	28	0.21013	223	21
40	52498	9.72014	20	85112	9.92999	8	61681	9.79015	28	0.20985	1.6212	20
41	522	9.72034	21	096	9.92991	8	721	9.79043	29	0.20957	202	19
42	547	9.72055	20	081	9.92983	7	761	9.79072	28	0.20928	191	18
43	572	9.72075	21	066	9.92976	8	801	9.79100	28	0.20900	181	17
44	597	9.72096	20	051	9.92968	8	842	9.79128	28	0.20872	170	16
45	52621	9.72116	21	85035	9.92960	8	61882	9.79156	29	0.20844	1.6160	15
46	646	9.72137	20	020	9.92952	8	922	9.79185	28	0.20815	149	14
47	671	9.72157	20	005	9.92944	8	962	9.79213	28	0.20787	139	13
48	696	9.72177	21	84989	9.92936	7	62003	9.79241	28	0.20759	128	12
49	720	9.72198	20	974	9.92929	8	043	9.79269	28	0.20731	118	11
50	52745	9.72218	20	84959	9.92921	8	62083	9.79297	29	0.20703	1.6107	10
51	770	9.72238	21	943	9.92913	8	124	9.79326	28	0.20674	097	9
52	794	9.72259	20	928	9.92905	8	164	9.79354	28	0.20646	087	8
53	819	9.72279	20	913	9.92897	8	204	9.79382	28	0.20618	076	7
54	844	9.72299	21	897	9.92889	8	245	9.79410	28	0.20590	066	6
55	52869	9.72320	20	84882	9.92881	7	62285	9.79438	28	0.20562	1.6055	5
56	893	9.72340	20	866	9.92874	8	325	9.79466	29	0.20534	045	4
57	918	9.72360	21	851	9.92866	8	366	9.79495	28	0.20505	034	3
58	943	9.72381	20	836	9.92858	8	406	9.79523	28	0.20477	024	2
59	967	9.72401	20	820	9.92850	8	446	9.79551	28	0.20449	014	1
60	992	9.72421		805	9.92842		487	9.79579		0.20421	003	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	52992	9.72421	20	84805	9.92842	8	62487	9.79579	28	0.20421	1.6003	60
1	53017	9.72441	20	789	9.92834	8	527	9.79607	28	0.20393	1.5993	59
2	041	9.72461	21	774	9.92826	8	568	9.79635	28	0.20365	983	58
3	066	9.72482	20	759	9.92818	8	608	9.79663	28	0.20337	972	57
4	091	9.72502	20	743	9.92810	7	649	9.79691	28	0.20309	962	56
5	53115	9.72522	20	84728	9.92803	8	62689	9.79719	28	0.20281	1.5952	55
6	140	9.72542	20	712	9.92795	8	730	9.79747	29	0.20253	941	54
7	164	9.72562	20	697	9.92787	8	770	9.79776	28	0.20224	931	53
8	189	9.72582	20	681	9.92779	8	811	9.79804	28	0.20196	921	52
9	214	9.72602	20	666	9.92771	8	852	9.79832	28	0.20168	911	51
10	53238	9.72622	21	84650	9.92763	8	62892	9.79860	28	0.20140	1.5900	50
11	263	9.72643	20	635	9.92755	8	933	9.79888	28	0.20112	890	49
12	288	9.72663	20	619	9.92747	8	973	9.79916	28	0.20084	880	48
13	312	9.72683	20	604	9.92739	8	63014	9.79944	28	0.20056	869	47
14	337	9.72703	20	588	9.92731	8	055	9.79972	28	0.20028	859	46
15	53361	9.72723	20	84573	9.92723	8	63095	9.80000	28	0.20000	1.5849	45
16	386	9.72743	20	557	9.92715	8	136	9.80028	28	0.19972	839	44
17	411	9.72763	20	542	9.92707	8	177	9.80056	28	0.19944	829	43
18	435	9.72783	20	526	9.92699	8	217	9.80084	28	0.19916	818	42
19	460	9.72803	20	511	9.92691	8	258	9.80112	28	0.19888	808	41
20	53484	9.72823	20	84495	9.92683	8	63299	9.80140	28	0.19860	1.5798	40
21	509	9.72843	20	480	9.92675	8	340	9.80168	27	0.19832	788	39
22	534	9.72863	20	464	9.92667	8	380	9.80195	28	0.19805	778	38
23	558	9.72883	19	448	9.92659	8	421	9.80223	28	0.19777	768	37
24	583	9.72902	20	433	9.92651	8	462	9.80251	28	0.19749	757	36
25	53607	9.72922	20	84417	9.92643	8	63503	9.80279	28	0.19721	1.5747	35
26	632	9.72942	20	402	9.92635	8	544	9.80307	28	0.19693	737	34
27	656	9.72962	20	386	9.92627	8	584	9.80335	28	0.19665	727	33
28	681	9.72982	20	370	9.92619	8	625	9.80363	28	0.19637	717	32
29	705	9.73002	20	355	9.92611	8	666	9.80391	28	0.19609	707	31
30	53730	9.73022	19	84339	9.92603	8	63707	9.80419	28	0.19581	1.5697	30
31	754	9.73041	20	324	9.92595	8	748	9.80447	27	0.19553	687	29
32	779	9.73061	20	308	9.92587	8	789	9.80474	28	0.19526	677	28
33	804	9.73081	20	292	9.92579	8	830	9.80502	28	0.19498	667	27
34	828	9.73101	20	277	9.92571	8	871	9.80530	28	0.19470	657	26
35	53853	9.73121	19	84261	9.92563	8	63912	9.80558	28	0.19442	1.5647	25
36	877	9.73140	20	245	9.92555	9	953	9.80586	28	0.19414	637	24
37	902	9.73160	20	230	9.92546	8	994	9.80614	28	0.19386	627	23
38	926	9.73180	20	214	9.92538	8	64035	9.80642	27	0.19358	617	22
39	951	9.73200	19	198	9.92530	8	076	9.80669	28	0.19331	607	21
40	53975	9.73219	20	84182	9.92522	8	64117	9.80697	28	0.19303	1.5597	20
41	54000	9.73239	20	167	9.92514	8	158	9.80725	28	0.19275	587	19
42	024	9.73259	19	151	9.92506	8	199	9.80753	28	0.19247	577	18
43	049	9.73278	20	135	9.92498	8	240	9.80781	27	0.19219	567	17
44	073	9.73298	20	120	9.92490	8	281	9.80808	28	0.19192	557	16
45	54097	9.73318	19	84104	9.92482	9	64322	9.80836	28	0.19164	1.5547	15
46	122	9.73337	20	088	9.92473	8	363	9.80864	28	0.19136	537	14
47	146	9.73357	20	072	9.92465	8	404	9.80892	27	0.19108	527	13
48	171	9.73377	19	057	9.92457	8	446	9.80919	28	0.19081	517	12
49	195	9.73396	20	041	9.92449	8	487	9.80947	28	0.19053	507	11
50	54220	9.73416	19	84025	9.92441	8	64528	9.80975	28	0.19025	1.5497	10
51	244	9.73435	20	009	9.92433	8	569	9.81003	27	0.18997	487	9
52	269	9.73455	19	83994	9.92425	9	610	9.81030	28	0.18970	477	8
53	293	9.73474	20	978	9.92416	8	652	9.81058	28	0.18942	468	7
54	317	9.73494	19	962	9.92408	8	693	9.81086	27	0.18914	458	6
55	54342	9.73513	20	83946	9.92400	8	64734	9.81113	28	0.18887	1.5448	5
56	366	9.73533	19	930	9.92392	8	775	9.81141	28	0.18859	438	4
57	391	9.73552	20	915	9.92384	8	817	9.81169	27	0.18831	.428	3
58	415	9.73572	19	899	9.92376	9	858	9.81196	28	0.18804	418	2
59	440	9.73591	20	883	9.92367	8	899	9.81224	28	0.18776	408	1
60	464	9.73611		867	9.92359		941	9.81252		0.18748	399	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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33°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	54464	9.73611	19	83867	9.92359	8	64941	9.81252	27	0.18748	1.5399	60
1	488	9.73630	20	851	9.92351	8	982	9.81279	28	0.18721	389	59
2	513	9.73650	19	835	9.92343	8	65024	9.81307	28	0.18693	379	58
3	537	9.73669	20	819	9.92335	9	065	9.81335	27	0.18665	369	57
4	561	9.73689	19	804	9.92326	8	106	9.81362	28	0.18638	359	56
5	54586	9.73708	19	83788	9.92318	8	65148	9.81390	28	0.18610	1.5350	55
6	610	9.73727	20	772	9.92310	8	189	9.81418	27	0.18582	340	54
7	635	9.73747	19	756	9.92302	9	231	9.81445	28	0.18555	330	53
8	659	9.73766	19	740	9.92293	8	272	9.81473	27	0.18527	320	52
9	683	9.73785	20	724	9.92285	8	314	9.81500	28	0.18500	311	51
10	54708	9.73805	19	83708	9.92277	8	65355	9.81528	28	0.18472	1.5301	50
11	732	9.73824	19	692	9.92269	9	397	9.81556	27	0.18444	291	49
12	756	9.73843	20	676	9.92260	8	438	9.81583	28	0.18417	282	48
13	781	9.73863	19	660	9.92252	8	480	9.81611	27	0.18389	272	47
14	805	9.73882	19	645	9.92244	9	521	9.81638	28	0.18362	262	46
15	54829	9.73901	20	83629	9.92235	8	65563	9.81666	27	0.18334	1.5253	45
16	854	9.73921	19	613	9.92227	8	604	9.81693	28	0.18307	243	44
17	878	9.73940	19	597	9.92219	8	646	9.81721	27	0.18279	233	43
18	902	9.73959	19	581	9.92211	9	688	9.81748	28	0.18252	224	42
19	927	9.73978	19	565	9.92202	8	729	9.81776	27	0.18224	214	41
20	54951	9.73997	20	83549	9.92194	8	65771	9.81803	28	0.18197	1.5204	40
21	975	9.74017	19	533	9.92186	9	813	9.81831	27	0.18169	195	39
22	999	9.74036	19	517	9.92177	8	854	9.81858	28	0.18142	185	38
23	55024	9.74055	19	501	9.92169	8	896	9.81886	27	0.18114	175	37
24	048	9.74074	19	485	9.92161	9	938	9.81913	28	0.18087	166	36
25	55072	9.74093	20	83469	9.92152	8	65980	9.81941	27	0.18059	1.5156	35
26	097	9.74113	19	453	9.92144	8	66021	9.81968	28	0.18032	147	34
27	121	9.74132	19	437	9.92136	9	063	9.81996	27	0.18004	137	33
28	145	9.74151	19	421	9.92127	8	105	9.82023	28	0.17977	127	32
29	169	9.74170	19	405	9.92119	8	147	9.82051	27	0.17949	118	31
30	55194	9.74189	19	83389	9.92111	9	66189	9.82078	28	0.17922	1.5108	30
31	218	9.74208	19	373	9.92102	8	230	9.82106	27	0.17894	099	29
32	242	9.74227	19	356	9.92094	8	272	9.82133	28	0.17867	089	28
33	266	9.74246	19	340	9.92086	9	314	9.82161	27	0.17839	080	27
34	291	9.74265	19	324	9.92077	8	356	9.82188	27	0.17812	070	26
35	55315	9.74284	19	83308	9.92069	9	66398	9.82215	28	0.17785	1.5061	25
36	339	9.74303	19	292	9.92060	8	440	9.82243	27	0.17757	051	24
37	363	9.74322	19	276	9.92052	8	482	9.82270	28	0.17730	042	23
38	388	9.74341	19	260	9.92044	9	524	9.82298	27	0.17702	032	22
39	412	9.74360	19	244	9.92035	8	566	9.82325	27	0.17675	023	21
40	55436	9.74379	19	83228	9.92027	9	66608	9.82352	28	0.17648	1.5013	20
41	460	9.74398	19	212	9.92018	8	650	9.82380	27	0.17620	004	19
42	484	9.74417	19	195	9.92010	8	692	9.82407	28	0.17593	1.4994	18
43	509	9.74436	19	179	9.92002	9	734	9.82435	27	0.17565	985	17
44	533	9.74455	19	163	9.91993	8	776	9.82462	27	0.17538	975	16
45	55557	9.74474	19	83147	9.91985	9	66818	9.82489	28	0.17511	1.4966	15
46	581	9.74493	19	131	9.91976	8	860	9.82517	27	0.17483	957	14
47	605	9.74512	19	115	9.91968	9	902	9.82544	27	0.17456	947	13
48	630	9.74531	18	098	9.91959	8	944	9.82571	28	0.17429	938	12
49	654	9.74549	19	082	9.91951	9	986	9.82599	27	0.17401	928	11
50	55678	9.74568	19	83066	9.91942	8	67028	9.82626	27	0.17374	1.4919	10
51	702	9.74587	19	050	9.91934	9	071	9.82653	28	0.17347	910	9
52	726	9.74606	19	034	9.91925	8	113	9.82681	27	0.17319	900	8
53	750	9.74625	19	017	9.91917	9	155	9.82708	27	0.17292	891	7
54	775	9.74644	18	001	9.91908	8	197	9.82735	27	0.17265	882	6
55	55799	9.74662	19	82985	9.91900	9	67239	9.82762	28	0.17238	1.4872	5
56	823	9.74681	19	969	9.91891	8	282	9.82790	27	0.17210	863	4
57	847	9.74700	19	953	9.91883	9	324	9.82817	27	0.17183	854	3
58	871	9.74719	18	936	9.91874	8	366	9.82844	27	0.17156	844	2
59	895	9.74737	19	920	9.91866	9	409	9.82871	28	0.17129	835	1
60	919	9.74756		904	9.91857		451	9.82899		0.17101	826	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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34°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	55919	9.74756	19	82904	9.91857	8	67451	9.82899	27	0.17101	1.4826	60
1	943	9.74775	19	887	9.91849	9	493	9.82926	27	0.17074	816	59
2	968	9.74794	18	871	9.91840	8	536	9.82953	27	0.17047	807	58
3	992	9.74812	19	855	9.91832	9	578	9.82980	28	0.17020	798	57
4	56016	9.74831	19	839	9.91823	8	620	9.83008	27	0.16992	788	56
5	56040	9.74850	18	82822	9.91815	9	67663	9.83035	27	0.16965	1.4779	55
6	064	9.74868	19	806	9.91806	8	705	9.83062	27	0.16938	770	54
7	088	9.74887	19	790	9.91798	9	748	9.83089	28	0.16911	761	53
8	112	9.74906	18	773	9.91789	8	790	9.83117	27	0.16883	751	52
9	136	9.74924	19	757	9.91781	9	832	9.83144	27	0.16856	742	51
10	56160	9.74943	18	82741	9.91772	9	67875	9.83171	27	0.16829	1.4733	50
11	184	9.74961	19	724	9.91763	8	917	9.83198	27	0.16802	724	49
12	208	9.74980	19	708	9.91755	9	960	9.83225	27	0.16775	715	48
13	232	9.74999	18	692	9.91746	8	68002	9.83252	28	0.16748	705	47
14	256	9.75017	19	675	9.91738	9	045	9.83280	27	0.16720	696	46
15	56280	9.75036	18	82659	9.91729	9	68088	9.83307	27	0.16693	1.4687	45
16	305	9.75054	19	643	9.91720	8	130	9.83334	27	0.16666	678	44
17	329	9.75073	18	626	9.91712	9	173	9.83361	27	0.16639	669	43
18	353	9.75091	19	610	9.91703	8	215	9.83388	27	0.16612	659	42
19	377	9.75110	18	593	9.91695	9	258	9.83415	27	0.16585	650	41
20	56401	9.75128	19	82577	9.91686	9	68301	9.83442	28	0.16558	1.4641	40
21	425	9.75147	18	561	9.91677	8	343	9.83470	27	0.16530	632	39
22	449	9.75165	19	544	9.91669	9	386	9.83497	27	0.16503	623	38
23	473	9.75184	18	528	9.91660	9	429	9.83524	27	0.16476	614	37
24	497	9.75202	19	511	9.91651	8	471	9.83551	27	0.16449	605	36
25	56521	9.75221	18	82495	9.91643	9	68514	9.83578	27	0.16422	1.4596	35
26	545	9.75239	19	478	9.91634	9	557	9.83605	27	0.16395	586	34
27	569	9.75258	18	462	9.91625	8	600	9.83632	27	0.16368	577	33
28	593	9.75276	18	446	9.91617	9	642	9.83659	27	0.16341	568	32
29	617	9.75294	19	429	9.91608	9	685	9.83686	27	0.16314	559	31
30	56641	9.75313	18	82413	9.91599	8	68728	9.83713	27	0.16287	1.4550	30
31	665	9.75331	19	396	9.91591	9	771	9.83740	28	0.16260	541	29
32	689	9.75350	18	380	9.91582	9	814	9.83768	27	0.16232	532	28
33	713	9.75368	18	363	9.91573	8	857	9.83795	27	0.16205	523	27
34	736	9.75386	19	347	9.91565	9	900	9.83822	27	0.16178	514	26
35	56760	9.75405	18	82330	9.91556	9	68942	9.83849	27	0.16151	1.4505	25
36	784	9.75423	18	314	9.91547	9	985	9.83876	27	0.16124	496	24
37	808	9.75441	18	297	9.91538	8	69028	9.83903	27	0.16097	487	23
38	832	9.75459	19	281	9.91530	9	071	9.83930	27	0.16070	478	22
39	856	9.75478	18	264	9.91521	9	114	9.83957	27	0.16043	469	21
40	56880	9.75496	18	82248	9.91512	8	69157	9.83984	27	0.16016	1.4460	20
41	904	9.75514	19	231	9.91504	9	200	9.84011	27	0.15989	451	19
42	928	9.75533	18	214	9.91495	9	243	9.84038	27	0.15962	442	18
43	952	9.75551	18	198	9.91486	9	286	9.84065	27	0.15935	433	17
44	976	9.75569	18	181	9.91477	8	329	9.84092	27	0.15908	424	16
45	57000	9.75587	18	82165	9.91469	9	69372	9.84119	27	0.15881	1.4415	15
46	024	9.75605	19	148	9.91460	9	416	9.84146	27	0.15854	406	14
47	047	9.75624	18	132	9.91451	9	459	9.84173	27	0.15827	397	13
48	071	9.75642	18	115	9.91442	9	502	9.84200	27	0.15800	388	12
49	095	9.75660	18	098	9.91433	8	545	9.84227	27	0.15773	379	11
50	57119	9.75678	18	82082	9.91425	9	69588	9.84254	26	0.15746	1.4370	10
51	143	9.75696	18	065	9.91416	9	631	9.84280	27	0.15720	361	9
52	167	9.75714	19	048	9.91407	9	675	9.84307	27	0.15693	352	8
53	191	9.75733	18	032	9.91398	9	718	9.84334	27	0.15666	344	7
54	215	9.75751	18	015	9.91389	8	761	9.84361	27	0.15639	335	6
55	57238	9.75769	18	81999	9.91381	9	69804	9.84388	27	0.15612	1.4326	5
56	262	9.75787	18	982	9.91372	9	847	9.84415	27	0.15585	317	4
57	286	9.75805	18	965	9.91363	9	891	9.84442	27	0.15558	308	3
58	310	9.75823	18	949	9.91354	9	934	9.84469	27	0.15531	299	2
59	334	9.75841	18	932	9.91345	9	977	9.84496	27	0.15504	290	1
60	358	9.75859		915	9.91336		70021	9.84523		0.15477	281	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

55°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	57358	9.75859	18	81915	9.91336	8	70021	9.84523	27	0.15477	1.4281	60
1	381	9.75877	18	899	9.91328	9	064	9.84550	26	0.15450	273	59
2	405	9.75895	18	882	9.91319	9	107	9.84576	27	0.15424	264	58
3	429	9.75913	18	865	9.91310	9	151	9.84603	27	0.15397	255	57
4	453	9.75931	18	848	9.91301	9	194	9.84630	27	0.15370	246	56
5	57477	9.75949	18	81832	9.91292	9	70238	9.84657	27	0.15343	1.4237	55
6	501	9.75967	18	815	9.91283	9	281	9.84684	27	0.15316	229	54
7	524	9.75985	18	798	9.91274	8	325	9.84711	27	0.15289	220	53
8	548	9.76003	18	782	9.91266	9	368	9.84738	26	0.15262	211	52
9	572	9.76021	18	765	9.91257	9	412	9.84764	27	0.15236	202	51
10	57596	9.76039	18	81748	9.91248	9	70455	9.84791	27	0.15209	1.4193	50
11	619	9.76057	18	731	9.91239	9	499	9.84818	27	0.15182	185	49
12	643	9.76075	18	714	9.91230	9	542	9.84845	27	0.15155	176	48
13	667	9.76093	18	698	9.91221	9	586	9.84872	27	0.15128	167	47
14	691	9.76111	18	681	9.91212	9	629	9.84899	26	0.15101	158	46
15	57715	9.76129	17	81664	9.91203	9	70673	9.84925	27	0.15075	1.4150	45
16	738	9.76146	18	647	9.91194	9	717	9.84952	27	0.15048	141	44
17	762	9.76164	18	631	9.91185	9	760	9.84979	27	0.15021	132	43
18	786	9.76182	18	614	9.91176	9	804	9.85006	27	0.14994	124	42
19	810	9.76200	18	597	9.91167	9	848	9.85033	26	0.14967	115	41
20	57833	9.76218	18	81580	9.91158	9	70891	9.85059	27	0.14941	1.4106	40
21	857	9.76236	17	563	9.91149	8	935	9.85086	27	0.14914	097	39
22	881	9.76253	18	546	9.91141	9	979	9.85113	27	0.14887	089	38
23	904	9.76271	18	530	9.91132	9	71023	9.85140	26	0.14860	080	37
24	928	9.76289	18	513	9.91123	9	066	9.85166	27	0.14834	071	36
25	57952	9.76307	17	81496	9.91114	9	71110	9.85193	27	0.14807	1.4063	35
26	976	9.76324	18	479	9.91105	9	154	9.85220	27	0.14780	054	34
27	999	9.76342	18	462	9.91096	9	198	9.85247	26	0.14753	045	33
28	58023	9.76360	18	445	9.91087	9	242	9.85273	27	0.14727	037	32
29	047	9.76378	17	428	9.91078	9	285	9.85300	27	0.14700	028	31
30	58070	9.76395	18	81412	9.91069	9	71329	9.85327	27	0.14673	1.4019	30
31	094	9.76413	18	395	9.91060	9	373	9.85354	26	0.14646	011	29
32	118	9.76431	17	378	9.91051	9	417	9.85380	27	0.14620	002	28
33	141	9.76448	18	361	9.91042	9	461	9.85407	27	0.14593	1.3994	27
34	165	9.76466	18	344	9.91033	10	505	9.85434	26	0.14566	985	26
35	58189	9.76484	17	81327	9.91023	9	71549	9.85460	27	0.14540	1.3976	25
36	212	9.76501	18	310	9.91014	9	593	9.85487	27	0.14513	968	24
37	236	9.76519	18	293	9.91005	9	637	9.85514	26	0.14486	959	23
38	260	9.76537	17	276	9.90996	9	681	9.85540	27	0.14460	951	22
39	283	9.76554	18	259	9.90987	9	725	9.85567	27	0.14433	942	21
40	58307	9.76572	18	81242	9.90978	9	71769	9.85594	26	0.14406	1.3934	20
41	330	9.76590	17	225	9.90969	9	813	9.85620	27	0.14380	925	19
42	354	9.76607	18	208	9.90960	9	857	9.85647	27	0.14353	916	18
43	378	9.76625	17	191	9.90951	9	901	9.85674	26	0.14326	908	17
44	401	9.76642	18	174	9.90942	9	946	9.85700	27	0.14300	899	16
45	58425	9.76660	17	81157	9.90933	9	71990	9.85727	27	0.14273	1.3891	15
46	449	9.76677	18	140	9.90924	9	72034	9.85754	26	0.14246	882	14
47	472	9.76695	17	123	9.90915	9	078	9.85780	27	0.14220	874	13
48	496	9.76712	18	106	9.90906	10	122	9.85807	27	0.14193	865	12
49	519	9.76730	17	089	9.90896	9	167	9.85834	26	0.14166	857	11
50	58543	9.76747	18	81072	9.90887	9	72211	9.85860	27	0.14140	1.3848	10
51	567	9.76765	17	055	9.90878	9	255	9.85887	26	0.14113	840	9
52	590	9.76782	18	038	9.90869	9	299	9.85913	27	0.14087	831	8
53	614	9.76800	17	021	9.90860	9	344	9.85940	27	0.14060	823	7
54	637	9.76817	18	004	9.90851	9	388	9.85967	26	0.14033	814	6
55	58661	9.76835	17	80987	9.90842	10	72432	9.85993	27	0.14007	1.3806	5
56	684	9.76852	18	970	9.90832	9	477	9.86020	26	0.13980	798	4
57	708	9.76870	17	953	9.90823	9	521	9.86046	27	0.13954	789	3
58	731	9.76887	17	936	9.90814	9	565	9.86073	27	0.13927	781	2
59	755	9.76904	18	919	9.90805	9	610	9.86100	26	0.13900	772	1
60	779	9.76922		902	9.90796		654	9.86126		0.13874	764	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

36°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	58779	9.76922	17	80902	9.90796	9	72654	9.86126	27	0.13874	1.3764	60
1	802	9.76939	18	885	9.90787	10	699	9.86153	26	0.13847	755	59
2	826	9.76957	17	867	9.90777	9	743	9.86179	27	0.13821	747	58
3	849	9.76974	17	850	9.90768	9	788	9.86206	26	0.13794	739	57
4	873	9.76991	18	833	9.90759	9	832	9.86232	27	0.13768	730	56
5	58896	9.77009	17	80816	9.90750	9	72877	9.86259	26	0.13741	1.3722	55
6	920	9.77026	17	799	9.90741	10	921	9.86285	27	0.13715	713	54
7	943	9.77043	18	782	9.90731	9	966	9.86312	26	0.13688	705	53
8	967	9.77061	17	765	9.90722	9	73010	9.86338	27	0.13662	697	52
9	990	9.77078	17	748	9.90713	9	055	9.86365	27	0.13635	688	51
10	59014	9.77095	17	80730	9.90704	10	73100	9.86392	26	0.13608	1.3680	50
11	037	9.77112	18	713	9.90694	9	144	9.86418	27	0.13582	672	49
12	061	9.77130	17	696	9.90685	9	189	9.86445	26	0.13555	663	48
13	084	9.77147	17	679	9.90676	9	234	9.86471	27	0.13529	655	47
14	108	9.77164	17	662	9.90667	10	278	9.86498	26	0.13502	647	46
15	59131	9.77181	18	80644	9.90657	9	73323	9.86524	27	0.13476	1.3638	45
16	154	9.77199	17	627	9.90648	9	368	9.86551	26	0.13449	630	44
17	178	9.77216	17	610	9.90639	9	413	9.86577	26	0.13423	622	43
18	201	9.77233	17	593	9.90630	10	457	9.86603	27	0.13397	613	42
19	225	9.77250	18	576	9.90620	9	502	9.86630	26	0.13370	605	41
20	59248	9.77268	17	80558	9.90611	9	73547	9.86656	27	0.13344	1.3597	40
21	272	9.77285	17	541	9.90602	10	592	9.86683	26	0.13317	588	39
22	295	9.77302	17	524	9.90592	9	637	9.86709	27	0.13291	580	38
23	318	9.77319	17	507	9.90583	9	681	9.86736	26	0.13264	572	37
24	342	9.77336	17	489	9.90574	9	726	9.86762	27	0.13238	564	36
25	59365	9.77353	17	80472	9.90565	10	73771	9.86789	26	0.13211	1.3555	35
26	389	9.77370	17	455	9.90555	9	816	9.86815	27	0.13185	547	34
27	412	9.77387	18	438	9.90546	9	861	9.86842	26	0.13158	539	33
28	436	9.77405	17	420	9.90537	10	906	9.86868	26	0.13132	531	32
29	459	9.77422	17	403	9.90527	9	951	9.86894	27	0.13106	522	31
30	59482	9.77439	17	80386	9.90518	9	73996	9.86921	26	0.13079	1.3514	30
31	506	9.77456	17	368	9.90509	10	74041	9.86947	27	0.13053	506	29
32	529	9.77473	17	351	9.90499	9	086	9.86974	26	0.13026	498	28
33	552	9.77490	17	334	9.90490	10	131	9.87000	27	0.13000	490	27
34	576	9.77507	17	316	9.90480	9	176	9.87027	26	0.12973	481	26
35	59599	9.77524	17	80299	9.90471	9	74221	9.87053	26	0.12947	1.3473	25
36	622	9.77541	17	282	9.90462	10	267	9.87079	27	0.12921	465	24
37	646	9.77558	17	264	9.90452	9	312	9.87106	26	0.12894	457	23
38	669	9.77575	17	247	9.90443	9	357	9.87132	26	0.12868	449	22
39	693	9.77592	17	230	9.90434	10	402	9.87158	27	0.12842	440	21
40	59716	9.77609	17	80212	9.90424	9	74447	9.87185	26	0.12815	1.3432	20
41	739	9.77626	17	195	9.90415	10	492	9.87211	27	0.12789	424	19
42	763	9.77643	17	178	9.90405	9	538	9.87238	26	0.12762	416	18
43	786	9.77660	17	160	9.90396	10	583	9.87264	26	0.12736	408	17
44	809	9.77677	17	143	9.90386	9	628	9.87290	27	0.12710	400	16
45	59832	9.77694	17	80125	9.90377	9	74674	9.87317	26	0.12683	1.3392	15
46	856	9.77711	17	108	9.90368	10	719	9.87343	26	0.12657	384	14
47	879	9.77728	16	091	9.90358	9	764	9.87369	27	0.12631	375	13
48	902	9.77744	17	073	9.90349	10	810	9.87396	26	0.12604	367	12
49	926	9.77761	17	056	9.90339	9	855	9.87422	26	0.12578	359	11
50	59949	9.77778	17	80038	9.90330	10	74900	9.87448	27	0.12552	1.3351	10
51	972	9.77795	17	021	9.90320	9	946	9.87475	26	0.12525	343	9
52	995	9.77812	17	003	9.90311	10	991	9.87501	26	0.12499	335	8
53	60019	9.77829	17	79986	9.90301	9	75037	9.87527	27	0.12473	327	7
54	042	9.77846	16	968	9.90292	10	082	9.87554	26	0.12446	319	6
55	60065	9.77862	17	79951	9.90282	9	75128	9.87580	26	0.12420	1.3311	5
56	089	9.77879	17	934	9.90273	10	173	9.87606	27	0.12394	303	4
57	112	9.77896	17	916	9.90263	9	219	9.87633	26	0.12367	295	3
58	135	9.77913	17	899	9.90254	10	264	9.87659	26	0.12341	287	2
59	158	9.77930	16	881	9.90244	9	310	9.87685	26	0.12315	278	1
60	182	9.77946		864	9.90235		355	9.87711		0.12289	270	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

53°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	60182	9.77946	17	79864	9.90235	10	75355	9.87711	27	0.12289	1.3270	60
1	205	9.77963	17	846	9.90225	9	401	9.87738	26	0.12262	262	59
2	228	9.77980	17	829	9.90216	10	447	9.87764	26	0.12236	254	58
3	251	9.77997	16	811	9.90206	9	492	9.87790	27	0.12210	246	57
4	274	9.78013	17	793	9.90197	10	538	9.87817	26	0.12183	238	56
5	60298	9.78030	17	79776	9.90187	9	75584	9.87843	26	0.12157	1.3230	55
6	321	9.78047	16	758	9.90178	10	629	9.87869	26	0.12131	222	54
7	344	9.78063	17	741	9.90168	9	675	9.87895	27	0.12105	214	53
8	367	9.78080	17	723	9.90159	10	721	9.87922	26	0.12078	206	52
9	390	9.78097	16	706	9.90149	10	767	9.87948	26	0.12052	198	51
10	60414	9.78113	17	79688	9.90139	9	75812	9.87974	26	0.12026	1.3190	50
11	437	9.78130	17	671	9.90130	10	858	9.88000	27	0.12000	182	49
12	460	9.78147	16	653	9.90120	9	904	9.88027	26	0.11973	175	48
13	483	9.78163	17	635	9.90111	10	950	9.88053	26	0.11947	167	47
14	506	9.78180	17	618	9.90101	10	996	9.88079	26	0.11921	159	46
15	60529	9.78197	16	79600	9.90091	9	76042	9.88105	26	0.11895	1.3151	45
16	553	9.78213	17	583	9.90082	10	088	9.88131	27	0.11869	143	44
17	576	9.78230	16	565	9.90072	9	134	9.88158	26	0.11842	135	43
18	599	9.78246	17	547	9.90063	10	180	9.88184	26	0.11816	127	42
19	622	9.78263	17	530	9.90053	10	226	9.88210	26	0.11790	119	41
20	60645	9.78280	16	79512	9.90043	9	76272	9.88236	26	0.11764	1.3111	40
21	668	9.78296	17	494	9.90034	10	318	9.88262	27	0.11738	103	39
22	691	9.78313	16	477	9.90024	10	364	9.88289	26	0.11711	095	38
23	714	9.78329	17	459	9.90014	9	410	9.88315	26	0.11685	087	37
24	738	9.78346	16	441	9.90005	10	456	9.88341	26	0.11659	079	36
25	60761	9.78362	17	79424	9.89995	10	76502	9.88367	26	0.11633	1.3072	35
26	784	9.78379	16	406	9.89985	9	548	9.88393	27	0.11607	064	34
27	807	9.78395	17	388	9.89976	10	594	9.88420	26	0.11580	056	33
28	830	9.78412	16	371	9.89966	10	640	9.88446	26	0.11554	048	32
29	853	9.78428	17	353	9.89956	9	686	9.88472	26	0.11528	040	31
30	60876	9.78445	16	79335	9.89947	10	76733	9.88498	26	0.11502	1.3032	30
31	899	9.78461	17	318	9.89937	10	779	9.88524	26	0.11476	024	29
32	922	9.78478	16	300	9.89927	9	825	9.88550	27	0.11450	017	28
33	945	9.78494	16	282	9.89918	10	871	9.88577	26	0.11423	009	27
34	968	9.78510	17	264	9.89908	10	918	9.88603	26	0.11397	001	26
35	60991	9.78527	16	79247	9.89898	10	76964	9.88629	26	0.11371	1.2993	25
36	61015	9.78543	17	229	9.89888	9	77010	9.88655	26	0.11345	985	24
37	038	9.78560	16	211	9.89879	10	057	9.88681	26	0.11319	977	23
38	061	9.78576	16	193	9.89869	10	103	9.88707	26	0.11293	970	22
39	084	9.78592	17	176	9.89859	10	149	9.88733	26	0.11267	962	21
40	61107	9.78609	16	79158	9.89849	9	77196	9.88759	27	0.11241	1.2954	20
41	130	9.78625	17	140	9.89840	10	242	9.88786	26	0.11214	946	19
42	153	9.78642	16	122	9.89830	10	289	9.88812	26	0.11188	938	18
43	176	9.78658	16	105	9.89820	10	335	9.88838	26	0.11162	931	17
44	199	9.78674	17	087	9.89810	9	382	9.88864	26	0.11136	923	16
45	61222	9.78691	16	79069	9.89801	10	77428	9.88890	26	0.11110	1.2915	15
46	245	9.78707	16	051	9.89791	10	475	9.88916	26	0.11084	907	14
47	268	9.78723	16	033	9.89781	10	521	9.88942	26	0.11058	900	13
48	291	9.78739	17	016	9.89771	10	568	9.88968	26	0.11032	892	12
49	314	9.78756	16	78998	9.89761	9	615	9.88994	26	0.11006	884	11
50	61337	9.78772	16	78980	9.89752	10	77661	9.89020	26	0.10980	1.2876	10
51	360	9.78788	17	962	9.89742	10	708	9.89046	27	0.10954	869	9
52	383	9.78805	16	944	9.89732	10	754	9.89073	26	0.10927	861	8
53	406	9.78821	16	926	9.89722	10	801	9.89099	26	0.10901	853	7
54	429	9.78837	16	908	9.89712	10	848	9.89125	26	0.10875	846	6
55	61451	9.78853	16	78891	9.89702	9	77895	9.89151	26	0.10849	1.2838	5
56	474	9.78869	17	873	9.89693	10	941	9.89177	26	0.10823	830	4
57	497	9.78886	16	855	9.89683	10	988	9.89203	26	0.10797	822	3
58	520	9.78902	16	837	9.89673	10	78035	9.89229	26	0.10771	815	2
59	543	9.78918	16	819	9.89663	10	082	9.89255	26	0.10745	807	1
60	566	9.78934		801	9.89653		129	9.89281		0.10719	799	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

38°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	61566	9.78934	16	78801	9.89653	10	78129	9.89281	26	0.10719	1.2799	60
1	589	9.78950	17	783	9.89643	10	175	9.89307	26	0.10693	792	59
2	612	9.78967	16	765	9.89633	9	222	9.89333	26	0.10667	784	58
3	635	9.78983	16	747	9.89624	10	269	9.89359	26	0.10641	776	57
4	658	9.78999	16	729	9.89614	10	316	9.89385	26	0.10615	769	56
5	61681	9.79015	16	78711	9.89604	10	78363	9.89411	26	0.10589	1.2761	55
6	704	9.79031	16	694	9.89594	10	410	9.89437	26	0.10563	753	54
7	726	9.79047	16	676	9.89584	10	457	9.89463	26	0.10537	746	53
8	749	9.79063	16	658	9.89574	10	504	9.89489	26	0.10511	738	52
9	772	9.79079	16	640	9.89564	10	551	9.89515	26	0.10485	731	51
10	61795	9.79095	16	78622	9.89554	10	78598	9.89541	26	0.10459	1.2723	50
11	818	9.79111	17	604	9.89544	10	645	9.89567	26	0.10433	715	49
12	841	9.79128	16	586	9.89534	10	692	9.89593	26	0.10407	708	48
13	864	9.79144	16	568	9.89524	10	739	9.89619	26	0.10381	700	47
14	887	9.79160	16	550	9.89514	10	786	9.89645	26	0.10355	693	46
15	61909	9.79176	16	78532	9.89504	9	78834	9.89671	26	0.10329	1.2685	45
16	932	9.79192	16	514	9.89495	10	881	9.89697	26	0.10303	677	44
17	955	9.79208	16	496	9.89485	10	928	9.89723	26	0.10277	670	43
18	978	9.79224	16	478	9.89475	10	975	9.89749	26	0.10251	662	42
19	62001	9.79240	16	460	9.89465	10	79022	9.89775	26	0.10225	655	41
20	62024	9.79256	16	78442	9.89455	10	79070	9.89801	26	0.10199	1.2647	40
21	046	9.79272	16	424	9.89445	10	117	9.89827	26	0.10173	640	39
22	069	9.79288	16	405	9.89435	10	164	9.89853	26	0.10147	632	38
23	092	9.79304	15	387	9.89425	10	212	9.89879	26	0.10121	624	37
24	115	9.79319	16	369	9.89415	10	259	9.89905	26	0.10095	617	36
25	62138	9.79335	16	78351	9.89405	10	79306	9.89931	26	0.10069	1.2609	35
26	160	9.79351	16	333	9.89395	10	354	9.89957	26	0.10043	602	34
27	183	9.79367	16	315	9.89385	10	401	9.89983	26	0.10017	594	33
28	206	9.79383	16	297	9.89375	11	449	9.90009	26	0.09991	587	32
29	229	9.79399	16	279	9.89364	10	496	9.90035	26	0.09965	579	31
30	62251	9.79415	16	78261	9.89354	10	79544	9.90061	25	0.09939	1.2572	30
31	274	9.79431	16	243	9.89344	10	591	9.90086	26	0.09914	564	29
32	297	9.79447	16	225	9.89334	10	639	9.90112	26	0.09888	557	28
33	320	9.79463	15	206	9.89324	10	686	9.90138	26	0.09862	549	27
34	342	9.79478	16	188	9.89314	10	734	9.90164	26	0.09836	542	26
35	62365	9.79494	16	78170	9.89304	10	79781	9.90190	26	0.09810	1.2534	25
36	388	9.79510	16	152	9.89294	10	829	9.90216	26	0.09784	527	24
37	411	9.79526	16	134	9.89284	10	877	9.90242	26	0.09758	519	23
38	433	9.79542	16	116	9.89274	10	924	9.90268	26	0.09732	512	22
39	456	9.79558	15	098	9.89264	10	972	9.90294	26	0.09706	504	21
40	62479	9.79573	16	78079	9.89254	10	80020	9.90320	26	0.09680	1.2497	20
41	502	9.79589	16	061	9.89244	11	067	9.90346	25	0.09654	489	19
42	524	9.79605	16	043	9.89233	10	115	9.90371	26	0.09629	482	18
43	547	9.79621	15	025	9.89223	10	163	9.90397	26	0.09603	475	17
44	570	9.79636	16	007	9.89213	10	211	9.90423	26	0.09577	467	16
45	62592	9.79652	16	77988	9.89203	10	80258	9.90449	26	0.09551	1.2460	15
46	615	9.79668	16	970	9.89193	10	306	9.90475	26	0.09525	452	14
47	638	9.79684	15	952	9.89183	10	354	9.90501	26	0.09499	445	13
48	660	9.79699	16	934	9.89173	11	402	9.90527	26	0.09473	437	12
49	683	9.79715	16	916	9.89162	10	450	9.90553	25	0.09447	430	11
50	62706	9.79731	15	77897	9.89152	10	80498	9.90578	26	0.09422	1.2423	10
51	728	9.79746	16	879	9.89142	10	546	9.90604	26	0.09396	415	9
52	751	9.79762	16	861	9.89132	10	594	9.90630	26	0.09370	408	8
53	774	9.79778	15	843	9.89122	10	642	9.90656	26	0.09344	401	7
54	796	9.79793	16	824	9.89112	11	690	9.90682	26	0.09318	393	6
55	62819	9.79809	16	77806	9.89101	10	80738	9.90708	26	0.09292	1.2386	5
56	842	9.79825	15	788	9.89091	10	786	9.90734	25	0.09266	378	4
57	864	9.79840	16	769	9.89081	10	834	9.90759	26	0.09241	371	3
58	887	9.79856	16	751	9.89071	11	882	9.90785	26	0.09215	364	2
59	909	9.79872	15	733	9.89060	10	930	9.90811	26	0.09189	356	1
60	932	9.79887		715	9.89050		978	9.90837		0.09163	349	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

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39°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	62932	9.79887	16	77715	9.89050	10	80978	9.90837	26	0.09163	1.2349	60
1	955	9.79903	15	696	9.89040	10	81027	9.90863	26	0.09137	342	59
2	977	9.79918	16	678	9.89030	10	075	9.90889	25	0.09111	334	58
3	63000	9.79934	16	660	9.89020	11	123	9.90914	26	0.09086	327	57
4	022	9.79950	15	641	9.89009	10	171	9.90940	26	0.09060	320	56
5	63045	9.79965	16	77623	9.88999	10	81220	9.90966	26	0.09034	1.2312	55
6	068	9.79981	15	605	9.88989	11	268	9.90992	26	0.09008	305	54
7	090	9.79996	16	586	9.88978	10	316	9.91018	25	0.08982	298	53
8	113	9.80012	15	568	9.88968	10	364	9.91043	26	0.08957	290	52
9	135	9.80027	16	550	9.88958	10	413	9.91069	26	0.08931	283	51
10	63158	9.80043	15	77531	9.88948	11	81461	9.91095	26	0.08905	1.2276	50
11	180	9.80058	16	51[illegible]	9.88937	10	510	9.91121	26	0.08879	268	49
12	203	9.80074	15	494	9.88927	10	558	9.91147	25	0.08853	261	48
13	225	9.80089	16	476	9.88917	11	606	9.91172	26	0.08828	254	47
14	248	9.80105	15	458	9.88906	10	655	9.91198	26	0.08802	247	46
15	63271	9.80120	16	77439	9.88896	10	81703	9.91224	26	0.08776	1.2239	45
16	293	9.80136	15	421	9.88886	11	752	9.91250	26	0.08750	232	44
17	316	9.80151	15	402	9.88875	10	800	9.91276	25	0.08724	225	43
18	338	9.80166	16	384	9.88865	10	849	9.91301	26	0.08699	218	42
19	361	9.80182	15	366	9.88855	11	898	9.91327	26	0.08673	210	41
20	63383	9.80197	16	77347	9.88844	10	81946	9.91353	26	0.08647	1.2203	40
21	406	9.80213	15	329	9.88834	10	995	9.91379	25	0.08621	196	39
22	428	9.80228	16	310	9.88824	11	82044	9.91404	26	0.08596	189	38
23	451	9.80244	15	292	9.88813	10	092	9.91430	26	0.08570	181	37
24	473	9.80259	15	273	9.88803	10	141	9.91456	26	0.08544	174	36
25	63496	9.80274	16	77255	9.88793	11	82190	9.91482	25	0.08518	1.2167	35
26	518	9.80290	15	236	9.88782	10	238	9.91507	26	0.08493	160	34
27	540	9.80305	15	218	9.88772	11	287	9.91533	26	0.08467	153	33
28	563	9.80320	16	199	9.88761	10	336	9.91559	26	0.08441	145	32
29	585	9.80336	15	181	9.88751	10	385	9.91585	25	0.08415	138	31
30	63608	9.80351	15	77162	9.88741	11	82434	9.91610	26	0.08390	1.2131	30
31	630	9.80366	16	144	9.88730	10	483	9.91636	26	0.08364	124	29
32	653	9.80382	15	125	9.88720	11	531	9.91662	26	0.08338	117	28
33	675	9.80397	15	107	9.88709	10	580	9.91688	25	0.08312	109	27
34	698	9.80412	16	088	9.88699	11	629	9.91713	26	0.08287	102	26
35	63720	9.80428	15	77070	9.88688	10	82678	9.91739	26	0.08261	1.2095	25
36	742	9.80443	15	051	9.88678	10	727	9.91765	26	0.08235	088	24
37	765	9.80458	15	033	9.88668	11	776	9.91791	25	0.08209	081	23
38	787	9.80473	16	014	9.88657	10	825	9.91816	26	0.08184	074	22
39	810	9.80489	15	76996	9.88647	11	874	9.91842	26	0.08158	066	21
40	63832	9.80504	15	76977	9.88636	10	82923	9.91868	25	0.08132	1.2059	20
41	854	9.80519	15	959	9.88626	11	972	9.91893	26	0.08107	052	19
42	877	9.80534	16	940	9.88615	10	83022	9.91919	26	0.08081	045	18
43	899	9.80550	15	921	9.88605	11	071	9.91945	26	0.08055	038	17
44	922	9.80565	15	903	9.88594	10	120	9.91971	25	0.08029	031	16
45	63944	9.80580	15	76884	9.88584	11	83169	9.91996	26	0.08004	1.2024	15
46	966	9.80595	15	866	9.88573	10	218	9.92022	26	0.07978	017	14
47	989	9.80610	15	847	9.88563	11	268	9.92048	25	0.07952	009	13
48	64011	9.80625	16	828	9.88552	10	317	9.92073	26	0.07927	002	12
49	033	9.80641	15	810	9.88542	11	366	9.92099	26	0.07901	1.1995	11
50	64056	9.80656	15	76791	9.88531	10	83415	9.92125	25	0.07875	1.1988	10
51	078	9.80671	15	772	9.88521	11	465	9.92150	26	0.07850	981	9
52	100	9.80686	15	754	9.88510	11	514	9.92176	26	0.07824	974	8
53	123	9.80701	15	735	9.88499	10	564	9.92202	25	0.07798	967	7
54	145	9.80716	15	717	9.88489	11	613	9.92227	26	0.07773	960	6
55	64167	9.80731	15	76698	9.88478	10	83662	9.92253	26	0.07747	1.1953	5
56	190	9.80746	16	679	9.88468	11	712	9.92279	25	0.07721	946	4
57	212	9.80762	15	661	9.88457	10	761	9.92304	26	0.07696	939	3
58	234	9.80777	15	642	9.88447	11	811	9.92330	26	0.07670	932	2
59	256	9.80792	15	623	9.88436	11	860	9.92356	25	0.07644	925	1
60	279	9.80807		604	9.88425		910	9.92381		0.07619	918	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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40°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	64279	9.80807	15	76604	9.88425	10	83910	9.92381	26	0.07619	1.1918	60
1	301	9.80822	15	586	9.88415	11	960	9.92407	26	0.07593	910	59
2	323	9.80837	15	567	9.88404	10	84009	9.92433	25	0.07567	903	58
3	346	9.80852	15	548	9.88394	11	059	9.92458	26	0.07542	896	57
4	368	9.80867	15	530	9.88383	11	108	9.92484	26	0.07516	889	56
5	64390	9.80882	15	76511	9.88372	10	84158	9.92510	25	0.07490	1.1882	55
6	412	9.80897	15	492	9.88362	11	208	9.92535	26	0.07465	875	54
7	435	9.80912	15	473	9.88351	11	258	9.92561	26	0.07439	868	53
8	457	9.80927	15	455	9.88340	10	307	9.92587	25	0.07413	861	52
9	479	9.80942	15	436	9.88330	11	357	9.92612	26	0.07388	854	51
10	64501	9.80957	15	76417	9.88319	11	84407	9.92638	25	0.07362	1.1847	50
11	524	9.80972	15	398	9.88308	10	457	9.92663	26	0.07337	840	49
12	546	9.80987	15	380	9.88298	11	507	9.92689	26	0.07311	833	48
13	568	9.81002	15	361	9.88287	11	556	9.92715	25	0.07285	826	47
14	590	9.81017	15	342	9.88276	10	606	9.92740	26	0.07260	819	46
15	64612	9.81032	15	76323	9.88266	11	84656	9.92766	26	0.07234	1.1812	45
16	635	9.81047	14	304	9.88255	11	706	9.92792	25	0.07208	806	44
17	657	9.81061	15	286	9.88244	10	756	9.92817	26	0.07183	799	43
18	679	9.81076	15	267	9.88234	11	806	9.92843	25	0.07157	792	42
19	701	9.81091	15	248	9.88223	11	856	9.92868	26	0.07132	785	41
20	64723	9.81106	15	76229	9.88212	11	84906	9.92894	26	0.07106	1.1778	40
21	746	9.81121	15	210	9.88201	10	956	9.92920	25	0.07080	771	39
22	768	9.81136	15	192	9.88191	11	85006	9.92945	26	0.07055	764	38
23	790	9.81151	15	173	9.88180	11	057	9.92971	25	0.07029	757	37
24	812	9.81166	14	154	9.88169	11	107	9.92996	26	0.07004	750	36
25	64834	9.81180	15	76135	9.88158	10	85157	9.93022	26	0.06978	1.1743	35
26	856	9.81195	15	116	9.88148	11	207	9.93048	25	0.06952	736	34
27	878	9.81210	15	097	9.88137	11	257	9.93073	26	0.06927	729	33
28	901	9.81225	15	078	9.88126	11	308	9.93099	25	0.06901	722	32
29	923	9.81240	14	059	9.88115	10	358	9.93124	26	0.06876	715	31
30	64945	9.81254	15	76041	9.88105	11	85408	9.93150	25	0.06850	1.1708	30
31	967	9.81269	15	022	9.88094	11	458	9.93175	26	0.06825	702	29
32	989	9.81284	15	003	9.88083	11	509	9.93201	26	0.06799	695	28
33	65011	9.81299	15	75984	9.88072	11	559	9.93227	25	0.06773	688	27
34	033	9.81314	14	965	9.88061	10	609	9.93252	26	0.06748	681	26
35	65055	9.81328	15	75946	9.88051	11	85660	9.93278	25	0.06722	1.1674	25
36	077	9.81343	15	927	9.88040	11	710	9.93303	26	0.06697	667	24
37	100	9.81358	14	908	9.88029	11	761	9.93329	25	0.06671	660	23
38	122	9.81372	15	889	9.88018	11	811	9.93354	26	0.06646	653	22
39	144	9.81387	15	870	9.88007	11	862	9.93380	26	0.06620	647	21
40	65166	9.81402	15	75851	9.87996	11	85912	9.93406	25	0.06594	1.1640	20
41	188	9.81417	14	832	9.87985	10	963	9.93431	26	0.06569	633	19
42	210	9.81431	15	813	9.87975	11	86014	9.93457	25	0.06543	626	18
43	232	9.81446	15	794	9.87964	11	064	9.93482	26	0.06518	619	17
44	254	9.81461	14	775	9.87953	11	115	9.93508	25	0.06492	612	16
45	65276	9.81475	15	75756	9.87942	11	86166	9.93533	26	0.06467	1.1606	15
46	298	9.81490	15	738	9.87931	11	216	9.93559	25	0.06441	599	14
47	320	9.81505	14	719	9.87920	11	267	9.93584	26	0.06416	592	13
48	342	9.81519	15	700	9.87909	11	318	9.93610	26	0.06390	585	12
49	364	9.81534	15	680	9.87898	11	368	9.93636	25	0.06364	578	11
50	65386	9.81549	14	75661	9.87887	10	86419	9.93661	26	0.06339	1.1571	10
51	408	9.81563	15	642	9.87877	11	470	9.93687	25	0.06313	565	9
52	430	9.81578	14	623	9.87866	11	521	9.93712	26	0.06288	558	8
53	452	9.81592	15	604	9.87855	11	572	9.93738	25	0.06262	551	7
54	474	9.81607	15	585	9.87844	11	623	9.93763	26	0.06237	544	6
55	65496	9.81622	14	75566	9.87833	11	86674	9.93789	25	0.06211	1.1538	5
56	518	9.81636	15	547	9.87822	11	725	9.93814	26	0.06186	531	4
57	540	9.81651	14	528	9.87811	11	776	9.93840	25	0.06160	524	3
58	562	9.81665	15	509	9.87800	11	827	9.93865	26	0.06135	517	2
59	584	9.81680	14	490	9.87789	11	878	9.93891	25	0.06109	510	1
60	606	9.81694		471	9.87778		929	9.93916		0.06084	504	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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41°

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	65606	9.81694	15	75471	9.87778	11	86929	9.93916	26	0.06084	1.1504	60
1	628	9.81709	14	452	9.87767	11	980	9.93942	25	0.06058	497	59
2	650	9.81723	15	433	9.87756	11	87031	9.93967	26	0.06033	490	58
3	672	9.81738	14	414	9.87745	11	082	9.93993	25	0.06007	483	57
4	694	9.81752	15	395	9.87734	11	133	9.94018	26	0.05982	477	56
5	65716	9.81767	14	75375	9.87723	11	87184	9.94044	25	0.05956	1.1470	55
6	738	9.81781	15	356	9.87712	11	236	9.94069	26	0.05931	463	54
7	759	9.81796	14	337	9.87701	11	287	9.94095	25	0.05905	456	53
8	781	9.81810	15	318	9.87690	11	338	9.94120	26	0.05880	450	52
9	803	9.81825	14	299	9.87679	11	389	9.94146	25	0.05854	443	51
10	65825	9.81839	15	75280	9.87668	11	87441	9.94171	26	0.05829	1.1436	50
11	847	9.81854	14	261	9.87657	11	492	9.94197	25	0.05803	430	49
12	869	9.81868	14	241	9.87646	11	543	9.94222	26	0.05778	423	48
13	891	9.81882	15	222	9.87635	11	595	9.94248	25	0.05752	416	47
14	913	9.81897	14	203	9.87624	11	646	9.94273	26	0.05727	410	46
15	65935	9.81911	15	75184	9.87613	12	87698	9.94299	25	0.05701	1.1403	45
16	956	9.81926	14	165	9.87601	11	749	9.94324	26	0.05676	396	44
17	978	9.81940	15	146	9.87590	11	801	9.94350	25	0.05650	389	43
18	66000	9.81955	14	126	9.87579	11	852	9.94375	26	0.05625	383	42
19	022	9.81969	14	107	9.87568	11	904	9.94401	25	0.05599	376	41
20	66044	9.81983	15	75088	9.87557	11	87955	9.94426	26	0.05574	1.1369	40
21	066	9.81998	14	069	9.87546	11	88007	9.94452	25	0.05548	363	39
22	088	9.82012	14	050	9.87535	11	059	9.94477	26	0.05523	356	38
23	109	9.82026	15	030	9.87524	11	110	9.94503	25	0.05497	349	37
24	131	9.82041	14	011	9.87513	12	162	9.94528	26	0.05472	343	36
25	66153	9.82055	14	74992	9.87501	11	88214	9.94554	25	0.05446	1.1336	35
26	175	9.82069	15	973	9.87490	11	265	9.94579	25	0.05421	329	34
27	197	9.82084	14	953	9.87479	11	317	9.94604	26	0.05396	323	33
28	218	9.82098	14	934	9.87468	11	369	9.94630	25	0.05370	316	32
29	240	9.82112	14	915	9.87457	11	421	9.94655	26	0.05345	310	31
30	66262	9.82126	15	74896	9.87446	12	88473	9.94681	25	0.05319	1.1303	30
31	284	9.82141	14	876	9.87434	11	524	9.94706	26	0.05294	296	29
32	306	9.82155	14	857	9.87423	11	576	9.94732	25	0.05268	290	28
33	327	9.82169	15	838	9.87412	11	628	9.94757	26	0.05243	283	27
34	349	9.82184	14	818	9.87401	11	680	9.94783	25	0.05217	276	26
35	66371	9.82198	14	74799	9.87390	12	88732	9.94808	26	0.05192	1.1270	25
36	393	9.82212	14	780	9.87378	11	784	9.94834	25	0.05166	263	24
37	414	9.82226	14	760	9.87367	11	836	9.94859	25	0.05141	257	23
38	436	9.82240	15	741	9.87356	11	888	9.94884	26	0.05116	250	22
39	458	9.82255	14	722	9.87345	11	940	9.94910	25	0.05090	243	21
40	66480	9.82269	14	74703	9.87334	12	88992	9.94935	26	0.05065	1.1237	20
41	501	9.82283	14	683	9.87322	11	89045	9.94961	25	0.05039	230	19
42	523	9.82297	14	664	9.87311	11	097	9.94986	26	0.05014	224	18
43	545	9.82311	15	644	9.87300	12	149	9.95012	25	0.04988	217	17
44	566	9.82326	14	625	9.87288	11	201	9.95037	25	0.04963	211	16
45	66588	9.82340	14	74606	9.87277	11	89253	9.95062	26	0.04938	1.1204	15
46	610	9.82354	14	586	9.87266	11	306	9.95088	25	0.04912	197	14
47	632	9.82368	14	567	9.87255	12	358	9.95113	26	0.04887	191	13
48	653	9.82382	14	548	9.87243	11	410	9.95139	25	0.04861	184	12
49	675	9.82396	14	528	9.87232	11	463	9.95164	26	0.04836	178	11
50	66697	9.82410	14	74509	9.87221	12	89515	9.95190	25	0.04810	1.1171	10
51	718	9.82424	15	489	9.87209	11	567	9.95215	25	0.04785	165	9
52	740	9.82439	14	470	9.87198	11	620	9.95240	26	0.04760	158	8
53	762	9.82453	14	451	9.87187	12	672	9.95266	25	0.04734	152	7
54	783	9.82467	14	431	9.87175	11	725	9.95291	26	0.04709	145	6
55	66805	9.82481	14	74412	9.87164	11	89777	9.95317	25	0.04683	1.1139	5
56	827	9.82495	14	392	9.87153	12	830	9.95342	26	0.04658	132	4
57	848	9.82509	14	373	9.87141	11	883	9.95368	25	0.04632	126	3
58	870	9.82523	14	353	9.87130	11	935	9.95393	25	0.04607	119	2
59	891	9.82537	14	334	9.87119	12	988	9.95418	26	0.04582	113	1
60	913	9.82551		314	9.87107		90040	9.95444		0.04556	106	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

48°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	
0	66913	9.82551	14	74314	9.87107	11	90040	9.95444	25	0.04556	1.1106	60
1	935	9.82565	14	295	9.87096	11	093	9.95469	26	0.04531	100	59
2	956	9.82579	14	276	9.87085	12	146	9.95495	25	0.04505	093	58
3	978	9.82593	14	256	9.87073	11	199	9.95520	25	0.04480	087	57
4	999	9.82607	14	237	9.87062	12	251	9.95545	26	0.04455	080	56
5	67021	9.82621	14	74217	9.87050	11	90304	9.95571	25	0.04429	1.1074	55
6	043	9.82635	14	198	9.87039	11	357	9.95596	26	0.04404	067	54
7	064	9.82649	14	178	9.87028	12	410	9.95622	25	0.04378	061	53
8	086	9.82663	14	159	9.87016	11	463	9.95647	25	0.04353	054	52
9	107	9.82677	14	139	9.87005	12	516	9.95672	26	0.04328	048	51
10	67129	9.82691	14	74120	9.86993	11	90569	9.95698	25	0.04302	1.1041	50
11	151	9.82705	14	100	9.86982	12	621	9.95723	25	0.04277	035	49
12	172	9.82719	14	080	9.86970	11	674	9.95748	26	0.04252	028	48
13	194	9.82733	14	061	9.86959	12	727	9.95774	25	0.04226	022	47
14	215	9.82747	14	041	9.86947	11	781	9.95799	26	0.04201	016	46
15	67237	9.82761	14	74022	9.86936	12	90834	9.95825	25	0.04175	1.1009	45
16	258	9.82775	13	002	9.86924	11	887	9.95850	25	0.04150	003	44
17	280	9.82788	14	73983	9.86913	11	940	9.95875	26	0.04125	1.0996	43
18	301	9.82802	14	963	9.86902	12	993	9.95901	25	0.04099	990	42
19	323	9.82816	14	944	9.86890	11	91046	9.95926	26	0.04074	983	41
20	67344	9.82830	14	73924	9.86879	12	91099	9.95952	25	0.04048	1.0977	40
21	366	9.82844	14	904	9.86867	12	153	9.95977	25	0.04023	971	39
22	387	9.82858	14	885	9.86855	11	206	9.96002	26	0.03998	964	38
23	409	9.82872	13	865	9.86844	12	259	9.96028	25	0.03972	958	37
24	430	9.82885	14	846	9.86832	11	313	9.96053	25	0.03947	951	36
25	67452	9.82899	14	73826	9.86821	12	91366	9.96078	26	0.03922	1.0945	35
26	473	9.82913	14	806	9.86809	11	419	9.96104	25	0.03896	939	34
27	495	9.82927	14	787	9.86798	12	473	9.96129	26	0.03871	932	33
28	516	9.82941	14	767	9.86786	11	526	9.96155	25	0.03845	926	32
29	538	9.82955	13	747	9.86775	12	580	9.96180	25	0.03820	919	31
30	67559	9.82968	14	73728	9.86763	11	91633	9.96205	26	0.03795	1.0913	30
31	580	9.82982	14	708	9.86752	12	687	9.96231	25	0.03769	907	29
32	602	9.82996	14	688	9.86740	12	740	9.96256	25	0.03744	900	28
33	623	9.83010	13	669	9.86728	11	794	9.96281	26	0.03719	894	27
34	645	9.83023	14	649	9.86717	12	847	9.96307	25	0.03693	888	26
35	67666	9.83037	14	73629	9.86705	11	91901	9.96332	25	0.03668	1.0881	25
36	688	9.83051	14	610	9.86694	12	955	9.96357	26	0.03643	875	24
37	709	9.83065	13	590	9.86682	12	92008	9.96383	25	0.03617	869	23
38	730	9.83078	14	570	9.86670	11	062	9.96408	25	0.03592	862	22
39	752	9.83092	14	551	9.86659	12	116	9.96433	26	0.03567	856	21
40	67773	9.83106	14	73531	9.86647	12	92170	9.96459	25	0.03541	1.0850	20
41	795	9.83120	13	511	9.86635	11	224	9.96484	26	0.03516	843	19
42	816	9.83133	14	491	9.86624	12	277	9.96510	25	0.03490	837	18
43	837	9.83147	14	472	9.86612	12	331	9.96535	25	0.03465	831	17
44	859	9.83161	13	452	9.86600	11	385	9.96560	26	0.03440	824	16
45	67880	9.83174	14	73432	9.86589	12	92439	9.96586	25	0.03414	1.0818	15
46	901	9.83188	14	413	9.86577	12	493	9.96611	25	0.03389	812	14
47	923	9.83202	13	393	9.86565	11	547	9.96636	26	0.03364	805	13
48	944	9.83215	14	373	9.86554	12	601	9.96662	25	0.03338	799	12
49	965	9.83229	13	353	9.86542	12	655	9.96687	25	0.03313	793	11
50	67987	9.83242	14	73333	9.86530	12	92709	9.96712	26	0.03288	1.0786	10
51	68008	9.83256	14	314	9.86518	11	763	9.96738	25	0.03262	780	9
52	029	9.83270	13	294	9.86507	12	817	9.96763	25	0.03237	774	8
53	051	9.83283	14	274	9.86495	12	872	9.96788	26	0.03212	768	7
54	072	9.83297	13	254	9.86483	11	926	9.96814	25	0.03186	761	6
55	68093	9.83310	14	73234	9.86472	12	92980	9.96839	25	0.03161	1.0755	5
56	115	9.83324	14	215	9.86460	12	93034	9.96864	26	0.03136	749	4
57	136	9.83338	13	195	9.86448	12	088	9.96890	25	0.03110	742	3
58	157	9.83351	14	175	9.86436	11	143	9.96915	25	0.03085	736	2
59	179	9.83365	13	155	9.86425	12	197	9.96940	26	0.03060	730	1
60	200	9.83378		135	9.86413		252	9.96966		0.03034	724	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

′	Nat. Sin	Log.	d.	Nat. Cos	Log.	d.	Nat. Tan	Log.	c.d.	Log. Cot	Nat.	
0	68200	9.83378	14	73135	9.86413	12	93252	9.96966	25	0.03034	1.0724	60
1	221	9.83392	13	116	9.86401	12	306	9.96991	25	0.03009	717	59
2	242	9.83405	14	096	9.86389	12	360	9.97016	26	0.02984	711	58
3	264	9.83419	13	076	9.86377	11	415	9.97042	25	0.02958	705	57
4	285	9.83432	14	056	9.86366	12	469	9.97067	25	0.02933	699	56
5	68306	9.83446	13	73036	9.86354	12	93524	9.97092	26	0.02908	1.0692	55
6	327	9.83459	14	016	9.86342	12	578	9.97118	25	0.02882	686	54
7	349	9.83473	13	72996	9.86330	12	633	9.97143	25	0.02857	680	53
8	370	9.83486	14	976	9.86318	12	688	9.97168	25	0.02832	674	52
9	391	9.83500	13	957	9.86306	11	742	9.97193	26	0.02807	668	51
10	68412	9.83513	14	72937	9.86295	12	93797	9.97219	25	0.02781	1.0661	50
11	434	9.83527	13	917	9.86283	12	852	9.97244	25	0.02756	655	49
12	455	9.83540	14	897	9.86271	12	906	9.97269	26	0.02731	649	48
13	476	9.83554	13	877	9.86259	12	961	9.97295	25	0.02705	643	47
14	497	9.83567	14	857	9.86247	12	94016	9.97320	25	0.02680	637	46
15	68518	9.83581	13	72837	9.86235	12	94071	9.97345	26	0.02655	1.0630	45
16	539	9.83594	14	817	9.86223	12	125	9.97371	25	0.02629	624	44
17	561	9.83608	13	797	9.86211	11	180	9.97396	25	0.02604	618	43
18	582	9.83621	13	777	9.86200	12	235	9.97421	26	0.02579	612	42
19	603	9.83634	14	757	9.86188	12	290	9.97447	25	0.02553	606	41
20	68624	9.83648	13	72737	9.86176	12	94345	9.97472	25	0.02528	1.0599	40
21	645	9.83661	13	717	9.86164	12	400	9.97497	26	0.02503	593	39
22	666	9.83674	14	697	9.86152	12	455	9.97523	25	0.02477	587	38
23	688	9.83688	13	677	9.86140	12	510	9.97548	25	0.02452	581	37
24	709	9.83701	14	657	9.86128	12	565	9.97573	25	0.02427	575	36
25	68730	9.83715	13	72637	9.86116	12	94620	9.97598	26	0.02402	1.0569	35
26	751	9.83728	13	617	9.86104	12	676	9.97624	25	0.02376	562	34
27	772	9.83741	14	597	9.86092	12	731	9.97649	25	0.02351	556	33
28	793	9.83755	13	577	9.86080	12	786	9.97674	26	0.02326	550	32
29	814	9.83768	13	557	9.86068	12	841	9.97700	25	0.02300	544	31
30	68835	9.83781	14	72537	9.86056	12	94896	9.97725	25	0.02275	1.0538	30
31	857	9.83795	13	517	9.86044	12	952	9.97750	26	0.02250	532	29
32	878	9.83808	13	497	9.86032	12	95007	9.97776	25	0.02224	526	28
33	899	9.83821	13	477	9.86020	12	062	9.97801	25	0.02199	519	27
34	920	9.83834	14	457	9.86008	12	118	9.97826	25	0.02174	513	26
35	68941	9.83848	13	72437	9.85996	12	95173	9.97851	26	0.02149	1.0507	25
36	962	9.83861	13	417	9.85984	12	229	9.97877	25	0.02123	501	24
37	983	9.83874	13	397	9.85972	12	284	9.97902	25	0.02098	495	23
38	69004	9.83887	14	377	9.85960	12	340	9.97927	26	0.02073	489	22
39	025	9.83901	13	357	9.85948	12	395	9.97953	25	0.02047	483	21
40	69046	9.83914	13	72337	9.85936	12	95451	9.97978	25	0.02022	1.0477	20
41	067	9.83927	13	317	9.85924	12	506	9.98003	26	0.01997	470	19
42	088	9.83940	14	297	9.85912	12	562	9.98029	25	0.01971	464	18
43	109	9.83954	13	277	9.85900	12	618	9.98054	25	0.01946	458	17
44	130	9.83967	13	257	9.85888	12	673	9.98079	25	0.01921	452	16
45	69151	9.83980	13	72236	9.85876	12	95729	9.98104	26	0.01896	1.0446	15
46	172	9.83993	13	216	9.85864	13	785	9.98130	25	0.01870	440	14
47	193	9.84006	14	196	9.85851	12	841	9.98155	25	0.01845	434	13
48	214	9.84020	13	176	9.85839	12	897	9.98180	26	0.01820	428	12
49	235	9.84033	13	156	9.85827	12	952	9.98206	25	0.01794	422	11
50	69256	9.84046	13	72136	9.85815	12	96008	9.98231	25	0.01769	1.0416	10
51	277	9.84059	13	116	9.85803	12	064	9.98256	25	0.01744	410	9
52	298	9.84072	13	095	9.85791	12	120	9.98281	26	0.01719	404	8
53	319	9.84085	13	075	9.85779	13	176	9.98307	25	0.01693	398	7
54	340	9.84098	14	055	9.85766	12	232	9.98332	25	0.01668	392	6
55	69361	9.84112	13	72035	9.85754	12	96288	9.98357	26	0.01643	1.0385	5
56	382	9.84125	13	015	9.85742	12	344	9.98383	25	0.01617	379	4
57	403	9.84138	13	71995	9.85730	12	400	9.98408	25	0.01592	373	3
58	424	9.84151	13	974	9.85718	12	457	9.98433	25	0.01567	367	2
59	445	9.84164	13	954	9.85706	13	513	9.98458	26	0.01542	361	1
60	466	9.84177		934	9.85693		569	9.98484		0.01516	355	0
	Nat. Cos	Log.	d.	Nat. Sin	Log.	d.	Nat. Cot	Log.	c.d.	Log. Tan	Nat.	′

44°

′	Nat. Sin	Log. Sin	d.	Nat. Cos	Log. Cos	d.	Nat. Tan	Log. Tan	c.d.	Log. Cot	Nat. Cot	′
0	69466	9.84177		71934	9.85693		96569	9.98484		0.01516	1.0355	60
1	487	9.84190	13	914	9.85681	12	625	9.98509	25	0.01491	349	59
2	508	9.84203	13	894	9.85669	12	681	9.98534	25	0.01466	343	58
3	529	9.84216	13	873	9.85657	12	738	9.98560	26	0.01440	337	57
4	549	9.84229	13	853	9.85645	12	794	9.98585	25	0.01415	331	56
5	69570	9.84242	13	71833	9.85632	13	96850	9.98610	25	0.01390	1.0325	55
6	591	9.84255	13	813	9.85620	12	907	9.98635	25	0.01365	319	54
7	612	9.84269	14	792	9.85608	12	963	9.98661	26	0.01339	313	53
8	633	9.84282	13	772	9.85596	12	97020	9.98686	25	0.01314	307	52
9	654	9.84295	13	752	9.85583	13	076	9.98711	25	0.01289	301	51
10	69675	9.84308	13	71732	9.85571	12	97133	9.98737	26	0.01263	1.0295	50
11	696	9.84321	13	711	9.85559	12	189	9.98762	25	0.01238	289	49
12	717	9.84334	13	691	9.85547	12	246	9.98787	25	0.01213	283	48
13	737	9.84347	13	671	9.85534	13	302	9.98812	25	0.01188	277	47
14	758	9.84360	13	650	9.85522	12	359	9.98832	26	0.01162	271	46
15	69779	9.84373	13	71630	9.85510	12	97416	9.98863	25	0.01137	1.0265	45
16	800	9.84385	12	610	9.85497	13	472	9.98888	25	0.01112	259	44
17	821	9.84398	13	590	9.85485	12	529	9.98913	25	0.01087	253	43
18	842	9.84411	13	569	9.85473	12	586	9.98939	26	0.01061	247	42
19	862	9.84424	13	549	9.85460	13	643	9.98964	25	0.01036	241	41
20	69883	9.84437	13	71529	9.85448	12	97700	9.98989	25	0.01011	1.0235	40
21	904	9.84450	13	508	9.85436	12	756	9.99015	26	0.00985	230	39
22	925	9.84463	13	488	9.85423	13	813	9.99040	25	0.00960	224	38
23	946	9.84476	13	468	9.85411	12	870	9.99065	25	0.00935	218	37
24	966	9.84489	13	447	9.85399	12	927	9.99090	25	0.00910	212	36
25	69987	9.84502	13	71427	9.85386	13	97984	9.99116	26	0.00884	1.0206	35
26	70008	9.84515	13	407	9.85374	12	98041	9.99141	25	0.00859	200	34
27	029	9.84528	13	386	9.85361	13	098	9.99166	25	0.00834	194	33
28	049	9.84540	12	366	9.85349	12	155	9.99191	25	0.00809	188	32
29	070	9.84553	13	345	9.85337	12	213	9.99217	26	0.00783	182	31
30	70091	9.84566	13	71325	9.85324	13	98270	9.99242	25	0.00758	1.0176	30
31	112	9.84579	13	305	9.85312	12	327	9.99267	25	0.00733	170	29
32	132	9.84592	13	284	9.85299	13	384	9.99293	26	0.00707	164	28
33	153	9.84605	13	264	9.85287	12	441	9.99318	25	0.00682	158	27
34	174	9.84618	13	243	9.85274	13	499	9.99343	25	0.00657	152	26
35	70195	9.84630	12	71223	9.85262	12	98556	9.99368	25	0.00632	1.0147	25
36	215	9.84643	13	203	9.85250	12	613	9.99394	26	0.00606	141	24
37	236	9.84656	13	182	9.85237	13	671	9.99419	25	0.00581	135	23
38	257	9.84669	13	162	9.85225	12	728	9.99444	25	0.00556	129	22
39	277	9.84682	13	141	9.85212	13	786	9.99469	25	0.00531	123	21
40	70298	9.84694	12	71121	9.85200	12	98843	9.99495	26	0.00505	1.0117	20
41	319	9.84707	13	100	9.85187	13	901	9.99520	25	0.00480	111	19
42	339	9.84720	13	080	9.85175	12	958	9.99545	25	0.00455	105	18
43	360	9.84733	13	059	9.85162	13	99016	9.99570	25	0.00430	099	17
44	381	9.84745	12	039	9.85150	12	073	9.99596	26	0.00404	094	16
45	70401	9.84758	13	71019	9.85137	13	99131	9.99621	25	0.00379	1.0088	15
46	422	9.84771	13	70998	9.85125	12	189	9.99646	25	0.00354	082	14
47	443	9.84784	13	978	9.85112	13	247	9.99672	26	0.00328	076	13
48	463	9.84796	12	957	9.85100	12	304	9.99697	25	0.00303	070	12
49	484	9.84809	13	937	9.85087	13	362	9.99722	25	0.00278	064	11
50	70505	9.84822	13	70916	9.85074	13	99420	9.99747	25	0.00253	1.0058	10
51	525	9.84835	13	896	9.85062	12	478	9.99773	26	0.00227	052	9
52	546	9.84847	12	875	9.85049	13	536	9.99798	25	0.00202	047	8
53	567	9.84860	13	855	9.85037	12	594	9.99823	25	0.00177	041	7
54	587	9.84873	13	834	9.85024	13	652	9.99848	25	0.00152	035	6
55	70608	9.84885	12	70813	9.85012	12	99710	9.99874	26	0.00126	1.0029	5
56	628	9.84898	13	793	9.84999	13	768	9.99899	25	0.00101	023	4
57	649	9.84911	13	772	9.84986	13	826	9.99924	25	0.00076	017	3
58	670	9.84923	12	752	9.84974	12	884	9.99940	25	0.00051	012	2
59	690	9.84936	13	731	9.84961	13	942	9.99975	26	0.00025	006	1
60	711	9.84949	13	711	9.84949	12	1.0000	10.00000	25	0.00000	000	0
	Nat. Cos	Log. Cos	d.	Nat. Sin	Log. Sin	d.	Nat. Cot	Log. Cot	c.d.	Log. Tan	Nat. Tan	′

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BALL BEARINGS

CONTENTS

49-55 Lafayette Street, New York City

CHAPTER I

PRINCIPLES OF METAL SPINNING*

Metal spinning, that process of sheet metal goods manufacturing which deals with the forming of sheet metal into circular shapes of great variety by means of the lathe, forms and hand-tools, is full of kinks and schemes peculiar to itself. It is the purpose of this treatise to give a description of spinning in general, and to outline some of the methods and tools used in spinning for rapid production.

The products of metal spinning are used in a great many lines of manufacture. Examples of this work are chandelier parts, cooking utensils, silver and brittania hollow-ware, automobile lamps, cane-heads and many other sheet metal specialties. Brass, copper, zinc, aluminum, iron, soft steel, and, in fact nearly all metals yield readily to the spinner's skill. At best spinning is physically hard work, and the softer the stock, the easier and quicker the spinner can transform it into the required product.

There are but two practical ways of forming pieces of sheet metal into hollow circular articles: by dies and by spinning. By far the cheapest and best method of producing quantities of this class of work is by the use of dies, but there are many cases where it is impractical or impossible to follow this course. Dies are expensive and there is constant danger of breakage, whereas spinning forms are easily and cheaply made and are almost never damaged by use beyond a reasonable amount of wear. Thus it will be seen that when the production is small, it does not pay to make costly dies. Again, the styles or designs of many articles that are spun are constantly being changed; if made by dies each change would necessitate a new die, while in spinning merely a new wooden form is required—and sometimes the old form can be altered, costing practically nothing. Still other advantages of spinning are that in working soft steel, a much cheaper grade may be spun than can be drawn with dies; beads may be rolled at the edges of shells at little expense; experimental pieces may be made quickly, and, added to these features comes the fact that very difficult work that cannot possibly be made with dies can be spun with comparative ease. It must not be construed from the above that spinning is to be preferred to die work in all or even in the majority of cases, because, on the contrary, die work is a more economical method of manufacture, and should always be used when possible on production work. The cases already cited are merely given to point out some of the instances in which, for economical reasons, spinning is to be preferred to die work.

* MACHINERY, December, 1909.

The Spinning Lathe

The principal tool used in the operation of spinning is the spinning lathe, shown in Fig. 1. While in many respects this machine is similar to any other lathe, it is built without back-gears, carriage or lead-screw, is very rigid in construction, and, on the whole, very much resembles a speed lathe. Like other lathes, the spinning lathe is fitted with a cone pulley (preferably of wood, because of its lightness and gripping qualities), allowing the use of four or five different speeds. Speed is an important factor in spinning. Arbitrary rules for spinning speeds cannot be given, as the thicker the stock the

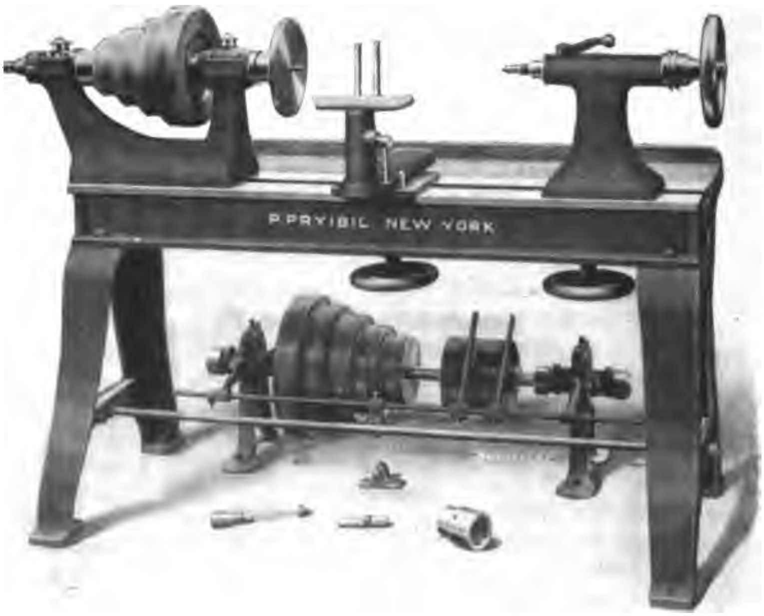


Fig. 1 Spinning Lathe

slower must be the speed; thus while 1/32-inch iron can be readily spun at 600 revolutions, 1/16-inch iron would necessitate reducing the speed to 400 revolutions per minute. Zinc spins best at from 1,000 to 1,400 revolutions; copper works well at 800 to 1,000; brass and aluminum require practically the same speed, from 800 to 1,200; while the comparatively slow speed of 300 to 600 revolutions is effective on iron and soft steel. Brittania and silver spin best at speeds from 800 to 1,000 revolutions.

One of the essential parts of the spinning lathe is the T-rest. The base of this rest is movable on the ways of the lathe, and it has at the side nearest the operator, a stud about four inches in diameter and six inches high, through which is swiveled the T-rest proper.

As the illustration shows, provision is made for raising and lowering the rest, and the entire rest may be clamped in any desired position by means of the hand-wheel shown beneath the ways. The rest proper consists of an arm, 12 to 15 inches long, similar to a wood turner's rest, and through the face of this arm are from twelve to sixteen closely spaced ⅜-inch holes. These holes are to receive the pin against which the hand tools are held while spinning. The pin is three inches long and of ¾-inch steel, turned down on one end to loosely fit the holes in the rest.

Another important part of the spinning lathe is the tail-center. This center is sometimes the ordinary dead center that is in general

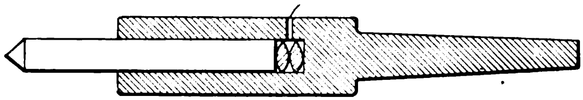


Fig. 2. Revolving Center

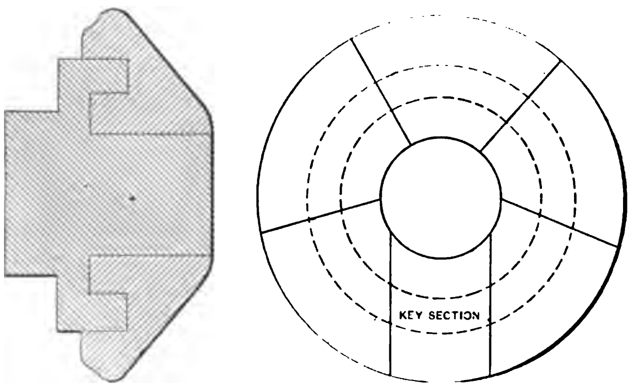


Fig. 3. Sectional Spinning Chuck

machine shop use, but nearly all spinners use the revolving center, shown in Fig. 2. The revolving center is ¾ inch diameter (without taper) and about six inches long, and is fitted into the socket in which it runs; this socket is, in turn, fitted to the taper hole in the tail-stock. At the bottom of the hole in the socket are two steel buttons, hardened and ground convex on their faces. These buttons act as ball bearings and reduce friction to a minimum.

Forms and Chucks for Spinning

The shape of a shell made by spinning is dependent on the form or chuck upon which the metal is spun. Forms are used for plain spinning where the shape of the shell will permit of its being readily taken from the form after the spinning has been completed; but when the shape of the shell is such that it will not "draw," as the molders say, it becomes necessary to employ sectional chucks, similar to the one shown in Fig. 3. Generally speaking, spinning forms are made of kiln dried maple. After being bored and threaded to fit the lathe spindle, the spinner turns the maple block to agree with a templet shaped in outline to the sample shell. When no sample is furnished, the templet must be laid out from a sketch or drawing; in either case proper allowance is made for the thickness of the stock. When large quantities of shells are to be spun, all alike, the form is sometimes made of lignum vitæ. Another method is to turn the maple form small enough so that one shell may be spun and cemented to it and then this metal-cased form is used to spin the balance of the shells. For continuous spinning, forms are made of cast iron or steel, which of course makes a most satisfactory surface to spin on and gives indefinite service.

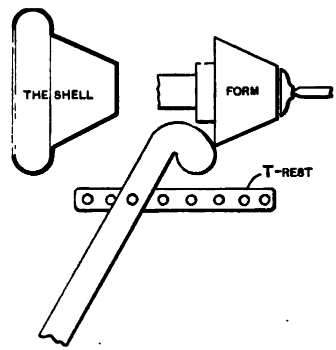
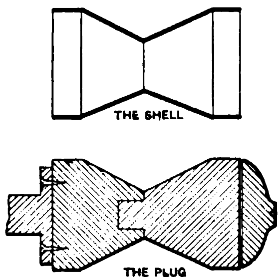


Fig. 4. Quick Method of Spinning Difficult Shell Without Sectional Chuck



Machinery, N.Y.

Fig. 5. Spinning on Plugs

A sectional or "split" chuck, as it is sometimes called, is, as the name implies, a spinning chuck or form which may be taken apart in sections after the shell has been spun over it. As before stated, this class of spinning chuck is only used when the finished shell could not be removed from an ordinary form after spinning. After a shell has been spun over a sectional chuck, the shell and the sections of the chuck are together pulled lengthwise from the core of the chuck. Then, starting with the key section, it is an easy matter to remove each section from the inside of the shell. As the sections are removed, they are replaced upon the core, slipped under the retaining flange and the chuck is ready for spinning a new shell. The whole operation of removing and replacing the sections of a chuck takes less time than it does to tell it, and, as the sections are of different sizes, it is easy to replace the them in the proper order. Like other forms, sectional chucks are made of wood or metal, according to the requirements of the job. The core and retaining ring are first made from one piece and then the sections are turned in a continuous ring and split with a fine saw. In some cases it is necessary to add a small piece to the last section to make up for the stock lost in splitting the sections.

Another kind of sectional chuck, known to the trade as a "plug" (shown in Fig. 5) is used extensively in some shops in cases where the shell must have projections or shoulders at both ends, and no bottom to the shell is required. In making the plug, which is always in two parts, the first half is turned to take the shell from one end to the center of the smallest diameter. Into the end of this part is bored a hole to which is fitted the end of the second part, which is afterwards turned to fit the shell. Over this two-part plug the shell is spun; then the bottom of the shell is cut out and the first half of the plug removed, thus allowing the shell to be withdrawn. The first part is then replaced and the plug is ready for use again. Fig. 4 shows a method of spinning difficult shells that ordinarily would require a sectional chuck. The shell shown at the left of Fig. 4 is first spun as far as the bulged part on an ordinary form that ends at this

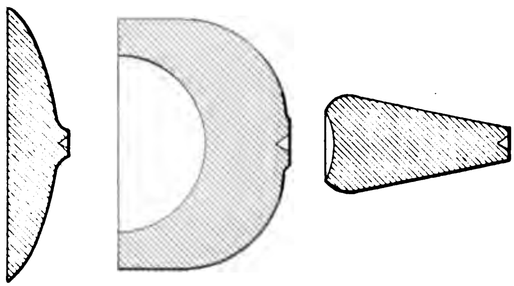


Fig. 6. Three Types of Followers

point. Then after annealing, it is replaced on the form and while another operator holds the wooden arm, supported with a pin in the T-rest, the spinner forms the metal around the bulge-shaped end of the arm. The arm, being stationary on the inside of the shell, acts as a continuation of the spinning form, and by this method as good a shell is obtained as could be spun with a sectional chuck.

For spinning operations upon tubing or press-drawn tubes, steel arbors are generally used. Tubing may be readily spun upon an arbor and it can be reduced or expanded to comply with the shape of shell required much more quickly than the shell could be spun from the blank.

Followers

For holding the sheet metal blank to the spinning form, a block of wood known as the follower, is used (see Fig. 6). Followers are made to suit the shape of the work with which they are to be employed, always being made with the largest possible bearing on the work; thus a shell with a flat bottom twelve inches in diameter would be turned with the aid of a follower having an 11¾-inch face, while a shell with a 4-inch face would take a follower with a 3⅞-inch face. All shells do not have flat bottoms, consequently, in spinning such as do not, it becomes necessary to employ hollow followers. Hollow followers have their bearing surfaces turned out to fit the ends of the

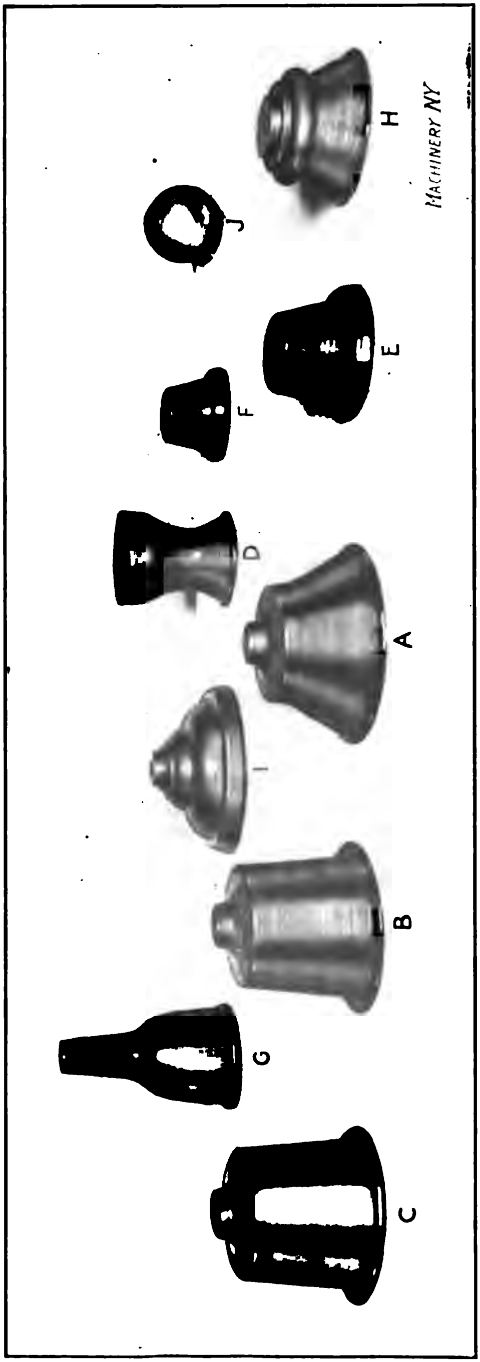


Fig. 7. Specimens of Metal Spinning

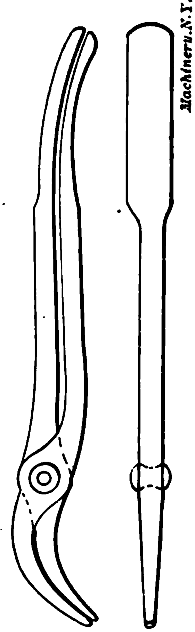
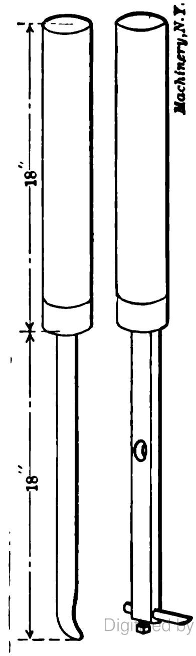


Fig. 10. Spinners' Pliers



Figs. 8 and 9. Spinning Tool and Swivel Cutter forms with which they are to be used. In practice, the blank is held against the end of the spherical form with a small flat follower until enough of the shell has been spun to admit of the hollow follower being used. All followers are made with a large center hole in one end to receive the revolving tail-center.

In starting to spin a difficult shell it sometimes happens that the necessarily small follower will not hold the blank. To prevent this slipping, the face of the follower is covered with emery cloth. Often, however, on rough work, the spinner will not stop to face the follower, but will make a large shallow dent at the center of the blank; the extra pressure required to force the metal against the form will usually overcome the slipping tendency.

Hand Tools

Hand tools, in great variety, form the principal asset of the spinner's kit. Spinning tools are made of tool steel forged to the required shapes, and are hardened and polished on the working end. The round steel from which they are made varies from ½ inch to 1½ inch in diameter, according to the class of work upon which they are to be used. The length of a spinning tool is about 2 feet, and it is fitted into a wooden handle 2 inches diameter and 18 inches long, making the total length of the handled tool about 3 feet, as shown in Fig. 8. As the spinner holds this handle under the right armpit, he secures a great leverage upon the work and is better able to supply the physical power required to bring the metal to the desired shape.

The commonest and by far the most useful of the spinning tools is the combination "point and ball" which together with a number of other tools, is shown in Fig. 11. This tool is used in doing the bulk of the spinning operations—for starting the work and bringing it approximately to the shape of the form. Its range of usefulness is large on account of the many different shapes that may be utilized by merely turning the tool in a different direction. Next in importance comes the flat or smoothing tool which, as the name implies, is for smoothing the shell and finishing any rough surfaces left by the point and ball tool. The fishtail tool, so named from its shape, is used principally in flaring the end of a shell from the inside, "spinning on air," as it is sometimes termed. This tool is used to good advantage in any place where it is necessary to stretch the metal to any extent, and its thin rounding edge proves useful in setting the metal into corners and narrow grooves. Other tools are the ball tool which is adapted to finishing curves; the hook tool, used on inside work; and the beading tool which is needed in rolling over a bead at the edge of a shell when extra strength or a better finish is desired.

When much beading of one kind is being done, a large heavy pair of round-nose pliers (Fig. 10) with the jaws bent around in a curve and sprung apart enough to allow for the thickness of the metal proves to be a handy tool. After the edge of the shell has been flared out to start the bead, the pliers are opened enough to admit the metal and then closed and the stock guided around to form the bead as far as possible. In this way the larger part of a bead is rapidly formed, one jaw of the pliers acting as a spinning tool and the other corresponding to the back-stick. During this operation, the pliers are, of course, supported by being held against the T-rest.

Closely allied with these spinning tools are two other tools (also shown in Fig. 11) known as the diamond point and the skimmer. The diamond point is for trimming the edges of the shell during the spinning operation and for cutting out centers or other parts of the work. The skimmer is for cleaning up the surface of a shell, removing a small amount of metal in doing so, the amount depending upon the skill the spinner used in the spinning proper.

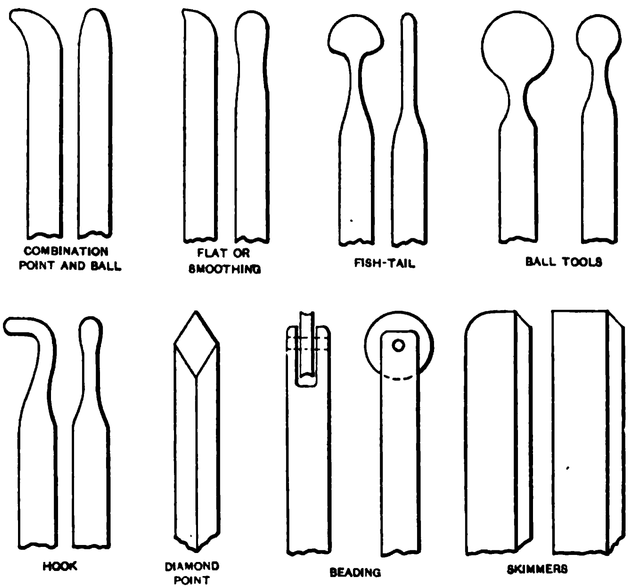


Fig. 11. Hand Tools of Various Forms used in Spinning

When the bottoms are to be cut from a large number of shells and it is necessary that they be cut exactly alike, a tool known as a swivel cutter is used. This tool (see Fig. 9) is simply an iron bar with a cutter on one end, which swivels near the center around a pin in the T-rest; thus by a slight movement of the arm the cutter is brought up to the work, cutting a piece from the shell of exactly the same size each time.

The Spinning Operation

In order to make clear the successive steps in spinning, let us briefly consider the making of a copper head-light reflector, and the way the work is handled when a few hundred pieces are to be made.

By trial spinning, the size of the blank required for one of the reflectors is determined, and with the square shears the copper sheets are cut into pieces an eighth of an inch larger each way. These squares are then taken to the circular shears and cut to round shapes ready for the spinning lathe. The spinning form, of kiln-dried maple, is screwed to the spindle and the belt thrown to that step of the cone pulley which will bring the speed nearest to 1,200 revolutions. From the stock-room a follower is selected whose face will nearly cover the bottom of the form. It is now "up to" the spinner. Holding a blank and also the follower against the end of the form, he runs the tail-center up to the center in the follower just hard enough to hold the blank in place. Then, starting the lathe, he centers the blank by lightly pressing against its edge a hard wood stick. As soon as it "lines up" he runs the center up a little harder and clamps it in place. Some spinners will "hop in" a blank with the lathe running, but this is dangerous practice and sometimes the blank will go sailing across the room. Often this happens in truing up the blank and for this reason it is considered advisable to have a wire grating at the further side of the lathe to prevent serious accidents; for a sheet metal blank is a dangerous missile traveling at the high rate of speed which is imparted to it by the lathe.

With a piece of beeswax (soap is sometimes used for economical reasons) the spinner lightly rubs the rapidly revolving blank and then adjusts the pin in the T-rest to a point near enough to the blank to obtain a good leverage with the spinning tool. Holding the handle of his point and ball tool under his right armpit and using the tool as a lever and the pin on the rest as a fulcrum, he slowly forces the metal disk back in the direction of the body of the form, never allowing the tool to rest in one spot, but constantly working it in and out, applying the pressure on the way out to the edge of the disk and letting up as he comes back for a new stroke. In the meantime his left hand is busy holding a short piece of hard wood (called the back-stick), firmly against the reverse side of the metal at a constantly changing point opposite the tool. The object of the back-stick is to keep the stock from wrinkling as it is stretched toward the edge of the disk. Wrinkles cause the metal to crack at the edges and for this reason they must be kept from the stock as much as possible.

After a few strokes of the spinning tool have been taken, the shell will appear about as shown at *B*, Fig. 12, and at this point it is necessary to trim the shell at the edges with the diamond-point tool. Trimming is required because spinning stretches the stock and the resulting uneven edge will cause splits in the metal if it is not trimmed occasionally. As a carpenter is known by his chips, so a spinner is known by the way his work stretches. While the even pressure of a good spinner will stretch the stock very little, the uneven pressure of the inexperienced man will lead him into all sorts of trouble on account of the way the stock will "go." In either case the metal always stretches least in the direction in which the sheet stock was originally rolled, consequently giving the edge a slight oval shape. In trimming zinc, the spinner holds a "swab" of cloth just above the diamond point, to prevent the chips from flying into his face and eyes—or those of his neighbors. With other metals the swab is unnecessary.

The reflector is now taking shape. With each successive stroke the spinner sets a little more of the metal against the form. Not only does spinning stretch the metal, but it hardens it as well; therefore, at the stage *C* it becomes necessary to anneal the partially completed reflector, which is done by heating it to a low red in a gas furnace. In running through a lot of shells, the common practice is to spin them all as far as possible without annealing, and after annealing the whole lot, to complete the spinning.

After replacing the shell upon the form, it is trimmed and worked further along the form, gradually assuming the appearance shown at

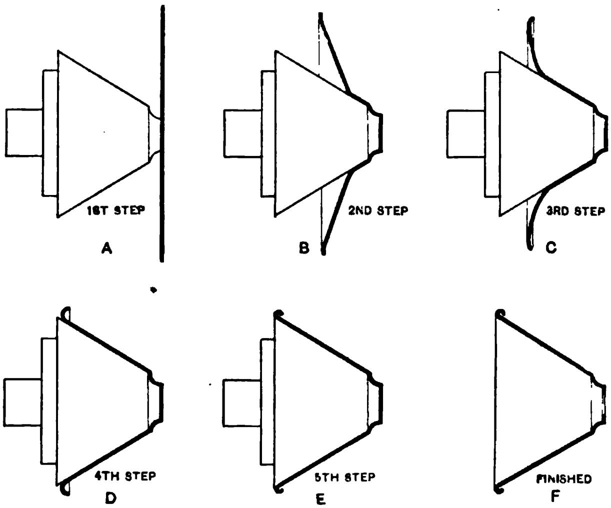


Fig. 12. Successive Steps in Spinning a Reflector

D. At this time, the spinner goes back to the small radius at the front end of the shell and with a ball tool he closes the annealed metal hard down against the form, for the spinning has tended to pull the stock slightly from the form at this point. The body of the reflector is now practically completed and the spinner directs his attention to rolling the bead at the outside edge. Slowly he begins to roll the edge of the shell back, using his hook tool to complete the bead as far as possible and exercising care to keep the back-stick firmly against the metal so as to keep the wrinkles out. Now, with the diamond point, he gives the edges a final trim, and with the beading tool closes down the bead snugly against the rest of the shell, as shown at *E*. Lastly, the swivel cutter is placed in the proper hole of the T-rest and a turn of the tool cuts out the center to the exact size, and the reflector is completed. If any burrs or rough places remain they are easily removed at this time with the skimmer or diamond point, and a little emery cloth gives the shell a finished appearance.

Referring to the illustration Fig. 7, *A*, *B* and *C* represent the three most important stages of spinning a shell like that shown at *C*. Annealing is necessary between steps *A* and *B*. *D* is a shell spun upon a form of the plug variety, and *E* and *F* are two views of a shell spun after the method shown in Fig. 4, *F* being the completed shell. *G* illustrates a very difficult shell to spin, on account of the small follower that must be used; the length of the small diameter also adds to the difficulty. *H* shows a shell that must be spun upon a sectional chuck, while *I* is a plain easy job of ornamental spinning. The ball shown at *J* was spun from one piece of aluminum and it is more of a curiosity than a specimen of practical spinning. It was first spun over a form that would leave one half of the ball complete and the stock for the other half straight out like a short tube. Next a wooden split chuck was made, hollowed out to receive the finished end of the ball and the open end was gradually spun down and in until the ball was complete with but a 1/16-inch hole at the end. This hole was plugged and the hollow ball was done.

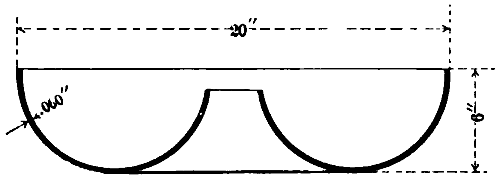


Fig. 13. An Interesting Example of Metal Spinning

As another example of metal spinning, assume the shape shown in Fig. 13. The shell is to be 20 inches in diameter, 6 inches deep, and 0.060 inch thick. The metal to be used is zinc. This is an interesting metal spinning job, and not a particularly difficult one. The shell can be best spun with the aid of two spinning forms, such as are illustrated in Figs. 14 and 15. These forms should be made of kiln-dried maple if there are comparatively few shells to be spun. If there are many, the forms should be made of cast iron. Fig. 14 shows the first form to be used, which conforms to the outside of the shell as far as the centers of the spherical ring. Beyond these points, the form is straight. The blank to be spun is placed as indicated by the dotted lines, and follower No. 1 is used to hold the work against the form. The chief trouble will be met in properly starting the shell, because of the small follower that must be employed. However, follower No. 2 may be substituted after working the metal back against the form a few inches, and as this gives a better grip on the shell, there will be no further danger of slipping. After spinning the zinc shell to the shape of the first form (Fig. 14) it will probably have to be annealed, but this can only be determined by trial. In annealing zinc, the flame should not be allowed to touch the metal. The half completed shell is then put on form No. 2 shown in Fig. 15. It is an easy matter to spin the metal round to complete the arc. The dotted line shows the position of the shell before starting the last part of the spinning. Of course, it will be understood that the shell must be trimmed several times during the spinning, and if the trimming is frequently done, a well-shaped shell should result. For spinning on form No. 2, follower No. 3 must be used. Either beeswax or soap should be frequently rubbed over the work while spinning. If it is necessary to cut out the center, it can be done before removing the shell from the last form by simply removing the follower and using a diamond point tool, or in large-product work the swivel cutter will work well. The shell will cling to the form without the follower. The spinning speed should be from 800 to 1,000 R. P. M.

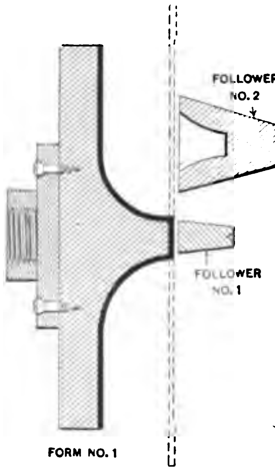


Fig. 14

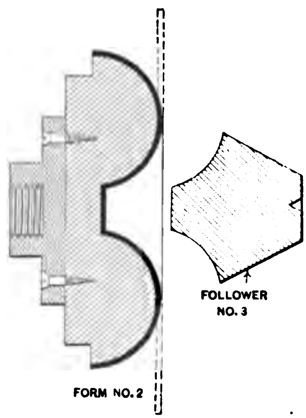


Fig. 15

While the operation of spinning is a comparatively simple one to describe, it is not easily learned, and to-day good all-around spinners are hard to find. The limits of accuracy are not as closely defined as in straight machine work, but there are times when good fits are absolutely necessary, as in cases where two shells must slip snugly together. In this chapter we have taken up only the plain every-day kind of spinning, and were we to follow its work in the gold and silversmith's trade, we would see it evolve into a fine art. In order to insure really good work coming from the spinning lathe, there is a wide range of knowledge that the spinner must have. That knowledge may be brought together and summed up by a single word—*judgment*.

CHAPTER II

TOOLS AND METHODS USED IN METAL SPINNING*

The principal object of this chapter is to describe in detail the various operations of spinning metal so that a tool-maker or machinist who has not access to a metal spinner, will be able to make his own tools, rig up an engine or speed lathe, and make the simple forms or models that are required in experimental work. To do this intelligently, it is necessary to follow in detail every step in metal spinning from the circular blank to annealing, pickling, dipping, burnishing, etc., and also to know how to make the simpler forms of spinning tools, what lubricants to use on the different kinds of metals, what material to make the spinning chuck of, and how far the metal can be worked before annealing.

Spinning metal into complicated and elaborate shapes, is an art fully as difficult as any craft, and the man is truly an artist that can make artistic and graceful outlines in metal, especially when only a few pieces are required and the cost will not allow of making special chucks to do the work on and with no outline chucks to govern his design, the forms being made by skill and manipulation of tools alone. Such skill is far superior to that of the Russian metal worker, who, instead of making a vase or ornament of one piece, cuts up several sections and soft solders them together, after covering them with crude "gingerbread" work to disguise his poor metal work.

The amateur can imitate the Russian work, but never the work of the skilled spinner. There are several grades of spinners, most of them never attaining the skill of the model-maker or the facility for handling the different metals. A man that has had several years of experience spinning brass or copper would not be able to spin britannia or white metal without stretching it to a very uneven thickness. As brass or copper is harder than the other metals mentioned, they resist the tool more and require more pressure in forming, and if the operator used the same pressure on the softer metals, he would stretch or distort them, so that they would be perhaps one-quarter of the original thickness at angles and corners where the strain in spinning would be greatest, which would ruin the articles. The best test for skill in ordinary spinning, is to take a long difficult shape, after being finished, and saw it in two lengthwise, and if the variation in thickness is less than 25 per cent of the original gage, it is good practice. Some spinners can keep within 10 per cent of the gage on ordinary work, but they are scarce.

The spinning trade in this country is mostly followed by foreigners, Germans and Swedes being the best. The American that has intelli-

* MACHINERY, March and April, 1910.

gence and skill enough to be a first class spinner, will generally look around for something easier about the time that he has the trade acquired. It is an occupation that cannot be followed up in old age, as it is too strenuous, the operator being on his feet constantly, and having to use his head as well as his muscles.

General Remarks on Metal Spinning Chucks

For common plain shapes, a patternmaker's faceplate, with a tapered center screw, is sufficient for holding the wood chuck. The hole in the wood should be the same taper as the screw, thus giving an even grip on the thread. If a straight hole only is used, and it is not reamed out before screwing to the plate, it will only have a bearing on one or two threads, and if the chuck is taken off and replaced on the faceplate, it will not run true. Care should also be taken to face off the end of the chuck flat, or to slightly recess it, so that it will screw up evenly against the faceplate, as a high center will cause it to rock and run out of true.

In large chucks (over five inches) it is best to have three or four wood screws, besides the center screw. The holes for these can be spaced off accurately on a circle in the iron faceplate, and drilled and countersunk. It is best to have twice as many holes as screws; that is, if four screws are used there should be eight holes, so that if the chuck has to be replaced at any time and the wood has shrunk, it can be turned one eighth of a revolution further than the original chucking.

Where a chuck has to be used several times, it is better practice to cut a thread in the wood and screw the chuck directly to the spindle of a lathe, not using the faceplate. This thread can be chased with a regular chasing tool, where the operator has the skill, or if not, the wood can be bored out and a special wood tap used. Such a tap has no flutes and it is bored hollow, there being a wall about 3/16 inch thick. One tooth does all the cutting, that is the one at the end of the thread. The chips go into the hollow part of the tap. The end of the tap for about ¼ inch should have the same diameter as the hole before threading to act as guide for the cutting tooth.

It is essential that a chuck should run very true and be balanced perfectly, as the high speed at which it runs will cause it to vibrate and run out of true, causing the finished metal to show chatter marks. The best wood for chucks is hard maple, and it should be selected for its even grain and absence of checks and cracks. It is best to paint the ends with paraffine or red lead, or to immerse the chucks in some vegetable oil after turning. Cottonseed oil is very good for this purpose, but care should be taken not to soak the chucks too long.

For a man not skilled in spinning, it is better to use metal chucks than wood, for if there are many shells of a kind, the operator is liable to bear too hard on the tool, thus compressing the chuck and making the last shells smaller than the first. Corners and angles not well supported might also be knocked off. The writer prefers cold rolled steel for chucks up to 6 inches in diameter and cast iron for the larger ones, but where good steel castings can be obtained, a good chuck can be made by turning roughly to shape a wood pattern, allowing enough for shrinkage and finishing, and hollowing out the back to lighten it. When the chuck is finished all over in the lathe, it should balance much better than a cast iron one, as there are not the chances of having blow holes in the iron, thus throwing the chuck out of balance.

Annealing

The distance that metal can be drawn without annealing, can only be learned by experience. A flat blank rotated in the lathe, being soft, will offer little resistance and it can be gradually drawn down by a tool held under the chuck and against the blank. This tool is pushed from the center outward and forward at the same time, and every time it passes over the blank or disk the metal becomes harder by friction, and the change of formation and the resistance at the point of the tool greater. This can be felt as the tool is under the operator's arm. When the spring of the metal is such that the tool does not gain any, but only hardens the metal, the shell should be taken off and annealed. If the metal has been under a severe strain, it should be hammered on the horn of an anvil or any metal piece that will support the inside. The hammer should be a wood or rawhide mallet, but never metal, the object being to put dents or flutes in the metal to relieve the strain when heating for annealing; if this is not done the shell will crack.

After annealing the shell it should be pickled to clean the oxide or scale from the surface; otherwise the metal will be pitted. When the scale is crowded into the metal and when it will not finish smooth after spinning to shape, the metal can be finished by skimming or shaving the outer surface which cuts out all tool marks; it can then be finished with medium emery cloth or the shell can be bright dipped, and be run over with a burnishing tool before buffing. Burnishing can be done on the spinning chuck, but the speed should be higher than for spinning; this requires some skill for a good job, and it can be done only on metal chucks.

Annealing is best accomplished in a wood or gas oven, where a forge fire is used. The metal should never touch the coke or other fuel, but it should be held in the flame above the fire. Where only part annealing is required, the shell can be immersed in water, the part to be annealed being exposed above the water, and a blowpipe used on it. The remainder of the shell will then be hard. This way of annealing is sometimes necessary on a special shapes.

Brass should be heated to a cherry red, and held at that point for a few minutes, in a muffle furnace. If an open furnace is used, just bring the metal to a cherry red and then dip it in water; this method is better than when waiting for it to cool, the action being just the opposite to that on steel. Brass such as the common yellow brass is not suitable for spinning, there being but 55 per cent copper and 45 per cent zinc. There are two grades of brass suitable for spinning. These are known as "spinning and drawing," having 60 per cent copper and 40 per cent zinc, and "extra spinning and drawing" having 67 per cent copper and 33 per cent zinc. There is also a better grade known as "low brass" having from 75 to 80 per cent copper; it has the color of bronze and is only used on very deep and difficult spinning.

The scale, after annealing, should be pickled off in an acid bath (described further on in this chapter), and the part thoroughly washed in running water. Brass, German silver and the harder metals should be hammered before annealing; it is not necessary to hammer zinc, copper, aluminum, etc.

A pyrometer in an annealing furnace would be an advantage where quantities of the softer metals such as zinc, aluminum, etc., are being heated. Copper is annealed the same as brass and is also pickled. Zinc is coated with oil before being put in the oven, and when the oil

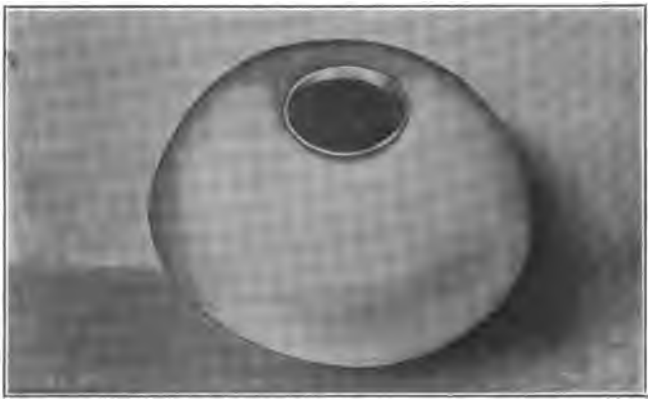


Fig. 16. Zinc Lamp Shade Spun in One Operation without Annealing

turns brown, which occurs when the temperature is about 350 degrees, the metal is ready to take out; it should then be plunged in water to shed the scale, but not pickled. The melting point of zinc is 780 degrees F. Aluminum can be annealed the same as zinc, as the melting point is 1,140 degrees F.

Steel should be annealed by heating to a cherry red and then allowing it to cool slowly; it should be scaled in a special pickle, thoroughly washed, and then put back in the fire long enough to evaporate every particle of acid that may have remained from the pickling operation. Any acid remaining on the steel will neutralize any lubricant that is applied when spinning. Annealing should be avoided wherever possible. Open hearth steel only should be used. It should be free from scale and preferably cold rolled. Bessemer steel is not suitable, except for very shallow spinnings. Tin plate made from open hearth steel can be spun about one-half as deep as its diameter where the shape is not too irregular. German silver is difficult to spin, especially when it contains over 15 per cent nickel; it has to be hammered before annealing, the same as brass, to avoid cracks.

Lubricants

Common yellow soap cut up in strips about ½ inch or ¾ inch square is a good lubricant for spinning most metals. It should be applied evenly to the disk or blank while it is revolving, by holding the soap in the hand and drawing it across the surface. Beeswax is the best for spinning steel, but it is expensive. Lard oil mixed with white lead is a fair substitute. Either mutton or beef tallow applied with a cloth swab is very good on most all metals; also vaseline and graphite mixed to a paste and applied the same as tallow.

Examples of Spinning Various Metals

The different metals are malleable, ductile and tenacious in the following order; white metal or britannia, aluminum, zinc, copper, low brass, high brass, German silver, steel, tin plate. White metal does not harden in spinning, but it requires special skill in handling,

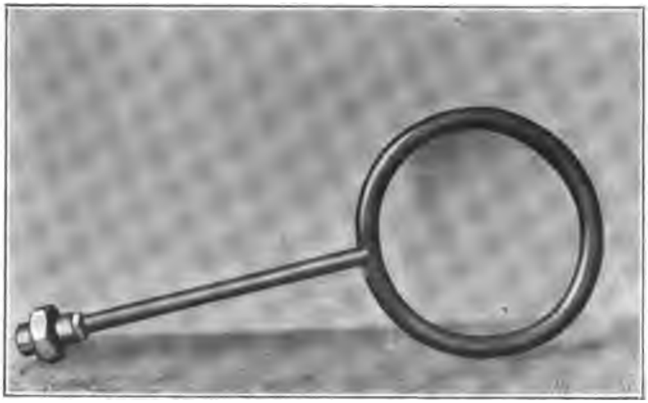
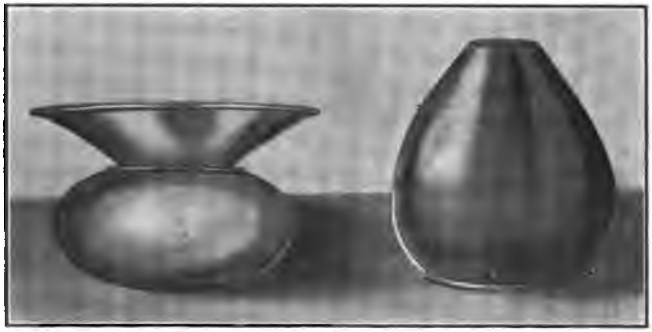


Fig. 17. Gas Burner for Heating Spinning Chuck

or the metal will be of very uneven gage. The best metal for an amateur to start on is copper, as it is both tenacious and ductile, and will stand much abuse in the fire and on the lathe. One of the peculiar properties of zinc is that it has a grain or texture, and when spinning, the two sides that went through the rolls lengthwise will be longer than the sides that have the cross grain, requiring the shell to be trimmed off quite a distance to even the edge.

To show the possibilities of working the different metals, and their relative spinning values, a number of articles made from different materials are illustrated herewith.

A zinc lamp shade is shown in Fig. 16 that is 14¼ inches in diameter and 4¾ inches deep. This shade was spun in one operation, without annealing, from a flat circular blank. All zinc should be warmed before spinning, either over a gas burner at the lathe or in hot soap water, and the chuck also should be heated, as otherwise the blank will soon chill, if spun on a cold metal chuck, as the chuck absorbs the heat long before the operation is finished. Of course this does not apply to wooden chucks. The chuck may be heated by using the burner shown in Fig. 17, which is located around the spindle of the lathe. The size of the burner should, of course, be in proportion to that of the chuck used. The burner illustrated is 8 inches in diameter. It has several small holes drilled for the gas on the side facing the chuck. The heat of the chuck is regulated by varying the supply of gas to the burner. The blank is heated before it is put on the



Figs. 18 and 19. Examples of Aluminum and Copper Spinning

chuck and the friction of the spinning tool helps to keep it warm until it comes in contact with the chuck. The metal retains its heat until the job is finished, and this sometimes saves an annealing operation.

In Fig 18 is shown an example of aluminum spinning. The article illustrated is a cuspidor having a top 7¾ inches in diameter, a neck with a 4-inch flare, a diameter at the top of 9½ inches, and a height

Fig. 20. German Silver Reflector **Fig. 21. Open Hearth Cold-rolled Steel Shell**

of 6½ inches. This shell was spun without annealing, which shows the extreme ductility of aluminum. The copper shell shown in Fig. 19, has a maximum diameter of 7 inches, and a depth of 8 inches; it was spun with four annealings. A German silver reflector, which is 10 inches in diameter at the largest end and 5 inches deep, is shown in Fig. 20. The spinning of such a reflector, when made from this material, is quite difficult. An open hearth cold-rolled steel shell with a maximum diameter of 3 inches and a depth of 4 inches is shown in Fig. 21. This shell was spun without annealing, which shows that the grade of steel used is well adapted for this work.

In Fig. 22 two finished brass shells are shown to the right, and also the number of operations required to change the form of the metal. The upper shell is 6 inches long and 3½ inches in diameter at the

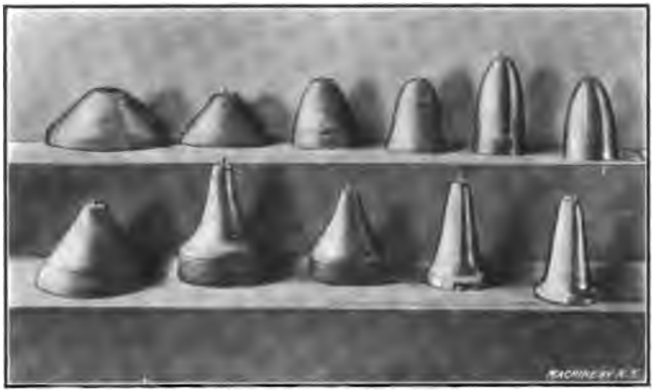


Fig. 22. Various Steps in Spinning the Two Brass Shells at the Right

large end, while the lower one is 7¼ inches long by 3¾ inches in diameter. It was necessary to anneal these shells between each operation, the upper shell being annealed four times and the lower one three times. These pieces were made in quantities sufficient to warrant the making of chucks for each operation, which enabled them to be spun with less skill than would be required if a finishing chuck

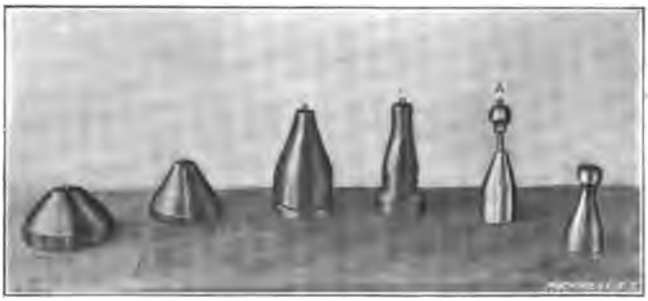


Fig. 23. Another Brass Spinning Operation; the Chuck used is shown at A

only were made. When a single finishing chuck is used, the various operations in spinning a shell of this kind would be left to the judgment of the spinner, who would decide the limit of the stretch of metal between the operations before annealing.

A brass shell that is made in five operations and with four annealings is shown in Fig. 23. The finishing chuck used is a split or key chuck on which it is necessary to cut out the end of the shell in order to withdraw the key after the shell is spun. This shell, which is shown finished to the right, is 5½ inches long. It is spun smooth on a machine steel chuck, and is not skimmed, but gone over with a planishing tool at the last operation. The two pieces shown in Fig. 22 were also finished in this way.

Fig. 24. An Example of "Air Spinning" and the Chucks used

Fig. 24 shows a brass shell, which is a good example of "air spinning," so called because the finishing or second operation on part of the shape is done in the air, thus avoiding the use of a sectional or split chuck. The shell shown is about 5½ inches in diameter. The first or breaking-down chuck is shown at *A*. The neck or small part

Fig. 25. Miscellaneous Collection of Spinning Chucks

of the piece, and also a portion of the spherical surface, is formed by the spinning tool without any support from the chuck. After the shell is spun or broken down on chuck *A*, it is annealed and pickled. It is then put back on chuck *A* and planished or hardened on the part that is to retain its present shape. The work is then placed on the chuck *B* and the soft part is manipulated by the tool until it conforms to the shape shown to the right. While this soft part of the metal is being formed, the part which was previously hardened retains its shape.

Various Types of Metal-spinning Chucks and their Construction

A miscellaneous collection of spinning chucks is shown in Fig. 25. As will be seen, the larger ones are machined out in the back to lighten them, and also to give them an even balance. The larger of those illustrated measure about 9½ inches in diameter, and they are made of cast iron, while the smaller chucks shown in this view are of machine steel. The chuck marked *A* is a key chuck. Another collection of spinning chucks of various shapes is shown in Fig. 26.

Fig. 26. Another Group of Spinning Chucks. Those in the Upper Row are of the Split or Key Type

Those in the upper row are all key or split chucks, and the keys are shown withdrawn from the sockets. All these chucks, up to 6 inches in diameter, are made of machine steel; those seen in the lower row are shapes which are comparatively easy to spin.

A collection of hard maple chucks is shown in Fig. 27, some of which represent shapes that are difficult to spin. The chuck *A* is 15 inches long, and the maximum diameter of *B* is 12½ inches. These figures will serve to give an idea of the proportions of the other chucks. All of the chucks shown have threads cut in them and they are screwed directly to the spindle of the lathe, the faceplate being dispensed with. Some of the larger wooden chucks used measure approximately 5 feet in diameter. A chuck of this size is built up of sections which are glued together.

A number of bronze sectional split chucks are shown in Fig. 28. When spinning over a sectional chuck, it is first necessary to break down the shell as far as is practicable on a solid chuck. Care should be taken, however, to leave sufficient clearance so that the work may be withdrawn. The shell is then annealed, after which it is put on the sectional chuck and the under cut or small end is spun down to the chuck surface. When the entire surface of the shell is spun down to a bearing, the shell is planished or skimmed to a smooth surface;

Fig. 27. Various Forms of Spinning Chucks made from Hard Maple

Fig. 28. A Group of Bronze Sectional Chucks

the open edge is also trimmed even and the shell is polished with emery cloth.

A large bronze chuck of seven sections, one of which is a key section, is shown at *A*. The largest diameter of this chuck is 10 inches. It has a cast iron center hub and a steel cap at the top for holding the sections in place. This cap, when in place in the retaining groove shown, is flush with the top of the chuck. Another large chuck having five sections and one key section is shown at *B*. The retaining cap in this case is of a different form. The lower parts of the sections of all these chucks fit in a groove at the bottom of the hub. A chuck of five sections that is without a binding cap, is shown at *C*. This is not a good design as the hub or center is too straight, and all of the grip or drive is from the bottom groove, which is not sufficient. The shape shown at *D* is more difficult to spin than any of the others, as it is smaller at the opening in proportion to its size. This chuck also requires more sections in order that it may be withdrawn from the shell after the latter is spun. The chuck *E* is intended for a small shell that is also difficult to spin. The drive pins which prevent the segments of the chuck *E* from turning may be seen projecting from its base. The centering pins at the outer end of chucks *D* and *E* and

Fig. 29. Sectional Chucks made from Wood

the binding caps may also be seen. The chuck *A*, because of its size, is hollowed out to reduce the weight. All of these chucks were made for hard service, and they have been used in spinning thousands of shells.

Another group of sectional chucks is shown in Fig. 29. They are mostly made from hard maple. The sections of chuck *A* are planed and fitted together and thin pieces of paper are glued to these sections before they are glued collectively for turning. By using the paper between the joints, the sections may be easily separated after they are turned to the proper size and form. If the different sections were glued without paper between them, the joint formed would be so good that the separation of the sections could not be controlled, and parts from opposite sections would be torn away. The use of the paper, however, between the glued joints, controls the separation of the sections. The chuck shown at *D* is also made with the paper between the sections. Chucks *B* and *E* are turned from the solid, care being taken to have the grain of the wood lengthwise. After they are turned to the required form, they are split into sections with a sharp chisel. Before doing this, the key section should first be laid out. There should be as few sections as possible, the number being just sufficient to enable the withdrawing of the chuck from the shell after the latter is spun to shape. This method of making a chuck, while quicker than the other, is not good practice, except for small work.

A lignum vitæ chuck is shown at *A* in Fig. 30; this was made with paper between the sections. The key-section is shown on top. This wood, while being more durable than hard maple, costs sixteen cents a pound in the rough and, counting the waste material, is not any cheaper than bronze, and is less durable. The hard maple chucks *B* and *C* were turned from the solid, after which the sections were split. The segments shown in the center of the illustration did not split evenly, owing to a winding or twisting grain.

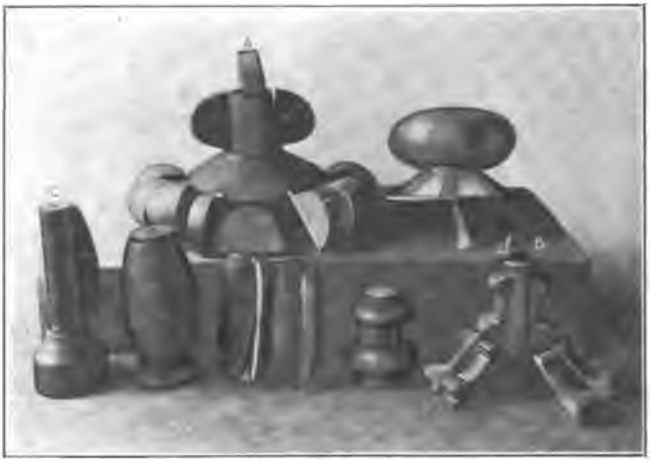


Fig. 30. Other Examples of Wooden Sectional Chucks

The construction of a sectional spinning chuck is shown in Fig. 31. This illustration also shows the proper proportion for the central hub and its taper. This hub should never be straight, but should have from 5 to 7½ degrees taper on the central part. There should also be a taper of 1½ degree on the other binding surfaces as indicated. These parts are made tapering so that the shell can be released from the lathe after spinning, without hammering or driving; when straight surfaces are used the work has to be pried off, and it is also harder to set up the sections for the next shell. Another disadvantage is that with straight fittings the wear cannot be taken up. An end cap or binder should be used wherever possible as it steadies the chuck. A drive pin should also be used and the hole for it drilled in the largest section; this is important, as it gives the sections a more positive drive. If they slip they will soon wear themselves loose and leave openings at the joints.

The plan view shows the method of laying out the various sections. The key should be laid out first. One key is enough for the particular form of chuck illustrated, but it is often necessary to use two key sections when the shell opening is small.

When a sectional chuck is to be made, it is important to decide first on the size of the central hub *A*, the number of sections *C*, and also the design of the cap or binder *B*. This cap must not exceed in size the opening in the finished shell, as it would be impossible to remove it after the chuck sections were taken out. After the size of the hub *A* has been decided upon, a wooden form should be turned that is a duplicate of *A*, except that a spherical surface *E* should be added. This spherical part should be slightly smaller than the inner diameter of the bronze sections in order to allow for machining them. In turning this wooden pattern on which the plaster patterns for the sections are to be formed, the shoulder *D* should be omitted, as a removable metal ring will take its place.

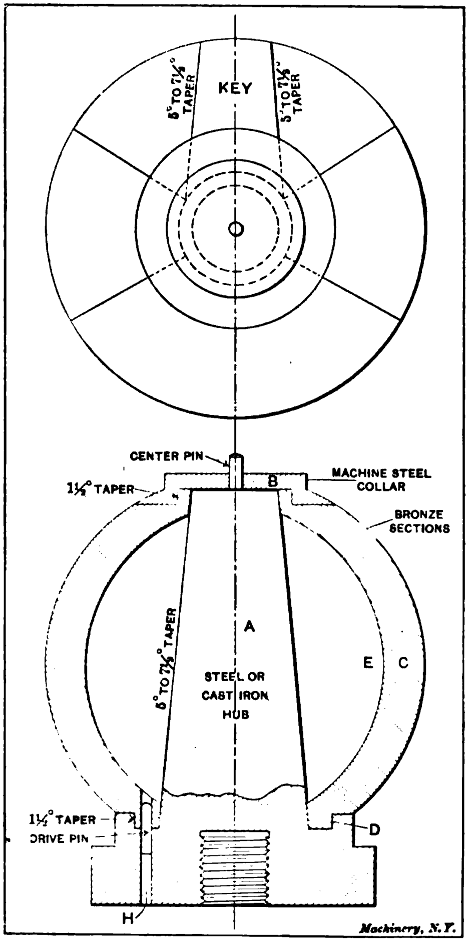


Fig. 31. Elevation and Plan showing Construction of Sectional Chuck

When the wooden hub is ready, two metal partitions or templets of the same outline as the chuck, though about one-half inch larger than its total diameter, for shrinkage and finishing, are fastened to the hub in the correct position for making a plaster pattern for the key section. These patterns should have extension ends so that the sections when cast may be held by them while they are being turned. The templets should be banked around with a wad of clay, and they should also be coated on the inside with sperm oil to keep the plaster from sticking. There should be two brads driven in the hub for each section of plaster to hold the sections in place while they are being turned. After the plaster for the key section has hardened, the templets should be located one on each side of the key section, so that the two adjacent sections may be made. In this way all the sections are finished. After about forty-eight hours the plaster will be hard enough to turn in the lathe with a hand tool. The form should be roughly outlined and plenty of stock left for shrinkage, as bronze shrinks considerably. Before taking the sections off the wooden frame, the metal band *D* should be removed to allow the sections to be separated. This should not be done, however, until they are numbered, so that they can be again placed in their proper positions. After the sections are cast, they should be surfaced on a disk grinder, or finished with a file, care being taken to remove as little metal as possible. Each section is next tinned on both contact faces, and then

Fig. 32. A Modern Spinning Lathe

all are assembled and sweated or soldered together by a blow-pipe. It is sometimes necessary to put a couple of strong metal bands around the sections to hold them firmly in place when soldering and also to support them during the turning operation.

The central hub *A* should be machined first; then the assembled outside shell should be machined to fit the hub *A*, both on the taper part and at the point *D*. While the segments are being bored and faced, they are held by the extension ends (not shown) which were provided for this purpose. This outer shell should also be machined all over the inside so that it will be in balance. It is then taken out of the chuck and a hole is drilled in the largest section for drive pin *H*. The hub *A* is then caught in the lathe chuck with the assembled sections on it, and a seat is turned for the cap *B*. After this is done the binder bands can be removed, but not before. The chuck can be finished with a hand tool and file after the roughing cut is taken. After the sections are removed from the hub and numbered at the bottom or inner ends, they can be separated by heating them. If the joints are properly fitted there will be only a thin film of solder, which can be wiped off when hot.

A twenty-four-inch metal spinning lathe that is rigged up in a modern way, is shown in Fig. 32. The hand wheel of the tailstock has been discarded for the lever *A*, which is more rapid and can be manipulated without stopping the lathe. This lathe has a roller bearing for the center *B* which is a practical improvement over types previously used. The pin *C*, which is used in the rest as a fulcrum for the spinning tools, is also an improvement, being larger than those ordinarily used. It is ¾ inch in diameter, 6 inches long, and it has

Fig. 33. View showing how the Tool is held when Spinning

a reduced end for the holes in the rest, ⅝ inch in diameter by 1 inch long. This pin is large enough so that the spinner can conveniently hold it with his left hand when necessary, and it can also be rapidly changed to different holes. The pins ordinarily used, because of their small size, do not have these advantages. The speed of a spinning lathe having a five-step cone should be about 2,250 to 2,300 revolutions per minute with the belt on the smallest step, and from 600 to 700 revolutions per minute with the belt on the largest step. The fastest speed given is suitable for all work under 5 inches in diameter, and the slowest for work within the capacity of the lathe. On large shells it is sometimes necessary to change from one speed to another as the work progresses. Figs. 33 and 34 show the spinner at work, and illustrate how the tool should be held, and also the proper position of the left hand.

Construction of the Tailstock and Back-center

Fig. 35 shows a spinning-lathe tailstock, which has been changed from the hand-wheel-and-screw type to one having a lever and a roller bearing. The spindle *A* which is withdrawn from the lever and turned one-quarter of a revolution to give a better view of the rollers, is made from 1¾-inch cold rolled steel. The rollers against which the center bears do not project beyond the spindle, so that the latter can be withdrawn through the tailstock. This eliminates the excessive overhang caused by ball bearings and other centers. When the center projects too far, the tailstock cannot be set close to the work, owing to the necessity of withdrawing the center when removing the

Fig. 34. Another View showing the Position of the Spinner and the Way the Tool is held when forming the Metal

spun part. The application of this principle to a spinning lathe is original and the type of center illustrated was used only after all other kinds had failed, including all the types of ball bearings and revolving pins. The best forms of ball bearing centers do not last over a year, if in constant use, and they will not always revolve on small work. Two other spindles are shown in this engraving, which were taken from other lathes in order to show different views of the parts. The cylindrical pieces *B* are the hardened friction rollers which belong in the slot of the spindle *F*, and *C* is the hardened pin upon which they revolve. The hardened center *D* has a threaded end on which the back-centers *E* of different lengths and shapes are screwed. The friction rollers should always be in a vertical position, and care should be taken to have them exactly central with the spindle.

and also gives the principal dimensions of a roller bearing for a 1¾-inch spindle. *A* is a hardened steel bushing, which is driven into the machine steel spindle. The parts *B* are the hardened steel rollers which travel in opposite directions. These rollers have a small amount of friction, and this is distributed over a large area. A spindle revolv-

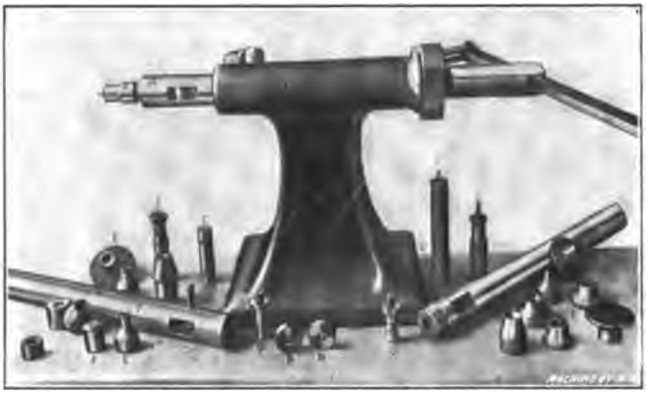


Fig. 35. Detailed View of a Spinning-lathe Tailstock

ing at 2,300 revolutions per minute will not cause these rollers to rotate very rapidly, while a ball bearing with balls traveling in a channel 1½ inch or 2 inches in diameter would be traveling at the same speed as the driving spindle. They also wear out rapidly as the end strain is very great, it being necessary to force the center against

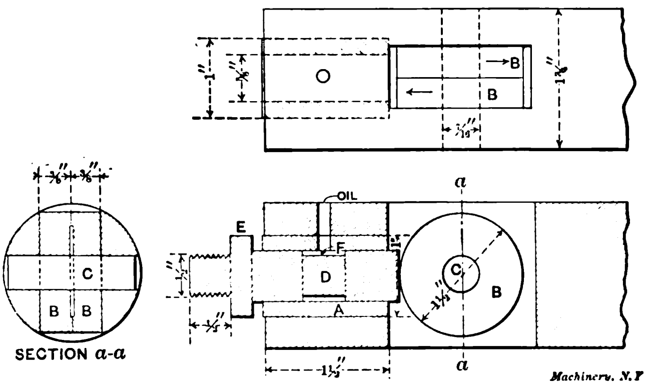


Fig. 36. Sectional View showing the Back-center and its Double Roller Bearing

the metal with considerable pressure to keep it from slipping. *C* is the hardened pin upon which the rollers revolve, and *D* is the hardened spindle on which the various back-centers are screwed. The collar *E* should either be flattened for a wrench, or a 5/16-inch hole, in which a wire can be inserted, should be drilled through the spindle, so that it can be kept from rotating when screwing on the back-centers. Some spinners prefer the spindle loose, so that it can be withdrawn when changing the centers, while others prefer one with considerable lateral motion, but not enough to permit of withdrawal. By inserting a screw-point in the recess *F*, the center has considerable lateral motion, but not enough to allow it to be withdrawn. This recess is useful in that it helps to distribute the oil. All parts should be hardened and drawn to a light straw color; they should also be ground or lapped to a true fit after hardening. Back-centers of this construction

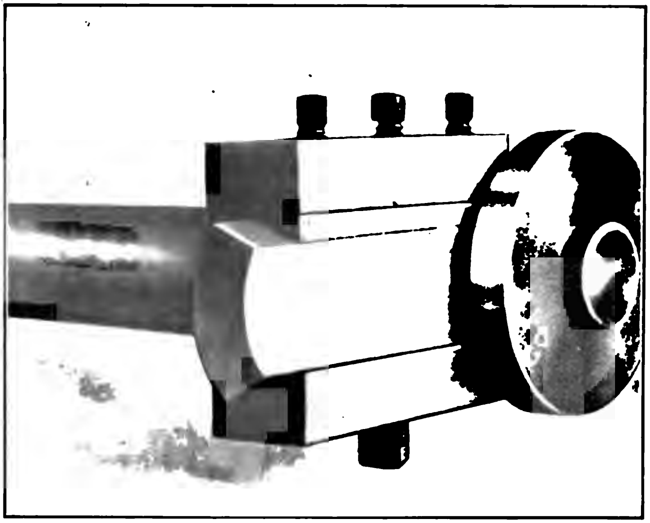


Fig. 37. Attachment used for Rolling Sharp Turns and Beads

have been in use for over three years in one establishment, and it has not been necessary to replace a single part.

Tools Used in Metal Spinning

Fig. 37 shows an attachment which is used to roll any bead or form. This tool, when in use, is inserted in the tailstock spindle in place of the regular center. It is adjustable for any diameter. The roll illustrated is for making a sharp turn, but rounds and other forms are used. The shell being spun by this tool should be held on a hollow chuck. The roll is set at a point where the metal is to be turned over, and by its use the curve may be governed and made uniform with less skill than when the work is done by "air spinning." In addition, the spinning may be done in less time. This attachment, for some shapes, makes the use of sectional chucks unnecessary.

Fig. 38 shows several spinning tools, the heads of which were turned in the lathe instead of being forged. This method of making spinning tools is believed to be original. The spinners prefer them to the tools which are forged in one piece, because the heads which are screwed to the shanks are made of the best quality of steel, such as the high-speed or self-hardening steel. The shapes are also better and the surfaces more true. The heads of these tools are all threaded with standard ¼-inch, ⅜-inch and ½-inch pipe taps, according to the size. Obviously, a spinner can have as many different shaped heads as may be required of each of the sizes given, and only one handle.

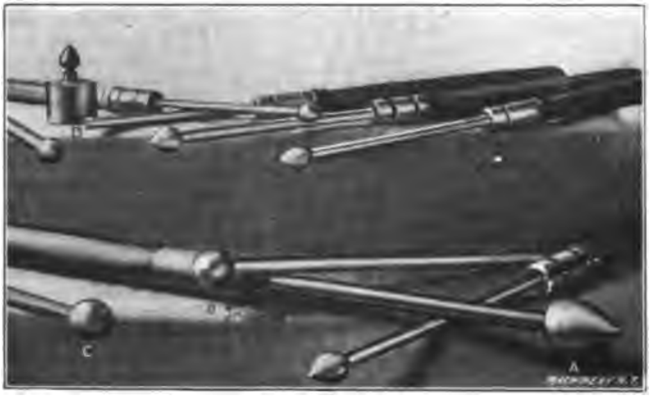


Fig. 38. Metal Spinning Tools with High-speed Steel Removable Heads

The tapering threads in these heads insure that they will always screw on the shanks tightly no matter how often they may be replaced. The ¼-inch size takes a ½-inch cold rolled holder; the ⅜-inch, a ⅝-inch holder, and the ½-inch, a ¾-inch holder. These will be found large enough for the heaviest work. The egg-shaped tool *A* is a good

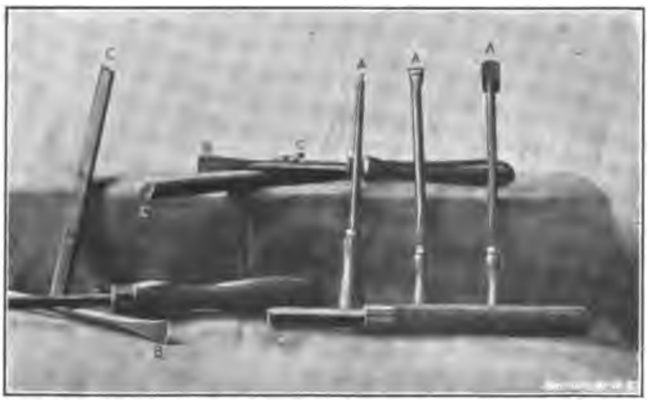


Fig. 39. Tools used for Trimming and Skimming Spun Work

form for roughing or breaking down, as it has plenty of clearance on the heel, and a blunt point that will not tear the metal. This tool is shown in four sizes. The ball or spherical tool *B* is a good one to to use on curves and large sweeps. The tool *C* is elliptic, and is slightly different from *A*, as it has a blunter point. One of these heads is shown at *D* screwed onto a reducer by which it is held in the lathe chuck while being turned. These heads or points can also be turned while on the handle by using a steady rest.

Fig. 40. A Group of Spinning Tools of Various Shapes

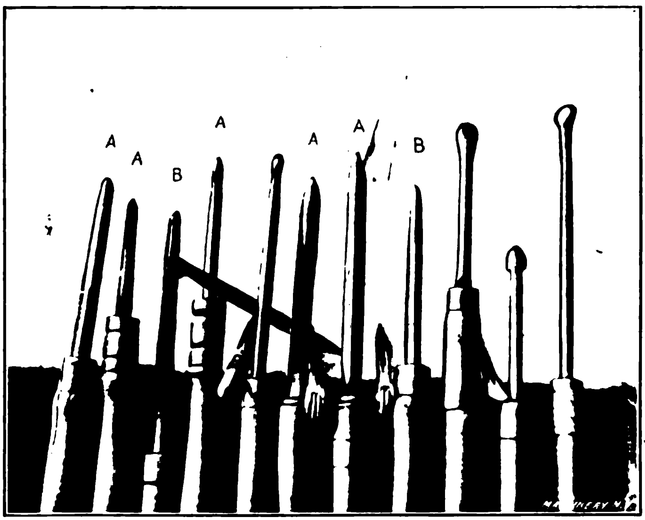


Fig. 41. Another Group of Spinning Tools

A group of trimmers, skimmers and edgers is shown in Fig. 39. Three skimmers of the built-up type are illustrated, the shanks being of machine steel and the blades being riveted to the holders. These blades are made of either high-speed or regular steel. Skimmers which are forged in the regular way from one piece of steel, are shown at *B*. A number of edgers *C*, which are made of high-speed or self-hardening steel, are also illustrated. These tools are used without handles until they are worn down short, after which tangs are forged on their ends and they are used in handles. Edgers are utilized on all kinds of work for trimming the ends of the shells. The skimmer is seldom used on metal chucks, but mostly in connection with wooden chucks, where the metal cannot be smoothed down with a planisher. The skimmer is run over the metal lightly, taking a thin shaving and smoothing the uneven surfaces. It requires con-

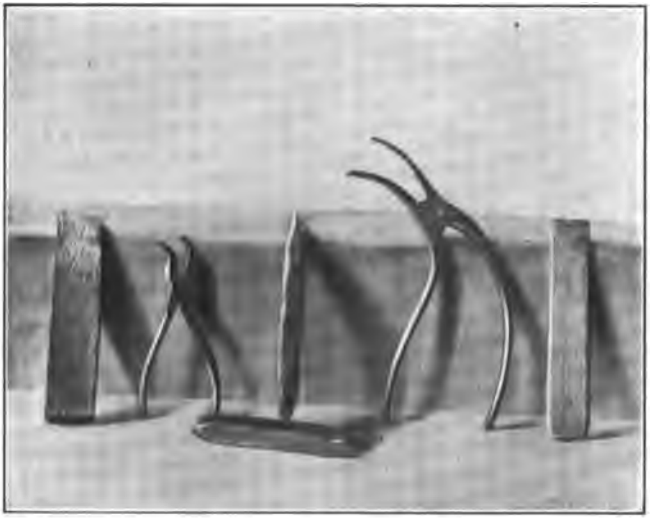


Fig. 42. Spinners' Pliers which are used for turning the Edge of the Metal when making a Large Bend

siderable skill to use this tool without wasting the metal. The surface of the work is finished with emery cloth after skimming.

Figs. 40 and 41 show a number of spinning tools of various shapes. The letters *A* indicate the breaking-down or round-nosed tools of different sizes. This type of tool, which is finished smooth and has a blunt point, is used for forming corners and sharp angles, and it is the tool most commonly used by spinners. The planishers and burnishers *B* are used on all convex surfaces and for finishing on metal chucks where there is to be no skimming done. The tools *C* are known as hook or poker tools, and they are used to turn up beads or curves from the inside of the shell. The holders having rollers are used for turning over beads, the metal first being trimmed and turned to a vertical position. The other shapes shown are irregular tools for special work and they are not in daily use.

Two pairs of spinners' pliers for turning over the edge of the metal when making large curves are shown in Fig. 42. The wedge-shaped pieces shown in this illustration are used when breaking down or roughing shells to give a bearing to the metal in order to prevent it from wrinkling or buckling when changing its formation. These pieces are made of hard wood with the exception of the one to the right, which is of steel. When one of these pieces is in use it is held in the left hand at a point directly opposite the spinning tool, the metal being between the two. Wood is preferable in most cases, as it does not harden the metal blank.

The tools shown in Fig. 43 are used in spinning steel. The round tools are of drawn brass, and they can be used where the steel tools

Fig. 43. Some Spinning Tools used in Working Steel

cannot, for while a steel tool is perfection on brass, a brass tool is the only thing on steel. It wears out, however, much more rapidly than one of steel. The rolls shown in the center are used for breaking down steel shells. These tools are hardened and have hardened roller bearings. The handles are made of one-inch iron pipe, which is filled with lead to give weight and strength.

Hard wood tools that are used for breaking down large thin copper blanks ranging from 2 to 5 feet in diameter are shown in Fig. 44. These tools are also used where the surface that the tool will cover without hardening the metal is important. Blanks which are broken down with these tools are finished with the regular types.

The handles of spinning tools vary in diameter from 1¼ to 1¾ inch, and in length from 16 inches to 20 inches. The tools should project from the handles from 9 to 18 inches, and the total length of the tool and handle should average from 30 to 34 inches.

A group of wood working tools is shown in Fig. 45. These tools are of the type commonly used by spinners for turning the various shapes of wooden spinning chucks. As the tools illustrated are the kind regularly used for wood turning by patternmakers and other wood-workers generally, they will need no description.

Preparation of the Metal

Brass, copper, and German silver should be pickled after annealing in order to get the scale or oxide from the surface. There are furnaces

Fig. 44. Wooden Tools which are used on Large Thin Copper Blanks

that anneal without scaling by excluding the air when heating, but they are not in general use. A pickling bath may be made by using one part of oil of vitriol (sulphuric acid) and five parts of water. The shells can be put in hot, or the bath can be heated by a coil of lead or copper pipe running through it. Steam in no case should enter the bath, as the iron in the feed pipe will spoil the pickle. Any basket or box that may be used to hold the shells in the pickle should not contain any iron. If a box is used it should be held together with copper nails. The pickle can be used cold, but it will take a little longer time to remove the scale. As soon as the scale is free, which will be in about half an hour, the shells should be removed or washed thoroughly in running water. The shells should be allowed to dry before the next operation, which is that of spinning. A lead-lined wooden tank or an earthen jar may be used for holding the pickle. The pickle which is used for steel should be about half as strong as that employed for brass. After the work is in this pickle, the latter should be brought to the boiling point, after which the pieces should

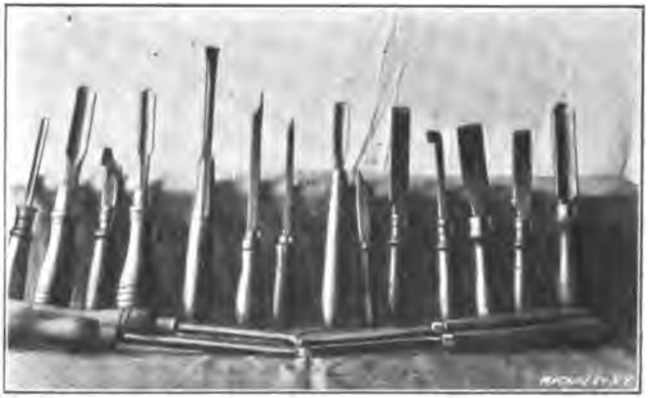


Fig. 45. Wood-turning Tools which are used in turning Spinning Chucks

be taken out and washed. They are then replaced in the fire for a short time to evaporate any acid that may remain after washing.

Finished brass articles may be given different shades by dipping them in a solution consisting of one part aqua fortis (nitric acid) and two parts oil of vitriol. This solution should stand seven or eight hours to cool after mixing, and be kept in a crock immersed in a water bath.

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NUMBER 58

HELICAL AND ELLIPTIC SPRINGS

CONTENTS

CHAPTER I

PRINCIPLES OF SPRING CALCULATIONS*

Although made in a great variety of shapes, the working and efficiency of any spring can be readily understood and investigated if a few fundamental principles determining the resistance to bending or twisting, and the deflection of elastic bodies are understood. Springs are generally made of steel or brass, and when under tension are either bent or twisted. Let us, therefore, first consider a flat piece of tempered tool steel of even thickness and width, firmly clamped at one end, and with a weight suspended at the free end, as shown in Fig. 1. The free length is a little over 12 inches, the width 1½ inch, the thickness 1/16 inch, and the suspended weight 10 pounds. The deflection at the free end will be about 4½ inches, and the curvature will be as

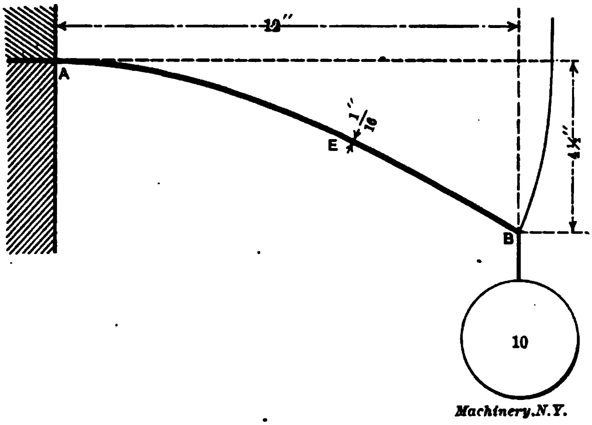


Fig. 1

shown in the figure. If made of high carbon crucible steel, properly tempered, 10 pounds is a safe load on this spring, but it may carry considerably more before the elastic limit is reached. These facts were obtained by calculation, by methods explained later.

It is obvious that the curvature of any part of the spring depends on the leverage or arm of bending, or rather on the "moment of bending," which is the weight multiplied by its arm of leverage. At *A* the arm is 12 inches, at *E* it is about 6 inches; the moment of bending at *E*, therefore, is only half the moment of bending at *A*; at *B* there is no arm, and therefore no bending. Consider a small part—an element—of the curve at *A*. This element will be bent to the arc of a circle, and the radius of this arc is called the radius of curvature at *A*; any ele

* MACHINERY, May, July and August, 1898.

ment nearer B will have a larger radius of curvature. At A the radius of curvature is about 11 inches, at E it is about 22 inches, and at B it is infinite.

Carrying Capacity of a Flat Spring

Considering any spring, we must first know whether it is strong enough to carry the load or will stand the work for which it is intended. The mistake is often made of using, or attempting to use, a spring which has not sufficient strength or endurance for the work it has to do, and which, consequently, gives out after being in use a short time. The spring shown in Fig. 1 being of even thickness through its entire length, is evidently weakest where it is bent most, that is at A. The moment of bending at this point is $10 \times 12 = 120$, and the bending brings forth a moment of resistance or internal resisting moment in the steel equal in magnitude to the bending moment of the extrane-

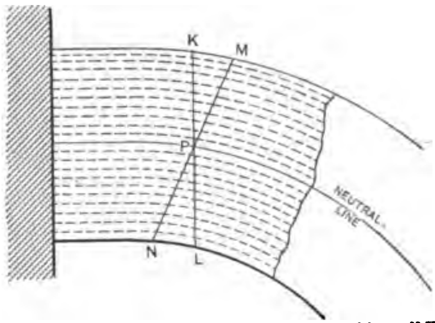


Fig. 2

ous force, and the question to be decided is: What is the greatest fiber stress for this moment? Is it within the safe limit?

Let Fig. 2 represent a small part of the spring greatly magnified and the bending greatly exaggerated, and let the dotted lines represent imaginary fibers or thin parallel strings or strips of steel. The upper half of these will be stretched, and the lower half will be compressed; but right in the center line of the thickness of the spring the fibers will neither be stretched nor compressed, and this line is therefore called the neutral line. We may consider any point in this line as a pivot for a double-armed lever to which the fibers are attached. Let P be the pivot and let KL represent the position of the lever before bending, and suppose that this lever, by the bending of the spring, is thrown in the position MN, and then KM represents the amount of stretching of the extreme upper fibers, and NL represents the compression of the extreme lower fibers, or rather of a small part of these. Steel is not fibrous, but we may call a string of molecules a fiber. All the fibers will be stretched or compressed in proportion to their distance from the neutral line, and they will therefore exert a certain resistance on the imaginary lever MN, and this collective resistance will exactly counterbalance the weight on the end of the spring acting on the lever *AB*, Fig. 1. The outside fibers will be stretched or compressed most, and if they are stretched beyond a certain limit, the spring will break or receive a permanent set. If we double the thickness of the spring, it can evidently only bend half as much before the limit of fiber stress is reached; but the average distance of the fibers from pivot *P* will, in this case, be doubled—that is, the leverage of resistance will be doubled and the number of fibers will also be doubled. The total resistance to bending at the same limit of fiber stress will therefore be twice doubled; that is, it will be $2 \times 2 = 2^2 = 4$ times as great, or, in other words, doubling the thickness of a spring quadruples its carrying capacity. If we had increased the thickness by one-half only, we should have $(1\frac{1}{2})^2 = 2\frac{1}{4}$ times greater strength. In general, let T and U represent the respective thicknesses of two similar springs of same width and length; then

$$\frac{\text{carrying capacity of spring } T}{\text{carrying capacity of spring } U} = \frac{T^2}{U^2}$$

or, the carrying capacities of otherwise similar springs are as the square of their respective thicknesses. This rule applies to bending only, and not to springs which are twisted. The strength of a flat spring is in simple proportion to its width, which is obvious without demonstration, and therefore, if thickness $= t$ and width $= b$, the moment of resistance for a given fiber stress $= cbt^2$, where c is a constant factor dependent on the allowable fiber stress. This factor can be found experimentally. Suppose, for instance, it is known that 10 pounds is the greatest load which the spring shown in Fig. 1 ought to carry, then in this case, the moment of bending $= 10 \times 12 = 120$, and the moment of resistance $= c \times 1\frac{1}{2} \times (1/16)^2 = \frac{3}{512} c$. Equating these two quantities we have $120 = \frac{3}{512} c$, or $c = 20{,}480$. Now c being a known constant factor, we can always find the moment of resistance from the formula cbt^2, and this product divided by the leverage of the load gives the carrying capacity or admissible load. For instance, let the length or leverage be 10 inches, the width 1 inch, and the thickness $\frac{1}{8}$ inch, then,

$$\frac{20{,}480 \times 1 \times (\frac{1}{8})^2}{10} = \frac{20{,}480}{640} = 32 \text{ pounds,}$$

which is the safe load on the free end of the spring. Great exactness is not necessary in such calculations, and the factor 20,500, being easier to remember, may be used instead of 20,480. If a spring is continually working, a smaller factor must be used than would be admissible if it were only occasionally in action, and a much higher factor may be used if it has only to exert a constant pressure without any bending motion. If a spring is sufficiently strong and durable under certain conditions, we may, from the formula here given, design any number of springs equally strong under similar conditions.

Deflection of a Flat Spring

We will now consider the amount of deflection of a flat spring. Referring to Fig. 3, suppose there be one flexible element at *A*, and suppose the rest of the spring to be perfectly stiff or unelastic, which part we will call the "arm," and suppose the deflection at *A* will bring the arm in the position *AF*. If there now, instead of one flexible element, be two such elements at *A*, the inclination of the arm will be on line *AG*, and deflection $GB = 2BF$. For three flexible elements the deflection would be three times *BF*, and so on, provided the length of the arm remains the same; that is, the deflection of the arm *AB* is directly proportional to the number of flexible elements at *A*. Now suppose we double the length of the arm, as shown by the dotted lines; then we also double the moment of bending, and the deflection at the end of the arm will therefore be twice doubled. Therefore, by doubling the number of elements at *A* and by doubling the arm, we increase the linear deflection at the free end $2 \times 2 \times 2 = 2^3 = 8$ times. In reality the

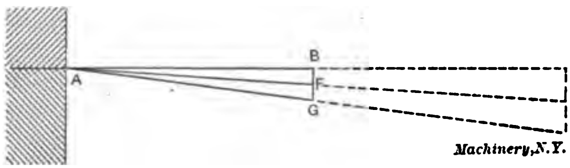


Fig. 3

arm itself is flexible, and considering it as made up of flexible elements, we may imagine the deflection at the free end as made up of a series of decreasing elementary deflections corresponding to a series of flexible elements of the spring and their respective arms.

Fig. 4 shows the curve of two similar springs of different lengths similarly loaded. *AC* represents the curve when the length is double that of *AE*. Suppose we divide *AE* in a number of small parts and call these elements, and divide *AC* into the same number of parts; each of these will contain two elements, that is, for each element of *AE* there will be two corresponding elements of *AC*, and the distance of any two such elements from the end of the spring will be twice the distance of the corresponding element of *AE* from *E*. That is, the arm and the moment of bending of any two elements of *AC* will be twice the arm and moment of bending of the corresponding element of *AE*. The deflection of spring *AC* will therefore be $2 \times 2 \times 2 = 2^3 = 8$ times the deflection of spring *AE*. If the deflection of spring *AE* is $\frac{3}{8}$ inch, the deflection of spring *AC* will be $2^3 \times \frac{3}{8} = 3$ inches. If the deflection of *AE* is $1\frac{1}{4}$ inch, the deflection of *AC* will be $2^3 \times 1\frac{1}{4} = 10$ inches, provided it is strong enough to carry the load.

If spring *AC* had been three times as long as *AE* there would, for each element of *AE*, be three corresponding elements of *AC*, and the moment of bending of any such group of elements would be three times the moment of bending of the corresponding single element of *AE*, and the distance of any group of three elements of *AC* from *C* would be three times the corresponding distance on AE; we should, therefore, in this case have a deflection at the free end of $AC = 3^3 = 27$ times the corresponding deflection of AE. If in this case the deflection of AE were ¼ inch, the deflection of AC would be $3^3 \times \frac{1}{4}$ inch $= 6\frac{3}{4}$ inches. If the length of AC were 1½ times the length of AE, we should have the deflection of $AC = (1\frac{1}{2})^3 \times \frac{1}{4}$ inch $= \frac{27}{32}$ inch.

In general, the deflection at the end is proportional to the third power of the length of the spring. The amount of deflection can be expressed by the formula al^3, in which l is the length and a is a factor dependent on the load, width, thickness and material of the spring. If we double the width, the bending moment for each element of the width will be halved, and the deflection will consequently be half of

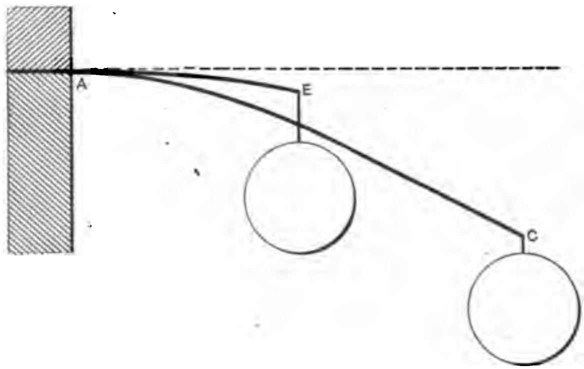


Fig. 4

that of the narrower spring. If we double the thickness of the spring, its area of cross-section is doubled, as is also the average distance of the fibers from the neutral axis, and besides for a given curvature or deflection the outer fibers will be stretched twice as much and will therefore offer twice the resistance (see Fig. 2). The total resisting moment is therefore increased $2 \times 2 \times 2 = 2^3 = 8$ times, or, in general, the moment of resistance is proportional to the third power of the thickness, and the deflection will be inversely proportional to this. If, for instance, we double the thickness and double the length of a flat spring, the deflection will remain the same for the same load. The deflection will also be directly proportional to the load.

It should be observed that we have here the moment of resistance in its general sense, that is, without any restriction in regard to the fiber stress. If we impose the condition that the fibers shall be stretched to a certain extent, as in the formula for strength, we have, in that case, the thickness squared in the moment of resistance, and it should be remembered that this is only in the formula for strength.

We are now able to calculate the deflection of any flat spring if we know the deflection of any other flat spring of the same material.

Let f = deflection of the free end, b = the width of the spring, t = the thickness, l = the length, w = the load, and k = a constant factor depending on the material, then,

$$f = \frac{wl^3}{kbt^3}$$

Suppose we have a spring 1½ inch wide, 1/16 inch thick and 12 inches long, and find that it deflects 4½ inches under a load of 10 pounds, then we have:

$$4\tfrac{1}{2} = \frac{10 \times 12^3}{k \times 1\tfrac{1}{2} \times (1/16)^3}$$

from which we deduce k = 10,500,000.

Suppose we have a spring 1 inch wide, ⅛ inch thick, 12 inches long and a load of 25 pounds at the end of it. If this spring is made of best high carbon steel, properly tempered, we may use the constant factor, 20,500, in the formula for carrying capacity, and have the greatest permissible moment of resistance = 20,500 × $(1/8)^2$ = 320.3, and the moment of bending = 12 × 25 = 300. Twenty-five pounds is therefore a safe load on this spring. For k = 10,500,000 we have the

$$\text{deflection} = \frac{25 \times 12^3}{10{,}500{,}000 \times 1 \times (\tfrac{1}{8})^3} = 2\tfrac{1}{8} \text{ inches.}$$

A spring ⅛ inch thick will carry four times as great a load as one 1/16 inch thick, and the deflection under this load will only be one-half of that of the thinner spring; or generally, *for the same fiber stress* and the same length, the deflection will be inversely proportional to the thickness.

It is an easy matter to find the deflection of a spring by actual trial and then obtain the correct value of the constant k by calculation, as here explained; but the thickness of the spring must be very carefully measured, for it will be observed that a small variation in the thickness has a great effect on the deflection, and particularly so if thin springs are used. If, for instance, the deflection of a spring 1/16 inch thick is 4 inches, then the deflection of a similar spring which is 1/100 inch thicker will only be two inches for the same load.

The Modulus of Elasticity

The deflection may also be found if the "modulus of elasticity" of the material is known. The modulus of elasticity is the ratio of a direct pulling force to the extension per unit of length of a rod of 1 square inch sectional area. The extension must be within the elastic limit of the material, and is a very small quantity which can only be found by very careful measurement in a testing machine, but as it is obtained by a straight pull, it cannot furnish so trustworthy a constant for the calculation of bending deflection as that obtained by the method just explained. If a steel bar 1 inch square and 10 inches long is stretched one-hundredth of an inch by a pull of 30,000 pounds, the modulus of elasticity is 30,000 ÷ 0.001 = 30,000,000 which is the approximately correct figure for unhardened steel. For hardened tool steel it is about 42,600,000 according to Reuleaux. The tensile strength of different steels varies considerably. The strength of high carbon steel is greatly increased by hardening. The so-called spring steel is probably more elastic, but less strong, and it is doubtful whether hardening changes its elasticity, while it no doubt increases the elastic limit and the tensile strength; but no spring steel can compare in strength with high-grade high carbon crucible tool steel, properly tempered. The allowable fiber stress depends to a great extent on the treatment of the steel; it may, according to Reuleaux, exceed 200,000 pounds per square inch at the elastic limit. In the formula for carrying capacity, the factor c should be one-sixth of the allowable fiber stress, and in

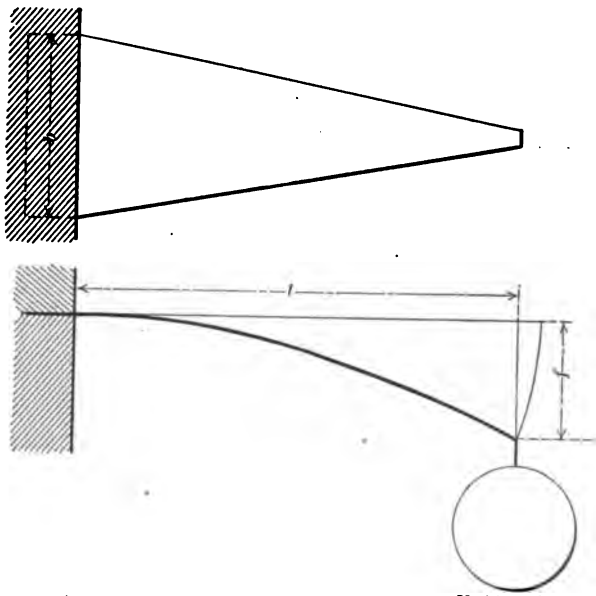


Fig. 5

the formula for deflection of a flat spring of even thickness and width the factor k should be one-fourth of the modulus of elasticity. The allowable fiber stress, is, of course, always less than the elastic limit.

Springs of Uniform Strength

A single steel band of even thickness and width does not always make a desirable spring, for if it is made just strong enough at its base, it will be stronger than necessary at other points, and the deflection at the free end will be less than if every part of the spring were equally strong—that is, if the fiber stress were uniform throughout the entire length. The spring shown in Fig. 5 is of nearly correct form; it is of even thickness and the edges converge nearly to a point.

It is obvious that, in practice, the end must be made a little blunt, but if it were continued to a sharp point, it will be seen that the width would be at any point of the length, proportional to the arm of leverage;

the radius of curvature would, therefore, be the same at any point—that is, the spring would bend to the arc of a circle. The strength is the same as that of a spring with parallel sides, but the deflection at the end will be one-half greater, and in the formula for deflection the factor k becomes two-thirds of that for parallel sides. Let the triangular spring be 2 inches wide at the base, 10 inches long, 1/16 inch thick, and made of high carbon steel, hardened, then,

$$\frac{2 \times 20{,}500}{16^2 \times 10} = 16 \text{ pounds}$$

is a safe load, and the deflection for this load is

$$\frac{16 \times 10^3 \times 16^3}{7{,}000{,}000 \times 2} = 4\ 11/16 \text{ inches.}$$

The factor 7,000,000 is here one-sixth of the supposed modulus of elasticity. All that has been said about springs with parallel sides is

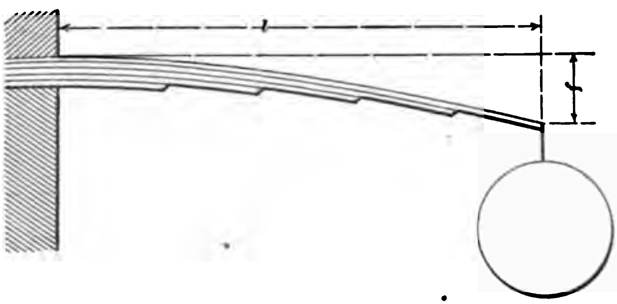


Fig. 6

also applicable to triangular springs, with the exception of the form of the curve and the factor k in the formula for deflection.

Built-up Leaf Springs

To get the most work out of a spring of given length or weight, it will often be found advantageous to use thin wide springs instead of thicker narrow ones, for it will be noticed that it is only the outside fibers which can be fully stretched or compressed, while all the others will be less useful in proportion to their proximity to the neutral axis. Instead of one broad triangular spring we may use a number of parallel springs, one on top of the other, as shown in Fig. 6. Each leaf or plate of this spring will be bent nearly to the same curve, and the deflection will be nearly equal to that of a triangular spring with a base equal to the collective width of all the leaves. Fig. 7 shows the leaves in the same plane laid side by side, and the dotted lines show the approximate size of the equivalent triangular spring. Suppose there be five leaves of tempered spring steel 2 inches wide and ⅜ inch thick, and let the working length of the main leaf be 18 inches; also suppose that the safe working fiber stress for this spring is 96,000 pounds per square inch; then we may, in the formula for strength, put

$$\text{factor } c = \frac{96{,}000}{6} = 16{,}000,$$

and the safe moment of resistance becomes $5 \times 2 \times (\frac{3}{8})^2 \times 16{,}000 = 22{,}500$, which, divided by 18 gives 1250 pounds as a safe working load

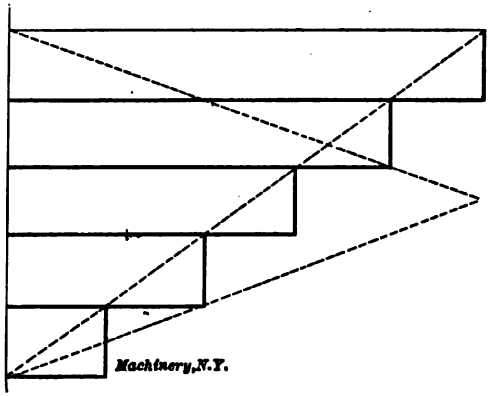


Fig. 7

on the end of the spring. Let the modulus of elasticity be 30,000,000, then we may in this case put $k = 5{,}000{,}000$, and the deflection equals

$$\frac{1250 \times 18^3}{5{,}000{,}000 \times 10 \times (\frac{3}{8})^3} = 2\tfrac{3}{4} \text{ inches.}$$

The factor 10 in the denominator is the total sum of the width of the leaves.

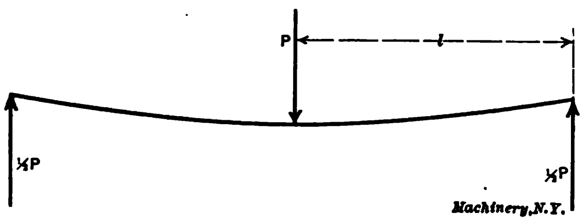


Fig. 8

It should be remarked that the deflection of such springs may vary considerably from that of the supposed equivalent triangular spring, and to get fairly correct results the factor k should be obtained by actual trial, and not from the supposed modulus of elasticity.

Fig. 8 represents a steel plate supported at both ends and a load P applied at the center. The upward pressure or reaction of each support is $\frac{1}{2}$ P, and it will readily be seen that the deflection of this spring must be exactly as if it had been supported at the center and loaded with $\frac{1}{2}P$ at each end; that is, the moment of bending at the center is $\frac{1}{2}Pl$.

Fig. 9 represents a so-called elliptic spring, of a type used on carriages, automobiles and railroad cars. It is made of steel plates 4 inches wide and ⅜ inch thick. The distance between centers is 30 inches, and there are five plates in each part. The following experimental data have been ascertained for this spring: light load = 2000 pounds; maximum working load = 7000 pounds; deflection due to a load of 5000 pounds $\doteq$ 3 inches. Comparing this case with that represented by Fig. 8, we take into account half of the ellipse only, and assuming the band b to be 3 inches wide we have $l = 13\frac{1}{2}$ inches,, and the moment of bending for maximum load $= 13\frac{1}{2} \times 3500 = 47{,}250$. Moment of resistance $= 5 \times 4 \times (\frac{3}{8})^2 \times c = \frac{45}{16}c$. These two quantities must be equal, therefore $47{,}250 = \frac{45}{16}c$, or $c = 16{,}800$. This

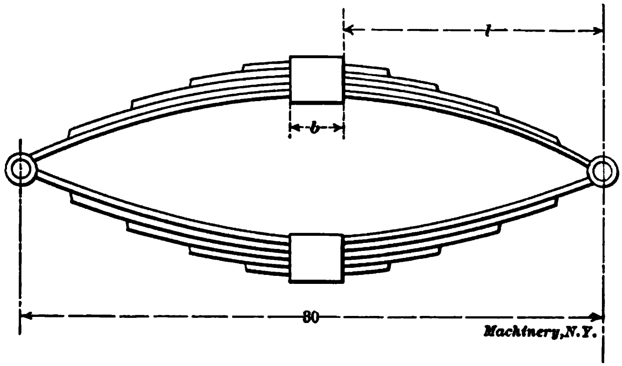


Fig. 9

value of c may correspond to a fiber stress of $6 \times 16{,}800 = 100{,}800$ pounds per square inch; but it would not be an absolutely safe assumption, for the theory of indirect molecular action is not yet fully substantiated by experimental data. The deflection of one-half of this spring is 1½ inch for 5000 pounds load; therefore:

$$1\frac{1}{2} = \frac{2500 \times (13\frac{1}{2})^3}{k \times 20 \times (\frac{3}{8})^3}$$

that is, $k = 3{,}888{,}000$. Assuming the curve of deflection similar to that of a single triangular spring, we should have, approximately, the modulus of elasticity $= 6 \times 3{,}888{,}000 = 23{,}328{,}000$, but this is probably too low a figure. By using the constant factor 3,888,000, sufficiently accurate results would be obtained for similar springs of similar material.

Miscellaneous Classes of Springs

The available space for a spring may determine its shape and size. A long straight spring cannot often be used. Fig. 10 shows a spring

which may be useful in a limited space. It is supposed to be made of a strip of high carbon crucible steel 1½ inch wide and 1/16 inch thick, and to be spring tempered. The moment of resistance is $1\frac{1}{2} \times (1/16)^2 \times c$, where c is supposed to be one-sixth of the allowable fiber stress per square inch, or the allowable unit-stress. For $c = 15{,}000$ we have the moment of resistance = 88. At *A* the lever arm is 4 inches, and the permissible load at *B* is therefore about $\frac{88}{4} = 22$ pounds. The moment of bending varies directly as the distance from *B*; at *E* and *F* it is $2 \times 22 = 44$ inch-pounds. If we imagine the spring divided into a number of small parts or elements, there will, for

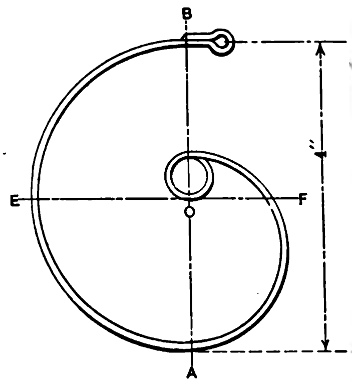


Fig.10

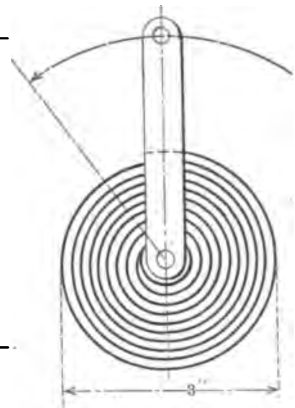


Fig.12

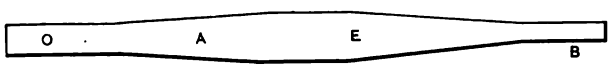


Fig.11 *Machinery,N.Y.*

Figs. 10 to 12

each of these, be a small deflection at *B* proportional to the square of its perpendicular distance from *B*. The horizontal deflection at *F* will be as if that element had been at *O*. But as the curve of the spring is longer than the straight line from *A* to *B*, and has a correspondingly greater number of elements, the entire horizontal deflection at *B* will be greater than that of a straight spring fixed at *A*. For a modulus of elasticity of 42,000,000, the deflection at *B* is about 1¼ inch, or about three times the deflection of a straight spring 4 inches long. A similar spring of the same thickness and width, but twice as large, would only carry 11 pounds, but it would deflect $2^2 \times 1\frac{1}{4} = 5$ inches under that load. Generally, for the same thickness and same unit stress the bending deflection of similar springs of this type varies as the square of their lengths.

It will be noticed that the bending moment for different parts of this spring varies considerably, while the moment of resistance is constant.

At *A* the lever arm is greatest and the unit stress is there at the safe limit, but at other points the spring will be stiffer than necessary; we may therefore improve it by varying the width in proportion to the bending moments; for then the same unit stress is obtained at any point of the length, whereby the deflection is increased without reducing the strength. Fig. 11 shows the spring when straightened out and shaped so as to give a nearly constant unit stress. This will make the deflection at *B* about one-third greater.

A great deal of potential energy may be stored in a small space by coiling a strip of steel like the main spring of a watch. If the ends are fixed and guided concentrically, the moment of bending will be constant for the whole length; and as the spring can be very long, it may be very efficient in a limited space. Fig. 12 represents a spring of this kind. Let W = bending force at end of the lever, R = length of lever, S = unit stress, b = width and t = thickness of spring; then

$$W = \frac{Sbt^2}{6R}$$

If the spring be made of $1 \times \frac{1}{8}$ inch spring steel and the length of the lever is 6 inches and the unit stress is 96,000 pounds, then,

$$W = \frac{96{,}000 \times (\frac{1}{8})^2}{6 \times 6} = 42 \text{ pounds, nearly.}$$

Let l = length of spring, E = modulus of elasticity, and F = deflection or length of arc described by the end of the lever; then,

$$F = \frac{12\, lWR^2}{Ebt^3}$$

$E = 28{,}000{,}000$ and $l = 56$ inches gives

$$F = \frac{12 \times 56 \times 42 \times 6^2}{28{,}000{,}000 \times (\frac{1}{8})^3} = 18\frac{1}{2} \text{ inches.}$$

Hence the lever turns nearly one-half of a revolution. This result may be found more directly from the formula

$$U = \frac{Sl}{\pi E t}$$

where U is the deflection expressed in revolutions and $\pi = 3.1416$. By substitution as above,

$$U = \frac{96{,}000 \times 56 \times 8}{\pi \times 28{,}000{,}000} = 0.49,$$

or nearly one-half revolution, which agrees with the former result. From this formula it appears that the deflection for a given unit stress varies directly as the length and inversely as the thickness, and is independent of the width of the spring. If we had this spring 1/16 inch thick, the lever could be turned nearly a whole revolution, but the force would be only $10\frac{1}{2}$ pounds. If we then had twice as many turns in the spiral, the lever would turn nearly two revolutions before the limit of stress would be reached. Such springs may also be useful when a nearly constant pressure through a shorter motion is desired, for this can be obtained by a considerable initial deflection. The great efficiency of watch springs is due to the high elastic limit and careful treatment of the steel.

It is sometimes preferable to coil the spring in a screw-line, as shown in Fig. 13. As in the former case, the motion is supposed to be about a fixed center, and the same formulas may be used in both cases. Let there be 72 inches of ¼ inch square spring steel, and let the lever be 3 inches long, then,

$$W = \frac{96{,}000 \times (\frac{1}{4})^2 \times \frac{1}{4}}{6 \times 3} = 83 \text{ pounds.}$$

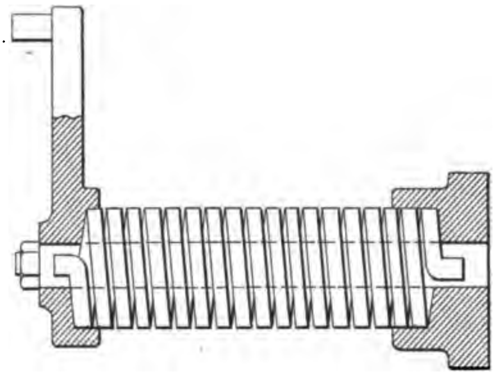


Fig. 13

We have further for this spring

$$U = \frac{96{,}000 \times 72 \times 4}{\pi \times 28{,}000{,}000} = 0.31,$$

or about 5/16 of one revolution of the lever, which is the maximum allowable motion for a unit stress of 96,000 pounds.

For round steel $W = \frac{S\pi t^3}{32\,R} = \frac{St^3}{10\,R}$, nearly. Therefore if this spring is made of ¼-inch round steel, then,

$$W = \frac{96{,}000}{10 \times 3 \times 4^3} = 50 \text{ pounds.}$$

Round steel has only 3/5 of the strength of square steel of the same diameter under bending action, but the value of U is the same in both cases.

The various springs treated of here are all of uniform thickness throughout their entire length. Good results may also be obtained by varying the thickness of a spring so as to correspond with a variable bending moment; but as such springs cannot be rolled to shape and can only receive the correct shape by skillful hand work, they are used very little. The forging down of the ends of flat springs is a simple matter and is often done. It improves the appearance of leaf springs and is preferable to blunt ends.

Torsional Springs

What really happens to the molecules of a bar when it is twisted within the elastic limit is a matter of conjecture, but all formulas for strength and deflection of torsional springs are based upon the assumption that the molecules receive a sort of lateral or sliding displacement, as if subjected to a shearing action. Whether or not this assumption is correct, it is certainly supported by experimental results. It is, for instance, known that the angle of deflection is directly proportional to the twisting force, which fact would hardly agree with other theories.

Fig. 15 represents a cross-section of a steel rod divided into a number of imaginary concentric rings of equal thickness. The torsional

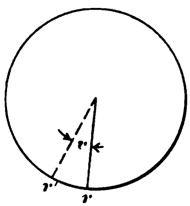


Fig. 14

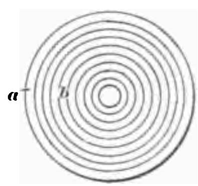
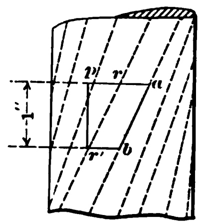


Fig. 15



Machinery,N.Y.

Fig. 16

strength of this rod depends on the resistance to shearing of the rings and on their respective distances from the center, which are their leverages of resistance. Ring *a* is twice as large as ring *b* and is twice as far removed from the center, and offers, therefore, $2 \times 2 = 2^2 = 4$ times the resistance to a twisting force. Suppose we have another rod twice as large in diameter, and divided it into the same number of rings, then each ring will be twice as thick, twice as long and twice as far removed from the center as the corresponding ring of the first rod; the torsional resistance will, therefore, be $2 \times 2 \times 2 = 2^3 = 8$ times that of the first rod, provided the resistance per unit area is the same in both cases. In other words, by increasing the diameter of the rod we increase both the thickness, length and leverage of resistance of the rings in the same proportion. The torsional strength, therefore, is proportional to the third power of the diameter.

The formulas for torsional strength are,

For round bars $RW = \frac{\pi}{16} Zd^3 = \frac{1}{5} Zd^3$, nearly. (1)

For square bars $RW = \frac{1}{3\sqrt{2}} Zd^3 = \frac{1}{4} Zd^3$, nearly. (2)

in which W = twisting force in pounds,
R = lever arm in inches,
Z = shearing unit stress of outside ring, in pounds,
d = diameter or size of bar, in inches,

For tool steel we may put $Z = 80{,}000$ pounds, and the moment of resistance of round steel $= 1/5 \times 80{,}000\ d^3 = 16{,}000\ d^3$. This gives for a ½-inch rod, the safe moment of resistance $= (½)^3 \times 16{,}000 = 2000$. If we twist the rod with a 6-inch lever, the safe load on the end of the lever $= \frac{2000}{6} = 333$ pounds. A ⅝-inch rod would carry $\frac{16{,}000}{6} \times (⅝)^3 = 650$ pounds on the end of a 6-inch lever. It will be noticed that a small increase of diameter greatly increases the strength, and that square steel will carry about one-fourth more than round steel of the same diameter.

We will now consider the torsional deflection. Fig. 14 is an end view or section of a twisted steel rod, r and r' are imaginary radial lines, and r is supposed to be in a plane above r' and is supposed to have just covered r' before the rod was twisted, that is, a small particle directly over r' is moved horizontally a distance $r'r$ through an angle v. Fig. 16 is an elevation of part of the rod where the dotted lines indicate the twisting of the surface much exaggerated. The planes of r and r' are supposed to be 1 inch apart and rp represents the transverse displacement of a small particle originally at p. The maximum unit stress in each transverse section of the rod is supposed to be equal to the product of this displacement and a certain constant multiplier. If the material be tool steel and $Z = 80{,}000$, the distance pr is about 1/150 inch. It varies directly as Z, and is independent of the diameter of tne rod. The multiplier is in this case 12,000,000, which, according to our hypothesis, is a constant for tool steel. It is a purely hypothetical quantity, which bears no rational relation to the modulus of elasticity of the material, but we may call it the torsional modulus of elasticity, because it takes the same place in the calculation of torsional deflection as the modulus of elasticity takes in the calculation of bending deflection. It will be seen that a rectangular area in the surface of the rod becomes a rhomboid when the rod is twisted. Area $rr'ba$ is a rhomboid, or deformed rectangle; suppose that pr' represents the unit of length, and let distance pr be the displacement caused for a torsional unit stress of one pound at the surface of the rod, then this displacement becomes the modulus or measure of deformation, which is the reciprocal of the torsional modulus of elasticity; but it will be readily inferred that such deformation does not produce a lateral or shearing stress, as if the surface had been stretched lengthwise of the rod a distance equal to pr, and that the torsional modulus of elasticity must be considerably less than the modulus of elasticity for bending. We have seen that for a given maximum unit stress Z, the moment of torsional resistance varies as the third power of the diameter; but without this limitation of stress the mean unit stress for any given angular deflection varies directly as the diameter of the rod, and under this condition the moment of resistance, therefore, becomes proportional to the fourth power of the diameter; and the deflection will be inversely proportional to this. That the entire angle of deflection must be proportional to the length of the rod requires no demonstration. It is also directly proportional to the load.

The following are convenient formulas for torsional deflection:

For round steel

$$F = \frac{32\,WR^2l}{\pi G d^4} = \frac{10\,WR^2l}{Gd^4}, \text{ nearly.} \qquad (3)$$

$$F = \frac{2ZlR}{Gd} \qquad (4)$$

For square steel

$$F = \frac{6\,WR^2l}{Gd^4} \qquad (5)$$

$$F = \frac{\sqrt{2}ZlR}{Gd} \qquad (6)$$

in which F = linear deflection at end of lever,
W = twisting force at end of lever,
R = length of lever,
l = length of rod,
G = torsional modulus of elasticity,
Z = unit shearing stress in periphery of cross-section,
d = diameter of rod.

For spring steel $G = 12{,}000{,}000$ is a nearly correct mean value. The proper value of Z depends on the working conditions. A spring that is continually working should be strained less than one whose action is intermittent or irregular; and it should be observed that shearing resistance at the elastic limit is somewhat less than tensile strength at the same limit. $Z = 80{,}000$ is probably not too much, unless the spring is continually working to its full capacity. But when the construction and circumstances are such as to admit of a lower stress, it is always preferable.

As a simple example of torsional springs, take a rod of ½-inch round steel 3 feet long, fixed solidly at one end, and the other end so guided as to prevent lateral motion, and let there be a 6-inch lever keyed to this end. How much will it be safe to load the end of the lever if the rod is twisted 100 times a minute? The rod is not supposed to be hardened, and though its ultimate strength is considerable, the elastic limit is comparatively low. Let $Z = 30{,}000$ and $E = 12{,}000{,}000$. Then, substituting in Formula (1) we have:

$$6\,W = 1/5 \times 30{,}000 \times (\tfrac{1}{2})^3 = \frac{6000}{8}, \text{ and } W = \frac{1000}{8} = 125 \text{ pounds}$$

which is the admissible force on the end of the lever. The deflection for this force can easily be found from Formula (4), because the value of

Z is known. We have

$$F = \frac{2 \times 30,000 \times 36 \times 6 \times 2}{12,000,000} = 2.16 \text{ inches.}$$

If this rod were of hardened steel we might put $Z = 70,000$, and would then have $W = 125 \times 7/3 = 292$ pounds, and $F = 2.16 \times 7/3 = 5.04$ inches.

Steel used for springs should have a high elastic limit and preferably a low modulus of elasticity, for the deflection is proportional to the quotient $\frac{Z}{G}$ and the greater efficiency of torsional springs is due to the smaller modulus of elasticity, as compared with that of bending. For the same unit stress at the surface of the rod the angular deflection will vary inversely as the diameter, which is an important rule easy to remember. But for the same load and varying diameters the deflection varies inversely as the fourth power of the diameter. The torsional deflection of a ⅝-inch rod, for instance, would only be about 2/5 of that of a ½-inch rod under the same load.

Helical Springs

The rod would in many cases have to be very long to give the desired deflection, and a straight rod would therefore often be impracticable; but fortunately it can be bent so as to make a comparatively short spring, easy to make and easy to harden. This is obtained by bending it in the form of a cylindrical helix, or screw-line, as shown in Figs. 17 and 18. One of these springs will be compressed and the other will be stretched, but the former may, by a slight change in the connections, be used both ways. These are true torsional springs, though it may not appear so at first sight. The following analogous case will explain it. Fig. 19 shows an open ring of steel wire firmly fixed and supported at A, and a radial lever firmly attached to the free end at B. A pressure exerted on this lever at the center of the ring perpendicular to its plane will twist the wire while it pushes point B back. This will be better understood by reference to the bent wire, shown in the dotted line. At a point N is drawn a tangent and from C a perpendicular CM. There will be a bending moment at N represented by line MN and a twisting moment represented by line CM; but when the curve becomes a circle with center at C the bending moment disappears and there is nothing but a twisting moment left, and this twisting moment is constant for any part of the concentric ring. We see that when the rod is coiled, the twisting lever is equal to the mean radius, and the deflection will be in line with the axis of the helix. The helical form is compact, and the weight of a helical spring of round steel is only about 5/12 of that of a leaf spring of the same capacity.

In the following are given a number of formulas for helical springs. Calculated values based on these formulas are given in MACHINERY'S Data Sheet No. 107, January, 1909.

The following formulas apply to helical springs:

$$\text{For round steel}\left\{\begin{array}{ll} W = \dfrac{40\,Zd^3}{100\,(D-d)} & (7) \\[2ex] F = \dfrac{8\,W\,(D-d)^3}{Gd^4} & (8) \\[2ex] F = \dfrac{314\,Z\,(D-d)^2}{100\,Gd} & (9) \end{array}\right.$$

$$\text{For square steel}\left\{\begin{array}{ll} W = \dfrac{47\,Zd^3}{100\,(D-d)} & (10) \\[2ex] F = \dfrac{47\,W\,(D-d)^3}{10\,Gd^4} & (11) \\[2ex] F = \dfrac{222\,Z\,(D-d)^2}{100\,Gd} & (12) \end{array}\right.$$

In these formulas F is the deflection of one coil, and D is the outside diameter of the coil, and the meaning of the other letters is the same as in Formulas (1) to (6). It appears from these formulas that square steel is about 17 per cent stronger than round steel, but for the same unit stress the deflection of square steel is about 30 per cent less. Round steel is, therefore, better adapted to helical springs. This may easily be perceived without any calculation, considering that when square steel is twisted, the corners cannot add very much to the strength on account of the smallness of their areas, which terminate in four points; but these points, being furthest removed from the center, will take the greatest strain, and will limit the angle of deflection as much as a full circle including the points, would do.

Fig. 18 shows a car spring of, say, 1-inch round steel, 5 inches outside diameter. How much will it carry? It must not close under the maximum static load, but it may close entirely by the jolting of the car, and we will therefore put $Z = 50{,}000$ pounds for the maximum static load, assuming the elastic limit to be above 100,000 pounds unit stress. Substituting these values in Formula (7) we have:

$$W = \frac{40 \times 50{,}000}{100 \times 4} = 5000 \text{ pounds,}$$

and assuming $Z = 100{,}000$ pounds when the spring is entirely closed, we have from Formula (9):

$$F = \frac{314 \times 100{,}000 \times 16}{100 \times 12{,}000{,}000} = 7/16 \text{ inch, nearly.}$$

That is, the coils should be 7/16 inch apart without load, and they will be 7/32 inch apart under maximum load.

The spring in Fig. 17 is, say, 3 inches in diameter and is made of ½-inch round steel, and there are 24 coils. How much may this spring be extended if used on a shaft governor? As its work is intermittent, and as it very seldom is fully extended, we may put $Z = 70{,}000$, and we have from Formula (9):

$$F = \frac{314 \times 70{,}000 \times (2\frac{1}{2})^2}{100 \times 12{,}000{,}000 \times \frac{1}{2}} = 0.23 \text{ inch.}$$

which is the allowable deflection of one coil, and $0.23 \times 24 = 5\frac{1}{2}$ inches is, therefore, the safe extension of this spring. From Formula (7) we find the maximum load to be 1400 pounds. Closed coil springs, as represented by Fig. 17, are sometimes distinguished by a considerable initial tension; that is, it takes some initial force to separate the coils, and the elongation cannot be calculated from the above formulas. The probabilities are that they are made from cold rolled wire, un-

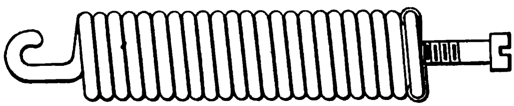


Fig.17

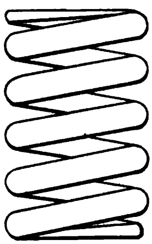


Fig.18

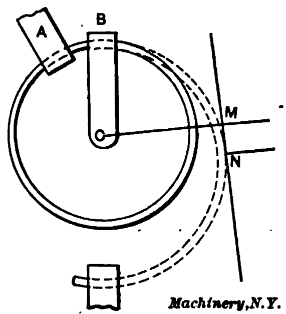


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Fig.19

Figs. 17 to 19

tempered, for the initial tension would be removed by the process of tempering. Such springs are easily distinguishable by their resistance to bending before they are stretched.

It will be noticed that in the calculations of springs the supposed elastic limit is approached closer than would be judicious in the calculation of other machine parts; but the results agree with the average common practice, and there are several reasons why this is so. In the first place, springs are made of tool steel of moderate dimensions, which is a most reliable material. In the second place, the form is such that no part can be subjected to unexpected or unaccountable strains, and on account of their great elasticity springs do not suffer materially by shocks or blows.

There seems to be considerable uncertainty or lack of knowledge as to the proper modulus of elasticity of hardened steel. The comparatively small demand for such knowledge except for the calculation of springs is a probable reason for its scarcity. According to various tests, the modulus of elasticity of untempered steel is from 28,000,000 to 32,000,000, and it appears from calculations of bending and twisting deflection of ordinary springs that the modulus of elasticity is not increased by tempering. Still it will hardly do to overlook the figures given by Reuleaux, which appear in his "Constructor." His figures for the elastic limit and ultimate tensile strength are also interesting. In the heading, he states that the figures are mean values of numerous experiments by various experimenters on materials of different make, and in actual use.

	Modulus of Elasticity	Elastic Limit	Ultimate Tensile Strength, Pounds per Square Inch
Spring steel, tempered	28,440,000	71,000 to 99,500	113,700
Tool steel, untempered	28,440,000	35,500	113,700
Tool steel, spring tempered	42,600,000	92,000 to 213,000	142,000

CHAPTER II

THE DESIGN OF HEAVY HELICAL SPRINGS*

A spring is usually specified by three dimensions, although some specifications complete the design by a fourth. The dimensions usually given are the outside diameter, free height, and diameter of bar. The fourth dimension, the solid height, is not generally given, so that the actual design of the spring is really left to the manufacturer. In some cases the number of coils or "rings" is specified, but this should never be done, as a tapered coil may be considered by one as a full coil and by another as a partial coil, thus causing confusion.

Investigation of such formulas as are found in the general text-books, hand-books, and books of reference, indicates the need of more direct formulas to facilitate the design of springs. It is the writer's intention to present the derivation of such formulas with parallel examples, showing the ease of application. For this purpose we adopt the following notation:

d = diameter of bar,
D = mean diameter of coil,
f = total deflection,
h = solid height,
H = free height,
L = blunt length of bar,
W = weight of bar, or spring,
P = capacity of coil,
P_1 = any load less than capacity,
h_1 = height of coil under load P_1,
S = maximum fiber stress,
G = torsional modulus,
w = weight of steel per cubic inch.

Only round bar coils will be considered.

I. Length of Bar when Solid Height is Given

Total number of coils $= \dfrac{L}{\pi D}$.

Total number of coils $= \dfrac{h}{d}$.

Hence,

$$\frac{L}{\pi D} = \frac{h}{d}.$$

$$L = \pi \left(\frac{D}{d}\right) h = 3.1416 \left(\frac{D}{d}\right) h$$

* MACHINERY, January, 1910, Railway Edition.

Example: Outside diameter = $4\frac{3}{8}$ inches,
Bar = 7/16 inch,
Solid height = 10 inches.

$$L = 3.1416 \times \left(\frac{3\frac{15}{16}}{\frac{7}{16}}\right) \times 10 = 282.74 \text{ inches.}$$

II. Deflection when Solid Height is Given

Fundamentally, as given in most text-books,

$$f = \frac{LDS}{Gd}.$$

But

$$L = \pi\left(\frac{D}{d}\right)h$$

Hence,

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h.$$

Or, for steel springs,

$$f = 0.019946\left(\frac{D}{d}\right)^2 h$$

Example: Outside diameter = $4\frac{1}{4}$ inches,
Diameter of bar = $\frac{3}{4}$ inch,
Solid height = 10 inches.

$$f = 0.019946\left(\frac{3\frac{1}{2}}{\frac{3}{4}}\right)^2 \times 10 = 4.34 \text{ inches.}$$

III. Ratio between Free and Solid Heights

$$H = h + f$$

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

Hence,

$$H = h + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

$$H = \left[1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2\right] h$$

Or, for steel springs,

$$H = \left[1 + 0.019946\left(\frac{D}{d}\right)^2\right] h$$

and

$$h = \frac{H}{1 + 0.019946\left(\frac{D}{d}\right)^2}$$

***Example* 1:** Outside diameter = 6 inches,
Diameter of bar = 1⅛ inch,
Free height = 13¾ inches.
Find solid height h.

$$h = \frac{13.75}{1 + 0.019946\left(\dfrac{4\frac{7}{8}}{1\frac{1}{8}}\right)^2} = 10 \text{ inches.}$$

***Example* 2:** Outside diameter = 7⅛ inches,
Diameter of bar = 1⅛ inch,
Solid height = 10 inches.
Find free height H.

$$H = \left[1 + 0.019946\left(\frac{6}{1\frac{1}{8}}\right)^2\right] \times 10 = 15.67 \text{ inches.}$$

IV. Deflection when only Free Height is Given

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

But

$$h = \frac{H}{1 + \dfrac{\pi S}{G}\left(\dfrac{D}{d}\right)^2}$$

Hence,

$$f = \frac{\dfrac{G}{\pi S}\left(\dfrac{D}{d}\right)^2 H}{1 + \dfrac{\pi S}{G}\left(\dfrac{D}{d}\right)^2}$$

$$f = \frac{H}{1 + \dfrac{G}{\pi S}\left(\dfrac{d}{D}\right)^2}$$

Or, for steel springs,

$$f = \frac{H}{1 + 50.1337\left(\dfrac{d}{D}\right)^2}$$

Example: Outside diameter = 5½ inches,
Diameter of bar = 1⅜ inch,
Free height = 11¾ inches.

$$f = \frac{11\frac{3}{4}}{1 + 50.1337\left(\dfrac{1\frac{3}{8}}{4\frac{1}{8}}\right)^2} = 1\tfrac{3}{4} \text{ very nearly.}$$

V. Weight when Solid Height is Given

$$\text{Area of cross section} = \frac{\pi d^2}{4}.$$

$$\text{Cubical contents of bar} = \frac{L \pi d^2}{4}.$$

$$\text{Then } W = \frac{L \pi d^2 w}{4}$$

$$\text{But } L = \pi \left(\frac{D}{d}\right) h$$

$$\text{Hence, } W = \frac{\pi^2 w}{4} d D h$$

For steel springs, where one cubic foot of steel weighs 486.6 pounds,

$$W = 0.694\ dDh.$$

Example: Outside diameter = 3¾ inches,
Diameter of bar = 15/16 inch,
Solid height = 10 inches.

$$W = 0.694 \times \frac{15}{16} \times 2\frac{13}{16} \times 10 = 18.3 \text{ pounds.}$$

VI. When Free and Solid Heights are Given to Determine Stress

$$h = \frac{H}{1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2}$$

$$S = \frac{(H - h)\,G}{\pi h} \times \left(\frac{d}{D}\right)^2$$

$$S = \frac{Gf}{\pi h} \times \left(\frac{d}{D}\right)^2$$

For steel springs,

$$S = 4{,}010{,}700\,\frac{f}{h}\left(\frac{d}{D}\right)^2$$

Example: Outside diameter = 4½ inches,
Diameter of bar = ½ inch,
Free height = 22¾ inches,
Solid height = 10 inches.

$$S = 4{,}010{,}700 \times \frac{12.75}{10}\left(\frac{0.5}{4}\right)^2 = 80{,}000 \text{ pounds.}$$

VII. When Free and Solid Heights are Given to Determine Capacity

$$P = \frac{S \pi d^3}{8 D}$$

and

$$S = \frac{G f}{\pi h}\left(\frac{d}{D}\right)^2$$

Hence,

$$P = \frac{G f d^5}{8 h D^3}$$

For steel springs,

$$P = 1{,}575{,}000 \frac{f d^5}{h D^3}$$

Example: Outside diameter = 2⅞ inches,
Diameter of bar = ½ inch,
Free height = 14½ inches,
Solid height = 10 inches.

$$P = 1{,}575{,}000 \times \frac{4.5 \times 0.5^5}{10 \times 2.375^3} = 1653 \text{ pounds.}$$

These last two formulas are very useful in ascertaining the stresses and loads of the separate coils of double and triple coil springs.

VIII. Given Free Height, Diameter of Spring and Bar, and Load Carried at Given Height. To Find Proper Solid Height

$$\frac{P_1}{P} = \frac{f_1}{f}$$

$$H = f + h$$

$$H = f_1 + h_1$$

Hence, $f_1 = f + h - h_1$

Then $P(f + h - h_1) = P_1 f$

Hence $h = \dfrac{P_1 f - P f + P h_1}{P}$

$$h = \frac{P_1 - P}{P} \times f + h_1$$

But $f = \dfrac{\pi S}{G}\left(\dfrac{D}{d}\right)^2 h$

Hence,

$$h = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2\left(\frac{P_1 - P}{P}\right) h + h_1$$

$$h = \frac{h_1}{1 + \dfrac{\pi S}{G}\left(\dfrac{P - P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$$

For steel springs,

$$h = \frac{h_1}{1 + 0.019946\left(\dfrac{P - P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$$

Example: Outside diameter = $5\frac{1}{2}$ inches,
Diameter of bar = $\frac{3}{4}$ inch,
Free height = 18 inches.

What solid height is required for carrying 1395 pounds at 14 inches?

$P = 2790$ pounds by formula $P = \dfrac{S \pi d^3}{8 D}$.

Then,

$$h = \frac{14}{1 + 0.019946\left(\dfrac{2790 - 1395}{2790}\right)\left(\dfrac{4\frac{3}{4}}{\frac{3}{4}}\right)^2} = 10 \text{ inches.}$$

IX. To Determine the Quality of the Steel

The value of G is the index to the quality of the steel, and upon this value depend all properties of the spring. By transposing either the formula given in (VII) for capacity, or that for load, we find a method for ascertaining this value, *i. e.:*

$$G = \pi S \frac{h}{f}\left(\frac{D}{d}\right)^2$$

or

$$G = 8 P \frac{h D^3}{f d^5}$$

Example: Outside diameter = $4\frac{7}{8}$ inches,
Diameter of bar = 11/16 inch,
Load = 1219 pounds,
Deflection = 3.7 inches,
Solid height = 10 inches.

$$G = 8 \times 1219 \times \frac{10 \times (4\frac{3}{16})^3}{3.7 \times (\frac{11}{16})^5} = 12{,}600{,}000.$$

General Remarks

Concentric coils, as shown in Fig. 21, are made generally of the same free and solid heights. Presuming that such coils are all made of the same quality of steel, the ratio of $\dfrac{D}{d}$ should be the same throughout, for the formula in (II) clearly shows that this is necessary to obtain equal stresses in all coils.

The formula in (I) shows that after all values of $\frac{D}{d}$ are made the same, the lengths of all bars will be the same before tapering. A study of all the formulas reveals the fact that the ratio of $\frac{D}{d}$ determines everything; this ratio might well be called the *spring index.*

Fig. 20. Types of Coil Springs for Railroad Cars

Fig. 21. Concentric Coil Springs for Railroad Cars

The absolutely perfect design of concentric springs is seldom possible where a scale of sixteenths inch for dimensions is used, with the customary one-eighth inch between inside diameter of one spring and outside diameter of the next. As cases of perfect design, however, the following springs are given as examples:

Spring No. 1

Outer: 5 inches outside diameter, 15/16 inch bar.
Inner: 3 inches outside diameter, 9/16 inch bar.

In this design $\frac{D}{d} = 4\ 1/3$.

Spring No. 2

Outer: 2⅝ inches outside diameter, ⅜ inch bar.
Inner: 1¾ inch outside diameter, ¼ inch bar.

In this design $\frac{D}{d} = 6$.

In concentric coil springs where perfect design is impossible, the coil having the least value of $\frac{D}{d}$ will be stressed the highest, as shown

Fig. 22. Groups of Coil Springs held together by Plates at Top and Bottom

by the formula in (VI); this coil may therefore be called the governing coil, inasmuch as the motion, or deflection, of the spring as a whole depends upon this coil. To estimate the capacity of such concentric coils we have recourse to the formula in (VII), while the formula in (VI) shows the separate stresses. The load which the concentric spring will carry at any height is then found by the fact that all loads are proportional to deflection.

In actual design adjacent coils are wound in opposite directions to prevent binding, as shown in Fig. 21. Instead of using concentric coils, groups of similar coils are sometimes used which are held together by pressed steel or cast spring-plates, as shown in Fig. 22. It is customary to suspend the static load at one-half the deflection.

A helical spring for railroad service is almost invariably made of round bar spring steel. The analysis of spring steel most frequently used is known as P. R. R. analysis, and its composition is as follows:

Carbon, 1.0 per cent (not under 0.90 per cent); phosphorous, 0.05 per cent (not over 0.07 per cent); manganese, 0.25 per cent (not over 0.50 per cent); silicon, not over 0.10 per cent; sulphur, not over 0.03 per cent.

For spring steel of this character the maximum fiber stress should not be over 80,000 pounds per square inch, and the torsional modulus should be taken as 12,600,000 pounds.

CHAPTER III

THE DESIGN OF ELLIPTIC SPRINGS*

It is doubtful if scientific calculations ever entered into the design of the original forms of such springs as are used under ordinary road carriages. Satisfactory as they are, they are not engineering results, but accepted standards born long ago of the cut-and-try methods of the blacksmith shop. Their manufacture belongs to such arts as are taught by father to son, or acquired through years of experience, during which have been gathered the "tricks of the trade." The manufacturer of this class of springs does not attempt to arrive at results by mathematics. He has learned as a part of his trade that certain styles of carriages should have certain springs.

Sufficient time did not exist during the development of railroad cars for a gradual development of definite types of springs for various types of cars. It devolved, therefore, upon the engineer to design these springs; but as soon as the spring maker found that the 70,000-, 80,000-, and 100,000-pound capacity car each had its own peculiar set of springs, and that any car could be fitted with springs according to its capacity, he adopted the engineer's designs as another class of standards. Railroad cars, while resting on springs whose dimensions were originally scientifically estimated, are now, therefore, suspended largely upon springs belonging to a few fixed classes.

With the advent of the automobile came a carriage traveling fast over uneven country roads, meeting severe usage in inexperienced hands, and demanding the extreme of comfort and safety. The question of springs and spring suspension thus becomes of primary importance, so that in these carriages each particular design requires a specially designed suspension. Automobile springs are fundamentally cantilevers, the same as all leaf springs. This class of springs more readily lends itself to an easy vibration, as well as to a better general design of the machine. It is possible to carry a load on a narrow-leafed elliptic leaf spring where there would not be room for a helical spring. Also, the addition of a leaf to an elliptic leaf spring adds to its capacity without changing its deflection, while the addition of a coil to a helical spring does not change its capacity but adds to its deflection.

Any leaf spring, tightly banded around the middle, should be considered as composed of two cantilevers of length l, where l is one-half the distance from center to center of the end bearings less one-half the width of the band. The length of each cantilever is then expressed (see Fig. 24):

$$l = \frac{c - w}{2}$$

* MACHINERY, January, 1910, Engineering Edition.

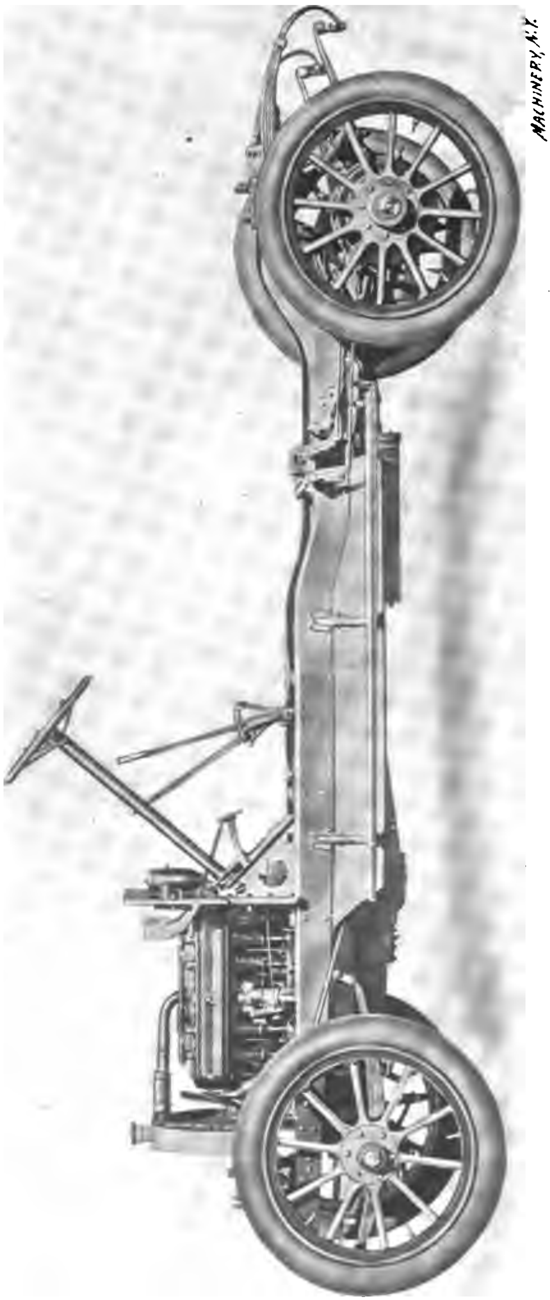

Fig. 23. Chassis of an F. B. Stearns Motor Car, showing Three-quarter Elliptic Spring Suspension in Rear and Semi-elliptic Springs in Front

To consider a spring as a simple beam of length c, is to overlook the effect of the band. It is easily demonstrated that variations in the width of the band cause corresponding variations in the strength and deflection of the spring. The elliptic spring, graduated throughout, with but *one* leaf in each section extending from end bearing to end bearing, is fundamentally a cantilever of *uniform strength;* and the formulas applicable are based on the fundamental formulas of that type of cantilever. An elliptic spring with *all* leaves in each section extending from end bearing to end bearing is, on the other hand, a cantilever of *uniform* section, and the formulas for this type of cantilever are then applicable.

The springs used in automobile practice are frequently combinations of these two forms, inasmuch as a considerable portion of the leaves extend the full length from bearing to bearing. It follows that neither of the above formulas will apply, but that the applicable formulas may be derived by combining the fundamental formulas for the

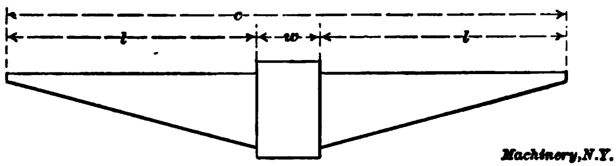

Fig. 24. Diagrammatical Sketch of Graduated Spring, giving Length Notation used in Formulas

two types of cantilevers. The load capacity of a cantilever is not affected by its form, for in either case:

$$P=\frac{S\,b\,h^2}{6\,l}$$

in which P = load,
S = allowable stress,
b = width of beam,
h = thickness of beam,
l = length of cantilever.

In other words, the load capacity is equal for like conditions, such as stress, size of beam, and length of span.

A great difference exists, however, in the deflections under the same load, one being fifty per cent more than the other:

$$f=\frac{4\,P\,l^3}{E\,b\,h^3}, \text{ for uniform section cantilevers,}$$

$$f=\frac{6\,P\,l^3}{E\,b\,h^3}, \text{ for uniform strength cantilevers,*}$$

in which f = deflection, and E = modulus of elasticity.

* The formula given is that for a cantilever of uniform strength, where the height h is uniform, but the width of the section of the cantilever decreases towards the outer end; b is the width at the support.

When such a difference as this exists, it is rather remarkable that many engineers calculate the properties of an elliptic spring no matter what the cantilever conditions, as though all elliptic springs were subject to the same rules and formulas; but, as a matter of fact, the proportion of back leaves, or the leaves on the longer side of the spring which commonly extends the full length, ranges from 5 to 50 per cent of the total number of leaves. It is not unusual to see attempts made through actual tests of the springs themselves to find the proper constant with which to modify the uniform strength equations so as to render them applicable to springs composed of uniform section cantilevers in combination with uniform strength cantilevers. The desired modifier, however, is a variable quantity, depending upon the relative size of the fundamental spring elements.

Lack of due consideration of this combination of different cantilevers accounts also for the different and conflicting formulas which various

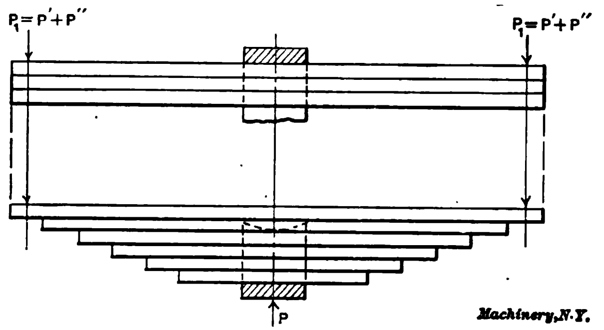

Fig. 25. Showing Division of Spring into Cantilevers of Uniform Section (Upper Portion) and Cantilevers of Uniform Strength (Lower Portion). One of the Full Length Leaves should always be considered as a Part of the Graduated Leaves

authorities advance. Thus Goodman, in "Mechanics Applied to Engineering"; Reuleaux, in his "Constructor"; and "Des Ingenieurs Taschenbuch" (Hütte), give formulas all of which reduce to uniform strength cantilevers. Molesworth and the Automotor Pocket Book base their formulas on uniform section cantilevers. Henderson, who assumed all semi-elliptic springs to contain one-fourth full length leaves, and made an approximation of the result, was the first to recognize the influence of the combination of cantilevers.

Deduction of General Formulas

For further consideration we will adopt the following notation, discussing only the semi-elliptic spring:

P = total load on spring,

P_1 = portion of load on one end of spring,

P' = portion of load on one end of full-length leaves, or on uniform section cantilever,

P'' = portion of load on one end of graduated leaves, or on uniform strength cantilever,

n = total number of leaves,
n' = number of full-length leaves,
n'' = number of graduated leaves,

$$r = \frac{n'}{n},$$

S = maximum fiber stress in spring,
S' = maximum fiber stress in full-length leaves,
S'' = maximum fiber stress in graduated leaves,
f = total deflection of banded leaves,
f' = total deflection of full-length leaves if unbanded,
f'' = total deflection of graduated leaves if unbanded,
b = width of leaves,
h = thickness of leaves,
l = length of cantilever,
L = net length of spring, *i. e.*, actual distance between end bearings, less width of band,
x = proper initial space between fundamental cantilevers before banding.

It is but reasonable to assume that the maximum fiber strain should be the same in both fundamental parts, or

$$S' = S''.$$

But

$$S' = \frac{6\,P'\,l}{n'\,b\,h^2},$$

$$S'' = \frac{6\,P''\,l}{n''\,b\,h^2}.$$

Hence,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

In a well-designed spring there should be, at full load, a division of the work proportional to the respective number of leaves in the two fundamental parts. The fundamental formulas of the two cantilevers have shown, however, that such proportional loads would produce different deflections in their respective carriers. This difference in deflection would cause a separation of the two portions of the spring were they initially together and unbanded. Were they initially together and banded the result would be internal stress under load which would mean that a division of the load proportional to the respective number of leaves in the two fundamental parts could not exist.

It is evident that by placing a space between the two fundamental parts when unloaded and unbanded, equal to the difference between the two deflections, there will result no space between the two fundamental parts at full load; and hence if banded in this position there will be no internal stress, so that the load on each part will be proportional to the number of leaves in that part. If then the load be removed, it follows that the band alone holds the two portions together and that there must exist a resulting stress upon the band and leaves.

Now

$$f' = \frac{4\,P'\,l^3}{E\,n'\,b\,h^3} \qquad (1)$$

and

$$f'' = \frac{6\,P''\,l^3}{E\,n''\,b\,h^3} \qquad (2)$$

But, as shown,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

or

$$P' = \frac{n'\,P''}{n''}$$

Hence $f' = \dfrac{4\,P''\,l^3}{E\,n''\,b\,h^3}$, as derived by substituting in (1).

Hence,

$$f'' - f' = \frac{2\,P''\,l^3}{E\,n''\,b\,h^3}.$$

Also, since

$$\frac{P'}{n'} = \frac{P''}{n''} = \frac{P_1}{n} = \frac{P}{2\,n},$$

we have

$$f'' - f' = \frac{P\,l^3}{E\,n\,b\,h^3}.$$

Also since

$$l = \frac{L}{2},$$

$$f'' - f' = \frac{P\,L^3}{8\,E\,n\,b\,h^3}$$

or

$$x = \frac{P\,L^3}{8\,E\,n\,b\,h^3}$$

This last expression is then a general expression of the proper initial distance between the two fundamental portions before banding, expressed in terms of total load on spring, total number of leaves in spring, and net span of spring. To find the actual working deflection of the entire spring it is only necessary now to ascertain how much either portion is deflected by the process of bending. For this purpose let us adopt the following notation:

P_x = force exerted by band,
f_x' = deflection of full-length leaves caused by band,
f_x'' = deflection of graduated leaves caused by band.

Then,

$$f_x' = \frac{2 P_x l^3}{E n' b h^3} \text{ and } f_x'' = \frac{3 P_x l^3}{E n'' b h^3}$$

Hence

$$\frac{P_x l^3}{E b h^3} = \frac{f_x' n'}{2} = \frac{f_x'' n''}{3}$$

or

$$f_x' = \frac{2}{3}\left(\frac{1-r}{r}\right) f_x''$$

But

$$f_x' + f_x'' = \frac{P l^3}{E n b h^3}$$

Hence

$$f_x'' + \frac{2}{3}\left(\frac{1-r}{r}\right) f_x'' = \frac{P l^3}{E n b h^3}$$

$$f_x'' = \left(\frac{3r}{2+r}\right)\frac{P l^3}{E n b h^3}$$

But

$$_x'' = \frac{3 P_x l^3}{E n'' b h^3}$$

Hence

$$\frac{3 P_x l^3}{E n'' b h^3} = \left(\frac{3 r}{2+r}\right)\frac{P l^3}{E n b h^3}$$

or

$$\frac{3 P_x l^3}{E (1-r) n b h^3} = \left(\frac{3 r}{2+r}\right)\frac{P l^3}{E n b h^3}$$

or

$$P_x = \left(\frac{r(1-r)}{2+r}\right) P$$

The expression inside the bracket in the above equation becomes zero for either extreme value of r, as would be expected, the extreme values of r being unity and zero. The formula gives the force exerted by the band, *i. e.*, the load upon the band.

The total deflection of the graduated leaves, as already developed, is,

$$f'' = \frac{3 P l^3}{E n b h^3}$$

The deflection of the graduated leaves, caused by the band, is

$$f_x'' = \left(\frac{3 r}{2+r}\right)\frac{P l^3}{E n b h^3}$$

The difference is, therefore, the deflection left in the graduated leaves after banding, or the general formula sought for the deflection of such a spring:

$$f'' - f_x'' = \left\{ 3 - \left(\frac{3r}{2+r} \right) \right\} \frac{P l^3}{E n b h^3}$$

or,

$$f = \left(\frac{6}{2+r} \right) \frac{P l^3}{E n b h^3}$$

or, since $l = \frac{L}{2}$ and

$$P = 2 P_1 = 2 \left(\frac{S n b h^2}{6 l} \right)$$

$$f = \left(\frac{6}{2+r} \right) \left(\frac{2 S n b h^2}{3 L} \right) \frac{L^3}{8 E n b h^3}$$

Hence

$$f = \frac{1}{2(2+r)} \times \frac{S L^2}{E h}$$

This last expression is then a general formula for the deflection of all semi-elliptic springs. If all the leaves are graduated, $r = 0$, and

$$f = 1/4 \times \frac{S L^2}{E h}.$$

If all the leaves are full length, $r = 1$, and

$$f = 1/6 \times \frac{S L^2}{E h}.$$

As was to be expected, the spring composed of all graduated leaves has a deflection, according to the above general formula, 50 per cent above that of a spring composed of all full-length leaves. For values of r above zero, the deflection will be found to decrease until r equals unity.

General Remarks

The general formulas given above were first deduced by the writer in the early part of 1905, at which time they were placed before Prof. C. H. Benjamin, then of the Case School of Applied Science, with a view of making extended experiments for the preparation of a thesis. It was the intention to have springs built with initial space as deduced, and compare the actual deflections of such springs with the estimated deflections. Although these experiments were not carried out, they are mentioned because it is believed that when such experiments are made, they will prove valuable. The deduction of the formulas was published for the first time in MACHINERY, in the January, 1910, issue, engineering edition. This deduction was made in connection with certain springs which were giving very poor service, although designed by the same formulas as other elliptic springs. It was the writer's conclusion that had the springs been built with the proper initial space between the fundamental parts, these springs would not have broken, and that the omission of this space caused an over-stress in the full-length leaves, and an under-stress in the graduated leaves, which caused the over-strained leaves to break, throwing an overload upon the previously under-stressed leaves which also broke when the stress became excessive. This conclusion seems to explain why springs of this type are frequently found with only the long leaves broken; the remaining leaves, all being of one type, divide the resultant overload evenly so that the over-stress is not so excessive. Perhaps the strongest indication of the correctness of the deduction lies in the well-known fact that the percentage of breakage is always much greater with semi-elliptic springs (of the combination type, usually) than with full elliptic springs. Also, it is generally found upon unbanding these springs that no initial space exists.

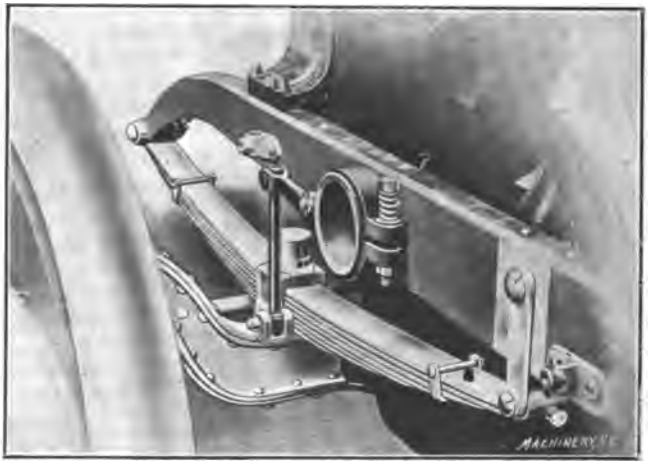

Fig. 26 Front Spring Arrangement of the 1910 Model Winton Six-cylinder Car

Comparison of deflections estimated from the above formulas, with actual deflections, has in some cases been quite satisfactory, while in other cases the actual deflections have appeared closer to those estimated by uniform strength formulas. In such cases where the writer has been able to make comparisons, however, the springs had been *made to specified deflections which evidently were estimated by the uniform strength formulas.* Experienced spring makers know that it is quite possible by putting a "pull" in the springs to vary the deflection and load. This trade term, "pull," is itself nothing more nor less than the introduction of an initial space between the leaves before banding.

Suspension of Automobiles

In road carriages, except in the heavier wagons, it is usual to find but two springs, one over each axle placed across the width of the carriage. In automobiles, one finds almost invariably at least the rear suspended upon two springs running lengthwise of the car, while, as is shown in the accompanying illustrations, it is the tendency to use the same suspension in the front. Such an arrangement takes up the forward and side lunges in a manner impossible with simple transverse springs. The further use of links and shackles, and of scroll ends, adds to the comfort, allowing the car to swing upon the springs rather than to be thrown upon them. In quite a few models, the two rear springs are attached in front to the frame and in the rear to a platform spring, which is itself attached to the center of the rear cross member of the frame. The three-quarter elliptic spring lends itself to both comfort and convenience of arrangement, and is

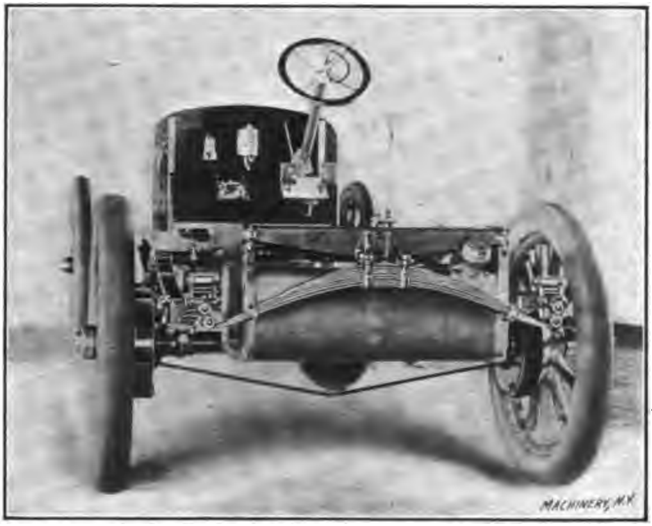

Fig. 27. Spring Support of the Lozier Motor Co.'s "Light Six" Car

rapidly coming into general use in this country, our manufacturers having apparently adopted it from foreign cars.

Steel Used in Automobile Springs

Automobile springs call for a high grade of steel, the ordinary spring steel lacking in strength and elasticity. Various grades of high carbon, silicon, manganese, nickel, chromium, and vanadium steels are used. Often such alloys are used as silico-manganese, chrome-nickel, and chrome-vanadium, the stiffening elements seeming to rank in the order given. Data as to the physical properties of such steels cannot well be given, as such properties must depend upon the proportions in the particular alloy used. Certain alloys of the vanadium group having an elastic limit of from 180,000 to 225,000 pounds per square inch, and tensile strength from 190,000 to 250,000 pounds, appear to be the most ideal steels yet produced.

Calculations of Springs

The calculation of spring properties by formulas is long and tedious. The writer appends, therefore, a table based on a modulus of elasticity of 25,400,000 and a fiber stress under maximum safe load of 80,000 pounds per square inch. Calculations of springs made of materials having other physical properties are made by simple proportion. This table is to be used only when all leaves are fully graduated.

The safe load on one leaf one inch wide is found by dividing the constant given under P_u by the net length. The corresponding deflection is found by multiplying the constant given under f_u by the square of the net length.

Example: What is the safe load on a semi-elliptic full graduated spring of five leaves if of one-quarter by two inch steel; length between end bearings, thirty-six inches; band or seat, three inches?

Net length = 36 — 3 = 33 inches.

$$\text{Load on one leaf one inch wide} = \frac{3333.33}{33} = 101.01 \text{ pounds.}$$

SEMI-ELLIPTIC SPRING TABLE

Giving safe load and deflection for 1 inch wide leaves, 1 inch net length. Used only when all leaves are fully graduated

Thickness of Leaf	P_u	f_u	Steel	P_u	f_u
$\frac{1}{32}$	52.08	0.02519	$\frac{9}{32}$	4218.75	0.00280
$\frac{1}{16}$	208.33	0.01260	$\frac{5}{16}$	5208.33	0.00252
$\frac{3}{32}$	468.75	0.00840	$\frac{11}{32}$	6302.08	0.00229
$\frac{1}{8}$	833.33	0.00630	$\frac{3}{8}$	7500.00	0.00210
$\frac{5}{32}$	1302.08	0.00504	$\frac{13}{32}$	8802.08	0.00194
$\frac{3}{16}$	1875.00	0.00420	$\frac{7}{16}$	10208.33	0.00180
$\frac{7}{32}$	2552.08	0.00360	$\frac{15}{32}$	11718.75	0.00168
$\frac{1}{4}$	3333.33	0.00315	$\frac{1}{2}$	13333.33	0.00157

Load on one leaf two inches wide = 2 × 101.01 = 202.02 pounds.

Load on five two-inch leaves = 5 × 202.02 = 1010.10 pounds.

Corresponding deflection is:

$$0.00315 \times (33)^2 = 3.43 \text{ inches.}$$

Formulas can easily be deduced making it possible to use the accompanying table for other classes of elliptic springs than those of the semi-elliptic type with all leaves fully graduated.

The formulas for the semi-elliptic spring with all leaves graduated are:

$$P = \frac{2\,S\,n\,b\,h^2}{3\,L} \text{ and } f = \frac{S\,L^2}{4\,E\,h}.$$

To find the values of P_u given in the table, insert $S = 80{,}000$, $n = 1$, $b = 1$, $h =$ the value given in the first column in the table, and $L = 1$. To find the values of f_u, insert in the second formula $S = 80{,}000$, $L = 1$, $E = 25{,}400{,}000$, and $h =$ the value given in the first column in the table.

Now if the values in the table are to be used for other springs, constants can be deduced by which the table values may be multiplied.

For a semi-elliptic spring with a portion of the leaves graduated the load P remains the same as for a spring with all leaves graduated. The formula for the deflection, however, is:

$$f = \frac{1}{2\,(2 + r)} \times \frac{S\,L^2}{E\,h}.$$

The values in the table, therefore, must be multiplied by the quantity $\frac{2}{(2 + r)} \times L^2$ to find the deflection for any given combination full leaf and graduated spring of effective length L.

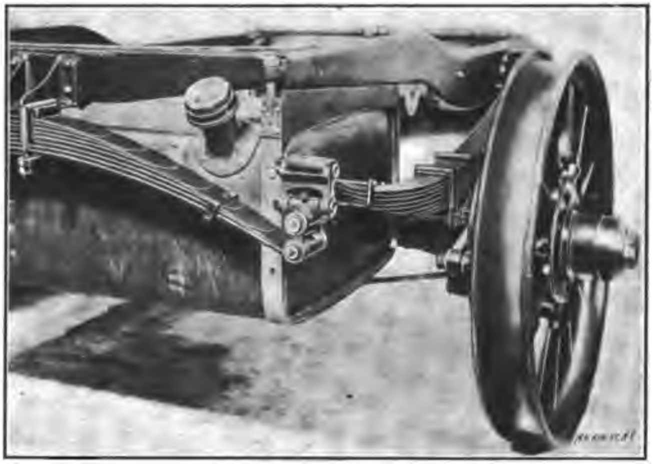

Fig. 28. Arrangement of Semi-elliptic Springs on the Lozier Motor Co.'s Four-cylinder Model

For a full elliptic spring with all leaves graduated, P still remains the same as for a semi-elliptic spring, but f doubles its value, or:

$$f = \frac{S\,L^2}{2\,E\,h}.$$

The values in the table, therefore, in this case must be multiplied by $2\,L^2$.

For the full elliptic spring with only part of the leaves graduated, the load P remains the same as before, but the deflection is twice that of a semi-elliptic spring:

$$f = \frac{1}{2\,(2 + r)} \times \frac{2\,S\,L^2}{E\,h} = \frac{S\,L^2}{(2 + r)\,E\,h}.$$

In this case, then, the values for the deflection in the table are to be multiplied by $\frac{4}{2 + r} \times L^2$.

The flexibility of a spring is the amount of deflection as compared to the load. This may be expressed as so many inches deflection per hundred pounds, or y.

Example: Assume a full-elliptic, fully graduated spring, where

$S = 80{,}000$,
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$n = 4$,
$h = \frac{1}{4}$ inch,
$L = 30$ inches.

Then the safe load equals:

$$P = 4 \times 1\tfrac{3}{4} \times \frac{3333.33}{30} = 778 \text{ pounds.}$$

And the deflection equals:

$$f = 30^2 \times 2 \times 0.00315 = 5.67 \text{ inches.}$$

Then,

$$y = \frac{5.67}{778} \times 100 = 0.73 \text{ inch.}$$

On the other hand, assume that the thickness and number of leaves are unknown. Then we have:

$P = 778$ pounds,
$S = 80{,}000$,
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$L = 30$ inches,
$y = 0.73$ inch.

Then

$$f = \frac{778}{100} \times 0.73 = 5.67 \text{ inches.}$$

But $f = 2 f_u L^2$, where f_u is the constant for deflection in the accompanying table.

Hence,

$$f_u = \frac{f}{2 L^2} = \frac{5.67}{1800} = 0.00315.$$

The thickness of steel in the table which corresponds to this value of f_u is one-fourth inch.

The number of leaves is found by using P_u.

Load on one leaf, one inch wide is:

$$\frac{3333.333}{30} = 111.11 \text{ pounds.}$$

Load on one leaf $1\frac{3}{4}$ inch wide is:

$$111.11 \times 1\tfrac{3}{4} = 194.25.$$

Number of leaves is then,

$$\frac{778}{194.25} = 4.$$

The present calculation makes no allowance for the leaves of a spring varying in thickness. Where such springs are used, the deflection of the different leaves will not be uniform. Hence, in such springs also a suitable initial "pull" should exist, and such springs should be estimated by a general formula based upon a combination of different cantilevers, thus making allowance for different depths of cantilevers.

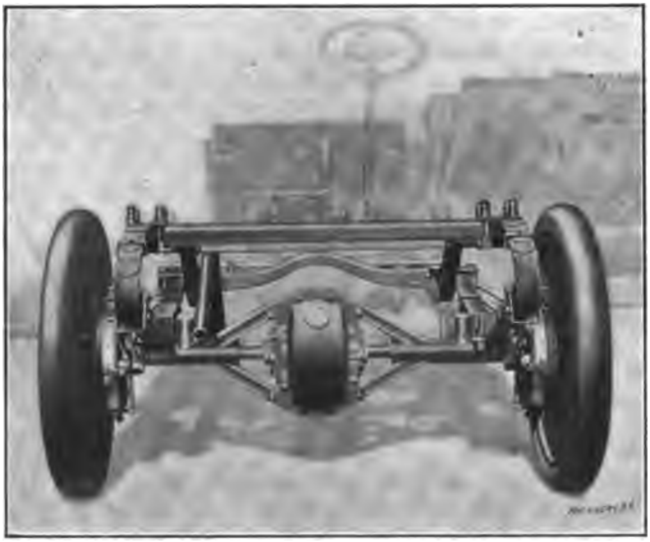

Fig. 29. Three-quarter Elliptic Spring Suspension on the F. B. Stearns Co.'s 15-30 H. P. Car

It is much better to use springs composed of but one thickness of leaves, as the combination of different thicknesses adds a complexity scarcely necessary.

Results obtained from fully graduated full elliptic springs would seem to show that the action of the friction between the leaves is not great enough to seriously affect the bending action, in that the formulas give results agreeing very closely with actual conditions.

CHAPTER IV

THE DESIGN OF SPRINGS FOR GAS ENGINE VALVES*

Springs for gas engines should be carefully designed, and if properly proportioned for the work they must do, should be just as reliable as any other part of the mechanism. While the general data for spring design are well known to engineers, yet attention may properly be given to some considerations specially applicable to gas engine valve springs. This chapter will consider compression springs of round steel wire only, as the writer knows of no valid reason for the use of any other material or section for this class of springs. It is well known that square steel is less desirable than round steel for springs, both on account of the higher cost of the springs per pound, and from the standpoint of efficiency.

The first consideration is the selection of the proper values for the fiber stress S and the torsional modulus of elasticity G. Experiments have shown that a fair value for G is 12,500,000, which value is fairly constant for the various grades and tempers of steel within their elastic limits. The safe value for S is not so easily determined, because the correct value for any given class of springs is largely a matter of experience. The highest normal value of S varies from about 120,000 pounds per square inch for 1/16-inch wire, to 90,000 pounds for ⅝-inch wire, which includes the range of sizes generally used on valves. The term "normal value" is used to distinguish these figures from the higher values which can be reached by spring makers, and which are sometimes necessary, but should never be used for rapidly vibrating springs, or for springs where safety and long life are primary considerations, as in this class of springs. In fact, even the above normal values are far too high for gas engine springs. These values are used very generally on machinery springs, etc., but should be reduced very materially to obtain springs which will give the maximum of service in gas engine work. A value of S of from 25,000 to 30,000 pounds per square inch has been found to give best results for gas engine valves.

The third variable is the length of the spring, which should be as long as practicable in order to keep the pressure on the lever or cam which operates the valve from being higher than necessary at the extreme lift of the valve. To illustrate this point we will take a valve on which a pressure of 40 pounds when closed is desired, and which opens ½ inch. If the spring is under ½ inch compression when the valve is closed, and holding 40 pounds, the pressure when the valve is open will be 80 pounds. But if we use a spring under 1¼ inch compression to hold 40 pounds when the valve is closed, when the valve is opened the ½-inch travel, the pressure will be

* MACHINERY, May, 1908.

increased to only 56 pounds. The diameter and assembled length of the spring will usually be determined by the general design of the engine. The diameter should be as large as convenient, which will lessen the tendency to buckle.

We will now design a spring for an exhaust valve, the lift of the valve being ½ inch, the assembled length of the spring 6 inches, the pitch diameter of the spring 2 inches, and the value of S at extreme compression 25,000 pounds per square inch. We will make the spring 7¼ inches long, thus giving a total compression of 1¾ inch, and a final pressure of 56 pounds. The following formulas will be used:

$$P = \frac{11d^3S}{28\,D} \tag{1}$$

$$f_1 = \frac{22D^2S}{7Gd} \tag{2}$$

in which

P = pressure at given compression,
d = diameter of wire in inches,
D = pitch diameter of spring in inches,
f_1 = deflection of one coil in inches,
S = fiber stress in pounds per square inch,
G = torsional modulus of elasticity..

The common forms of the Formulas (1) and (2) are:

$$P = \frac{S\pi d^3}{16R} \tag{3}$$

$$f = \frac{32PR^2l}{G\pi d^4} \tag{4}$$

In these formulas P, d, S, and G denote the same quantities as in Formulas (1) and (2), and

R = pitch radius of spring in inches,
f = deflection of the whole spring under load,
l = full length of wire in spring.

The Formulas (3) and (4) can easily be transformed to the form in (1) and (2) by writing $\pi = 22/7$, $R = D/2$, and $l = \pi Dn$ (n being the number of coils in the spring).

We use Formula (1) to determine the size of the wire. Substituting the known values, we have:

$$56 = \frac{11d^3 \times 25{,}000}{28 \times 2}, \text{ or } d = 0.225.$$

We therefore will use No. 4 Washburn & Moen gage wire, which is 0.225. To determine the deflection per coil, we will substitute the known values in Formula (2), as follows:

$$f_1 = \frac{22 \times 4 \times 25{,}000}{7 \times 12{,}500{,}000 \times 0.225} = 0.112 \text{ inch.}$$

The free length of the spring is 7¼ inches, and the length with the valve open is 5½ inches; the compression therefore is 1¾ inch. Then 1¾ ÷ 0.112 (the compression per coil) gives 15¾ acting coils approximately, and adding one coil on each end, for a flat bearing to be ground at right angles to the axis of the spring, gives 17¾ total coils. Therefore the spring will be 2 inches pitch diameter, 7¼ inches free length, No. 4 W. & M. gage wire, 17¾ total coils, squared and ground ends, holding 40 pounds at 6 inches long, and 56 pounds at 5½ inches long, with a fiber stress at 5½ inches long of 25,000 pounds per square inch.

If it is desirable that the pressure, when the valve is open, rise *as little as possible* above 40 pounds, we must make the spring as long as possible and still compress to the closed length given. We will assume a spring 2 inches pitch diameter, to hold 40 pounds when 6 inches long, and as little over 40 pounds as possible at 5½ inches long. As we do not know the pressure at 5½ inches long, we will take the fiber stress 25,000 pounds at 6 inches long, instead of at total compression. Using Formula (1): $40 = \frac{11d^3 \times 25{,}000}{28 \times 2}$, or $d^3 = \frac{224}{27{,}500}$, and $d = 0.200$. We will therefore use No. 5 W. & M. gage wire, which is 0.207. Using Formula (2): $f_1 = \frac{22 \times 4 \times 25{,}000}{7 \times 12{,}500{,}000 \times 0.207} = 0.1215$ inch compression per coil when holding 40 pounds. Then 5½ inches solid length less twice 0.207 gives the length occupied by the acting coils when solid, or 5.086 inches, and 5.086 ÷ 0.207 = 24.5 acting coils. Further, 24.5 × 0.1215 = 2.975 inches compression, which added to 6 inches gives 8.975 inches free length of the spring, say 9 inches. The spring therefore compresses 3 inches when holding 40 pounds, with a value of S of 25,000 pounds and at 5½ inches long, being compressed 3½ inches, holds 46 2/3 pounds, with a value of S of $\frac{46\ 2/3}{40} \times 25{,}000$ or 29,166 2/3 pounds.

In these examples we have not corrected the values of S to allow for the variation in sizes of wire used, from the theoretical sizes obtained, as it is not necessary to do so in practice. It is interesting to note, however, the difference in this value at final compression, obtained by the above method of proportion based on 25,000 pounds at 40 pounds pressure, from that obtained by using the original formula with the final pressure at 46 2/3 pounds, and wire of 0.207 inch diameter. The first method gives 29,166 pounds, while the second method gives 26,782 pounds, this difference being caused by the difference of 0.007 in the size of wire.

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CHAPTER I

ORGANIZATION AND EQUIPMENT OF AN AUTOMOBILE FACTORY*

The Leland, Faulconer & Norton Co., of Detroit, Mich., was formed in 1890 for the purpose of building machine tools and special machinery. Special milling machines, a lathe center grinder, a wet tool grinder, and some special machinery were built. Later the manufacture of wood trimmers for pattern shop use was undertaken; and next, during the development period of the bicycle industry, a line of machinery for making hardened and ground bicycle gears was developed. As the bicycle business declined, the company began building gas engines for motor boats, which were then rapidly rising in popularity. The natural step from the marine to the automobile type of gas engine was made in 1901 to 1902, when the motor now used in the Cadillac car was produced. In 1905 the company was united with the automobile firm building the Cadillac car to form the present Cadillac Motor Car Co. From 1902 until March, 1909, about 21,000 cars had been turned out, 17,000 of which were single cylinder 10 H. P. machines, and the rest four cylinder cars, rated at 30 H. P.

The Plant and Its Organization

The main or Cadillac plant has a double siding connected with the Belt Line Railroad, thus giving ample shipping facilities. The factory buildings are of brick and reinforced concrete construction, lighted by large windows. Heat is supplied by a live steam system. The boiler-room contains three water tube boilers, with room for another if it is needed. Light and power are furnished by electric current supplied by the Detroit Edison Co. Electric driving is used throughout the plant, with motors connected with each line shaft, and occasional installations with direct connected tools. A large compressor furnishes air at 125 pounds pressure for the pneumatic hammers in the frame department, and for use in the various assembling departments, for cleaning parts, running air drills, etc. Five large elevators in fire-proofed brick shafts convey materials and parts between the various floors. An automatic sprinkler system is installed, supplied by four tanks on the roof. These tanks are filled by a large fire pump which operates whenever the level of water in the tanks is reduced. This same system supplies water for lavatory and wash-room use. There are two large wash-rooms, each having 600 bowls and 1,000 lockers.

The old Leland & Faulconer plant comprises a foundry building of brick, steel and glass, supplied with cupolas and a hydraulic jib crane; a pattern shop and pattern storage building; a brass foundry build-

* MACHINERY, March, 1909.

ing; a brick building for the case-hardening department; and a large three story brick building for the power plant and the sheet metal and brass working departments. The building is lighted by both gas and electricity, has a hot-air heating system, and is provided with large wash-rooms on each floor.

The organization of the plant is divided into the following departments: First, the general manager; second, the secretary; third, the sales department; fourth, the advertising department; fifth, the purchasing department; sixth, the time-keeping and cost-keeping department; seventh, the superintendent and his assistants; eighth, the engineering and designing departments, which produce the new models, tools and fixtures, and in conjunction with the experimental department, test the new cars before placing them on the market; ninth, the foremen and their assistants in the forty-four manufacturing departments; and six other special departments, some of which will be mentioned later. While the reader will be most interested in the departments devoted strictly to manufacturing, the work of the engineering and purchasing departments is worthy of some notice.

The designing-room is separate from the general drawing-room and is used by the chief engineer and two designers. Suggestions for new designs and improvements in old ones may be made by any one on suitable blanks. They are all considered and passed upon by a mechanical committee, consisting of the general manager, the chief engineer, and the two designers. When approved, such changes are made immediately on the tracings, and new blue-prints are made and sent to the departments concerned in producing those parts. This keeps the blue-prints up-to-date, and avoids loss in the carrying through of parts of obsolete design. A well-organized experimental department is provided, having the necessary apparatus for testing new designs. The work of the general drawing-room includes the detailing of new designs, and the drafting work on the necessary tools, gages, jigs and fixtures needed to produce new parts or models. Filing cabinets are provided for current drawings, as well as for those which are obsolete, of which a full record is kept.

The Purchasing Department, the Stock-rooms and the Gasoline Storage

The purchasing agent has final authority on all matters concerning the actual buying of material used in the cars, and the care of this material until it goes to the machine or assembling departments. Purchasing orders are made out in quadruplicate. One copy goes to the seller, one to the receiving office, one to the bookkeeping department, and one to the file in the purchasing office. Small commercial parts, such as nuts, rivets, etc., are stored in bins in the general stock-room, which also receives the finished and inspected parts turned out by the manufacturing departments. The stock-room record is kept on a card index system, and material is delivered by the stock-keeper only on presentation of a requisition from the foreman of the department where it is to be used. Bulky parts and materials are kept in a large warehouse, which is also under the care of the purchasing department. A separate stock-room is required for repair parts. These are kept in stock for all models, clear back to the first one placed on the market, and they are replaced as fast as sold out.

The gasoline used in testing the cars is also considered as stock, and a very carefully planned storage system is provided for it. Four cylindrical tanks of 15,000 gallons capacity each are buried in concrete near the siding, with the tops of the tanks about five feet below the street level. They are connected at top and bottom by separate cross piping. The system of storage is such that these tanks are always full of water or gasoline, or both, so that air is always excluded, making explosion impossible. The upper cross pipe permits the free passage of gasoline between the tanks, while the lower pipe performs the same function for the water. A suitable arrangement of automatic valves lets in water as fast as gasoline is removed, or permits the escape of water as gasoline is introduced.

A notable safety provision in the outlet piping for the water positively prevents the escape of gasoline into the sewer. The outlet pipe is formed into a long U-bend, which extends vertically to a depth of 70 feet, inside of an 18-inch casing. From this it returns and discharges through a trap into the sewer. The depth of this bend is such that the column of water on the outlet side will balance a column of gasoline having a height corresponding to the head obtainable from a tank car on a grade 5 feet higher than the present siding. The water thus furnishes a permanent seal against the discharge of gasoline.

The distribution of the gasoline is also carefully safe-guarded. It is supplied to the various testing rooms and to the factory garage through piping from the storage system. It is retailed by Bowser registering pumps which are kept locked when not in use. As a further safeguard, all the piping is enclosed in concrete, and the whole system is so arranged that it may be flooded with water to a depth of five feet in case of fire in any building which might later be built over it.

Tool and Tool Supply Departments

The tool department is located on the top floor and at the north side of the building, where the best light is obtainable. It is devoted to the manufacture of the jigs and fixtures and many of the gages employed in the factory. The equipment consists largely of Reed and Hendey & Norton lathes, Hendey shapers, Brown & Sharpe milling machines, and Brown & Sharpe universal and surface grinders. The high degree of interchangeability required in the product demands a high standard of workmanship in this department. At the time Fig. 1 was taken, some manufacturing was being done here. A wire enclosure at the right contains the tool inspecting department. The tool steel stock and tool grinding rooms are at the further end of the picture.

The tool supply department is closely allied with the tool-room. Its work is principally that of caring for, sharpening and recording the various jigs, fixtures and cutting tools. All these tools are looked out for by a card index system, which shows where they are used, and what repairs, if any, have been necessary. This department orders all the small commercial tools, and keeps a debit account with each branch tool-room for the supplies furnished it, giving credit for all tools worn out in legitimate use or broken in unavoidable accidents. A perpetual inventory is thus kept of all the special and commercial tools kept on hand. A card index inventory of the machine tools is kept in the purchasing department.

Forge, Foundry and Sheet Metal Departments

It will not be possible to more than briefly mention that part of the equipment of the forty-four manufacturing departments which is concerned with the actual work on the parts. The blacksmith shop is small, owing to the extensive use of drop forgings, but it is finely

Fig. 1. A Partial View of the Tool-making Department

fitted up with Buffalo down-draft forges, a tool forge with a coke magazine, gas furnaces, water jacketed dipping tanks, and an electric welding machine. The bulk of the work consists of tool dressing, and the making of forgings for jigs and fixtures and for special car equipment. The case-hardening department has ten large Frankfort gas furnaces equipped with pyrometers, connected by a switch board with a galvanometer graduated to degrees Fahrenheit. Oil and water dipping tanks with steam and cooling water jackets are provided. These are piped to a steam pump to give positive circulation. Square and oblong pots are used for small machine parts, while round pots with central holes, to insure uniform heat, are used for the large rear axle bevel gear.

The iron foundry is provided with a large and a small cupola. The latter is used largely for heats of a special nature. The most approved methods for testing and chemical analysis are employed to keep track of the output. This is necessitated by the fact that the foundry furnishes castings for other motor car builders besides the

Cadillac Company. The brass foundry furnishes the necessary castings for the bronze bushings, carburetor and lubricator parts, small valves and fittings, etc. These are finished in the brass machine shop, which is equipped with forty Warner & Swasey screw machines, besides several Fox lathes, drill presses, milling machines and several special lathes. All the lubricators, gasoline valves, carburetors and bearings used are produced here.

In the sheet metal department are made the vertical tubular radiators, gasoline tanks, dashes, fenders, etc., as well as small punchings, such as washers, clips, etc. The press-room has a complete equipment, ranging from foot presses up to 20-ton power presses, capable of cutting and forming parts up to 36 by 48 inches. Gas furnaces are used for heating the soldering irons and work when assembling the radiators. The radiators and tanks are tested by compressed air, while

Fig. 2. The Chassis Drilling and Milling Departments

submerged in water. The frame department is equipped with gas furnaces and pneumatic hammers for riveting and heading.

Equipment of the Machine Departments

For convenience in handling the work, all the engine parts are drilled and milled in two separate departments in one large room, while the similar operations on the chassis parts are performed in another room, which is shown in part in Fig. 2. The equipment of this department includes a large number of Cincinnati drill presses, Cincinnati and Brown & Sharpe milling machines, and a Beaman & Smith cylinder boring machine, arranged for handling transmission cases and axle housings. The engraving shows the large use of multiple spindle drills, quick change drill sockets and jigs.

The equipment of the motor drilling department is somewhat similar, ranging from a sensitive bench drill to a 24-spindle motor-driven Baush machine. This is used in drilling the 24 holes for studs, cap screws, etc., in the lower half of the motor frame. These holes are all drilled at one time, and have to accurately match similar holes in the upper half. This, it will be seen, requires a high grade of workmanship. The milling department for motor parts employs several Whitney hand millers, Brown & Sharpe horizontal millers of various sizes, several vertical machines of the same make, and six heavy motor-driven Cincinnati machines. There are also to be found here two milling machines built by Leland & Faulconer, which are unusual in that the table has longitudinal and cross feeds only, the vertical adjustment being applied to the spindle. High-speed steel inserted tooth cutters are in general use.

The screw machine department is one of the largest in the factory, occupying a floor space of 80 by 200 feet, and containing 62 machines, exclusive of the tool grinders. Brown & Sharpe, National, Acme, Davenport and Cleveland machines are used for making cap screws, nuts, studs and other parts up to one inch in diameter. Gridley machines are employed for larger work. Jones & Lamson flat turret lathes are used for shafts, spindles and some gear blanks. The Potter & Johnston automatic machine is employed for much of the chucking work in combination with the Gisholt and Steinle machines, which are used mostly for machining clutch and gear mounts. A group of Bardons & Oliver machines are used on certain engine parts, which have to be held in face-plate fixtures and finished largely by hand labor. The larger Acme machines are direct connected.

While most of the round parts are finished complete on the screw machine, a lathe department is necessary for some work which has to be turned on arbors. Fly-wheels and some long axle shafts are also finished here. The equipment includes Reed lathes, a Bullard boring machine for finishing fly-wheels, and two Beaman & Smith double-spindle horizontal boring machines for roughing out the cylinders. The latter are provided with turntable fixtures, so that two cylinders may be set up while two others are being bored. After the cylinders are roughed out, they are tested under hydraulic pressure and sent to the grinding department.

The grinding department finishes practically every round part on the car except the crank-shaft, which comes finished from a firm making a specialty of that work. Heavy Norton and Brown & Sharpe grinders are used for finishing long parts. Medium sized Landis and Brown & Sharpe grinders take care of work up to 3 inches in diameter and 8 inches long. Special Brown & Sharpe and Heald grinders are used for finishing the cylinders, which are held exactly as they will be on the assembled engine, so that clamping strains are duplicated. The pistons are finished in one of the heavy Norton machines. The group of Heald machines is used exclusively on internal work, and an equipment of face grinders finishes the washers and flat disks used in the cars. The square shafts which carry the sliding members of the transmission are ground to size on a group of Brown & Sharpe surface machines, fitted with suitable index fixtures. In contrast to the heavy Norton grinders with their 24-inch wheels, is a bench grinder purchased from the Waltham Watch Tool Co., for finishing internal ball races. This little machine uses a wheel about the size of a five-cent piece, and may be set to grind to a radius of 1/8 inch. Careful attention is given to providing suitable racks for ground work to avoid injury in handling.

In Fig. 3 is seen a partial view of the gear-cutting department. A Gleason bevel gear generating machine is here shown at work cutting a rear axle gear. The complete equipment includes thirty standard machines, and four others of special design, besides four testing machines. The list includes fifteen Brown & Sharpe automatic gear cutters, one large Gould & Eberhardt machine, two Fellows gear shapers for internal gears, one Bilgram and three Gleason bevel gear planers, two imported French machines for special pinion work, and a Pratt & Whitney worm milling machine. One of the testing machines, that

Fig. 3. A Corner of the Gear-cutting Room

for bevel gears, is seen at the extreme right of Fig. 3. The testing machine for spur gears is provided with a vernier scale for reading center distances to thousandths of an inch.

Inspection and Assembling Departments

The inspection department consists of a chief inspector and his foremen, and the men under them, who together form a corps of over one hundred men. These men inspect commercial parts as they go through the receiving department, the output of each manufacturing department as it goes to the assembling, the final assembling of the parts in the chassis, and the finish of the completed machine on both the mechanism and the body. The inspectors are furnished with all necessary appliances for doing this work accurately. Drop forgings are examined for visible flaws, and sounded for invisible ones. Springs are tested on machines especially built for the purpose. Every machine department has its inspection bench, provided with the necessary plug and snap gages for the entire range of its output. Micrometer calipers up to the 6-inch size are in general use. Thread micrometers are used in place of ring thread gages wherever possible. For testing turned and bored parts for concentricity, Brown & Sharpe testing centers and indicators are used. Suitable surface plates, V-blocks and height gages are provided. The inspectors in the grinding department are furnished with strong reading glasses for use on certain work. These inspectors are outside the jurisdiction of the other department-heads, and have full authority to throw out all parts and materials not up to the standard.

The work of assembling is divided between several gangs, each of which does it own particular work. One group of assemblers scrapes the crank-shaft bearings to fit, and "runs them in" by a belt on the fly-wheel. Another assembles the cam-shaft members. Still another assembles the piston, its rings, pins, connecting-rod and bearings,

Fig. 4. The Four-cylinder Engine Assembling Department

while the "cylinder gang" assembles the cylinder and cylinder head and copper water jacket. The final assembling is then done on stands as shown in Fig. 4. This consists merely in bolting the various parts together, setting the cam gears (which are marked in a jig), timing the valves, adjusting the bearings, and testing the water connections. The points of valve opening and closing are marked on the rim of the fly-wheel, and a fixed pointer shows the central position.

The Testing Department and Its Equipment

From the assembling room the engines are taken to the testing department, where they are placed on iron stands and connected with the gasoline and water supplies, and to the electrical connections for the ignition, as shown plainly in Fig. 5. The engines are run at moderate speed until they get down to work, when the speed is gradually brought up to the maximum. A brake-horsepower test of each engine is made, and those which fail to come up to the requirements are returned to the assembling department for reconstruction. As a check on this test, stock engines are sent to the experimental department at regular intervals, and tested there by connection with a dynamo fitted with suitable electrical measuring instruments. After the testing the engines go into stock, or to the chassis assembling department.

All the parts necessary for the completed chassis are brought to this assembling department. The order of assembling is as follows: The frames are first laid on horses and the mechanism dust shield is put on. The springs and axles are next attached, and then the engine and transmission gearing are set and lined up. The engine is supported at three points, and is connected by a universal sliding joint with the transmission gearing, thus permitting "weaving" of the frame without danger of disalignment. The universal joint between the transmission and the differential gearing is practically straight when the car is loaded, and runs at a very slight angle when the car is light. The

Fig. 5. Testing the Four-cylinder Engines

exhaust pipe and muffler are next connected, and then the controlling and brake levers and the pedals. The radiator and water connections come next, followed by the steering gear. The placing of the mahogany dash in position permits the mounting of the electrical apparatus; and the bolting on of the gasoline tank and its connections completes the chassis, except for the wheels and tires. An old set of these are put on the car in the assembling department, to be used for the road test. The method of assembling is practically the same for the single cylinder car.

Two separate testing departments are provided—one for the single-cylinder cars, and the other for the four-cylinder cars. The former were given road tests for the first two years of their manufacture, until all the weak points in the construction had been eliminated. The testing room shown in Fig. 6 was then built, and the cars have since been tested here. Fifteen stands are provided. The rear wheels rest on a pair of 48-inch pulleys, mounted on a shaft which carries a fan about 72 inches in diameter by 36 inches wide, projecting through the floor in the sheet iron casing shown. In addition to the resistance thus offered by the fan, a brake is mounted on the shaft between the pulleys, controlled by the hand-wheel on the stand shown projecting through the floor at the rear of each machine. By this means it is possible to work the engine against any desired resistance, even to the extent of stalling it. The chassis are held by padded hooks, fastened by ropes or chains to the brake wheel stands. The blast of air produced by each fan is led through a sheet metal conduit and directed against the radiator of the engine, thus giving the same cooling effect that would be experienced at corresponding speeds on the road on a still day. The speed in miles per hour is read from Schaffer & Budenburg tachometers.

Fig. 6. The Single-cylinder Chassis Testing Stand, arranged for Fan and Brake Resistance

The four-cylinder testing stands are similar in principle, though somewhat differently arranged, as the fans are placed beneath the front of the machine, being connected with the driving shafts by sprockets and chains. After being run here a sufficient time to make sure of their adjustment and running condition, temporary bodies are placed on the chassis and each car given a thorough test by reliable men on the country roads outside the city. After this has been done to the satisfaction of the foreman of the department, the testing body is removed and the chassis is washed successively in water and gasoline, and dried by an air jet.

Finishing

The painting and finishing of the chassis, bodies and wheels is done in separate departments. The bodies receive one coat of rough filler, and fifteen more coats of filler color and varnish, before completion. A view of the trimming department for the bodies is shown in Fig. 7. Fenders, hoods, brackets, etc., are enameled and baked. Fig. 8 shows some of the pipe frame trucks used to hold these sheet metal parts during the baking.

The chassis, bodies, hoods, fenders, etc., finally go to the large finishing-room on the ground floor, where the final assembling and test-

Fig. 7. The Body Trimming Department

ing of the complete car is done. Each complete car is driven out by a final inspector to make sure that all adjustments are correct. Before shipping, a detailed record is made of each car, beginning with the motor number, and giving the dates of motor assembling, motor

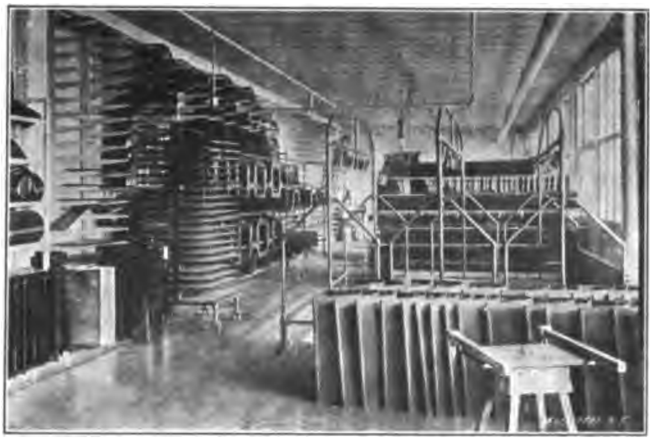

Fig. 8. Storage of Enamelled Parts, showing Wheeled Stands used in the Baking Ovens

testing, all the various painting, finishing and shipping dates, together with any information of a special kind, such as size and color of body, etc. This record has been found of the greatest assistance to the repair order department in filling poorly written orders.

Interchangeability

In connection with this subject of repair orders, mention should be made of the high degree of interchangeability attained by the Cadillac Co. This was illustrated by a test made in March, 1908, by a committee of the Royal Auto Club of England, who selected by lot three Cadillac cars of the same 10-horsepower model, disassembled them under the eyes of an inspector of their own appointment, placed the disassembled parts (721 from each car) in a pile, and mixed them up indiscriminately; 81 parts were then taken out and replaced by 81 repair parts from stock. The cars were thereupon reassembled from this mixed pile by the use of wrenches, screw-drivers, etc., but without the use of scrapers, files or even emery cloth. Only one part, a cotter pin, was injured in reassembling. These three heterogeneously reassembled cars were each given a 500-mile reliability run on the Brooklands track, at an average speed of 33 to 34 miles per hour, without developing the slightest defect.

CHAPTER II

MACHINES AND TOOLS FOR AUTOMOBILE MANUFACTURE*

The Cadillac Engine

In order to make clear the manufacturing operations which will be referred to in the following, a brief description of the Cadillac engine will here be given. The first automobile made by the Cadillac Motor Car Co., of Detroit, Mich., in 1902, was a runabout containing a 10

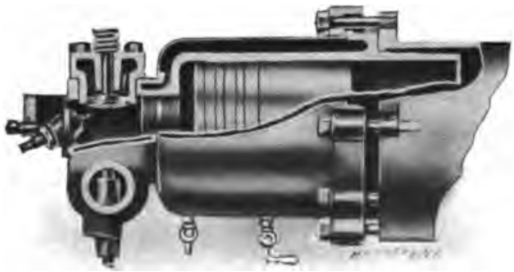

Fig. 9. Section through Cylinder showing the Water Jacket Construction

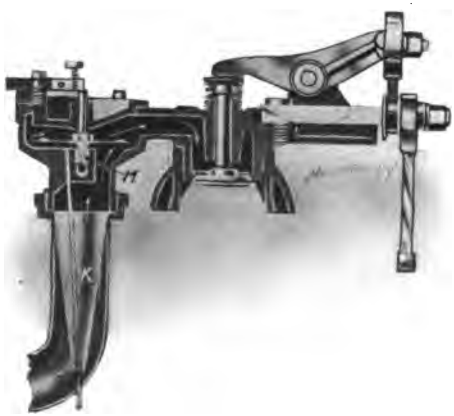

Fig. 10. The Cadillac Carburetor

H. P., single cylinder, four cycle, horizontal engine, of 5-inch bore by 5-inch stroke. This engine was found to be so satisfactory that it has been retained practically unchanged up to the present time, and its general features have been adopted, so far as possible, for

* MACHINERY, March and June, 1909.

the vertical four-cylinder engines of the 30 H. P. machine. A number of original features were employed on this engine which have proved their value in actual practice. One of the most interesting of these is the cylinder construction, best seen in Fig. 9. This cylinder, which is a fine-grained gray iron casting, has a flange near the forward end, which enters and fits a bored and faced seat in the frame. The copper water jacket slips over the cylinder, and is flanged to match

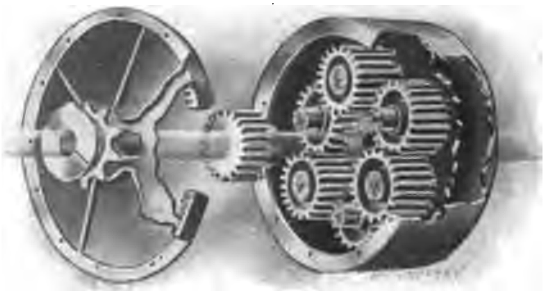

Fig. 11. Planetary Transmission used on the Single-cylinder Engine

its outer face. Both it and the cylinder are held in place by a ring which passes around the outside of the copper jacket, and is tightened down by the studs shown screwed into the frame. In this way the copper jacket forms its own gasket. The cylinder head or valve chamber is held in place by a hollow steel nut (or nipple, rather) which is threaded externally right- and left-hand, and screws into

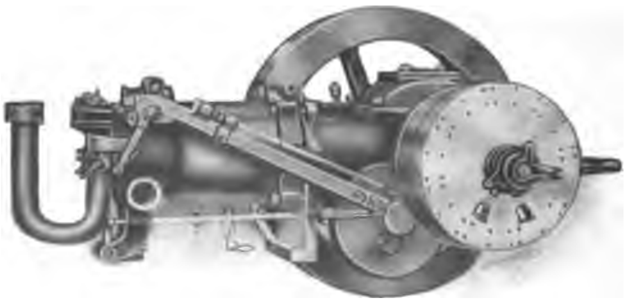

Fig. 12. The 10 H. P. Engine of the Single-cylinder Car

both the cylinder and the valve chamber. The upper end of the copper jacket is clamped between the two, and thus serves for a gasket at this joint also, forming the only packing needed. Parts are kept in alignment by a dowel, and suitable openings connect the jacket space of the cylinder and the head. Among the advantages of this construction over the usual cored jacket are lighter weight, greater water space, more uniform thickness of cylinder walls, facility in cleaning the jacket space, elimination of trouble from freezing the cooling water, and low repair cost for broken parts.

The exhaust valve is placed in the cylinder head with its axis vertical, and it is operated from the cam shaft by a push-rod and bell-crank. The inlet valve is of the inverted type, located directly above

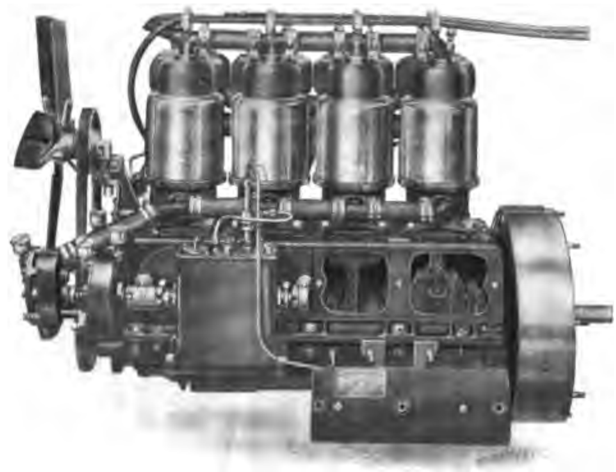

Fig. 13. Left Side of Cadillac 30-horsepower Engine

Fig. 14. Right Side of the Engine, showing Carburetor, Commutator, etc.

the exhaust valve. It is operated by a lever with a roller on its outer end which, in turn, is actuated by a push rod riding on a roller mounted on one arm of a short lever. The push-rod is connected with an eccentric on the cam shaft. The lever on which it rides is under the control of the driver, so that the timing of the valve and the amount of lift may be varied according to the work required. The throttling is thus effected by the inlet valve gear. The carburetor (shown in Fig. 10) is formed in one piece with the inlet valve mechanism. As may be seen, the inrush of air lifts valve *M* and allows the escape of the oil, which falls into the wire mesh basket *K*, where it is vaporized. The lift of the valve may be regulated to give the desired richness of mixture.

The motor frame is made in three parts—the frame proper, and the top and bottom plates. The main shaft, which is offset, is a nickel steel, center-crank forging, finished all over by grinding. It is carried in babbitt lined bronze bearings, fitted in bored and reamed seats in the motor frame. These are held in place by cap plates, which can be adjusted without opening the motor. The cam-shaft is

Fig. 15. Front View of the Four-cylinder Engine

carried in bronze bushings inserted in the bottom plate. This plate and the cam-shaft may be removed at any time without disturbing the crank-shaft.

The transmission of the 10 H. P. machine is of the planetary type, providing for two speeds forward and a slow reverse. As shown in Fig. 11, the gearing is all enclosed in an oil-tight casing. On the high-speed forward gear the whole transmission revolves as a unit. The driving pinion is of 40-point carbon steel and is case-hardened, as are also the idler pinions, which have bronze bushings pressed into them after hardening, and run on hardened and ground pins pressed into the gear case. Power is transmitted to the rear axle sprocket by a Whitney roller chain. An assembled view of the engine is shown in Fig. 12.

The later vertical four-cylinder engine for the 30 H. P. machine is shown in Figs. 13 to 19 inclusive. This engine has been built, as far as practicable, on the lines of the horizontal machine. As may be seen in Fig. 17, the same arrangement is used for clamping together the cylinder, the copper jacket and the cylinder head, although a somewhat different joint is used at the lower end of the jacket. In this engine also the crank-shaft is offset; the construction of the crank

Fig. 16. The Cadillac Steering Gear

Fig. 17. The Cylinder and Piston

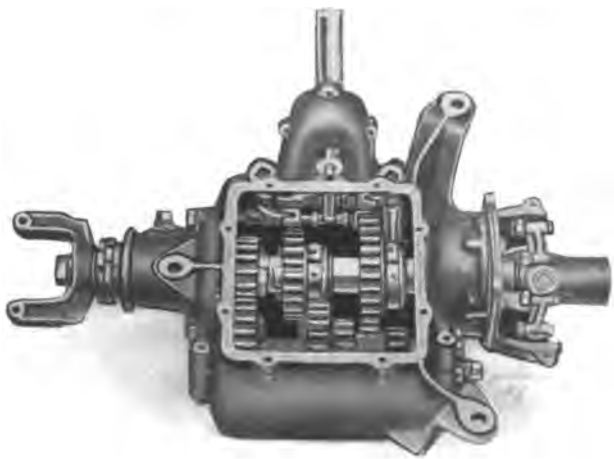

Fig. 18. Selective Type Sliding Gear Transmission

case and base is different, of course, as shown in Fig. 19. A leather-faced cone clutch in the fly-wheel transmits power to the sliding gear transmission (see Fig. 18) which gives three speeds forward and one reverse. The gears and shafts are of oil-treated chrome-nickel steel, and are carried on ball bearings. The gear case is oil-tight, as is also the universal joint housing and the rear axle casing.

The rear axle carries an oil-treated chrome-nickel steel bevel gear and pinion, and the gear mounts are adjustable for wear of the teeth. The steering gear (see Fig. 16) is of the worm and sector type, treated in the same ways as the transmission and differential gearing.

Machines and Tools for Automobile Manufacture

Upon first thought the design and construction of tools and jigs for automobile manufacture may not appear to present any problems radically different from those involved in the manufacture of any other power producing and transmitting machinery; but after a thorough consideration of the conditions under which a motor car necessarily operates, the importance of a standardized, interchangeable, simple and strong construction is realized. As one of the requirements of a car is maximum power with minimum weight, the use of nickel and other steel alloys is required, which, in turn, necessitates the use of high-speed steel in the machine tools. As an automobile engine is

Fig. 19. Top View of Motor Case and Crank-shaft

necessarily a high-speed engine, the provisions for adjustment of wearing parts and the cheap replacement of them when worn out, are of primary importance.

As the great majority of automobile owners are not mechanically inclined and wish the greatest amount of service with the least possible attention to their cars, the necessity of simple and reliable construction is apparent; and, as the motor car is forced by road conditions to do its hardest work on the poorest roads (which are usually farthest from the best repair facilities), under which conditions breakages are most likely to occur, the advantages of interchangeable construction, the parts of which are so designed that they can *not* be incorrectly assembled, are apparent, especially when road repairs must be made by men not thoroughly familiar with the construction of all cars. These are facts that the motor car designer must have seriously in mind, and which must reflect themselves to some extent in the tool design.

It is the purpose of this chapter to show how these ideas are carried out in practice, in the factory of the Cadillac Motor Car Company, and, while space permits showing only a few of the several thousand special tools, jigs and fixtures, it is thought that those shown will illustrate the care taken to secure absolute interchangeability and perfect alignment of parts. As the construction of the motor includes some very interesting tools, these together with some testings jigs are shown and described.

Engine Frames

As the engine frame is in two parts, divided horizontally at the shaft center, accurate milling and drilling is required. Heavy Brown

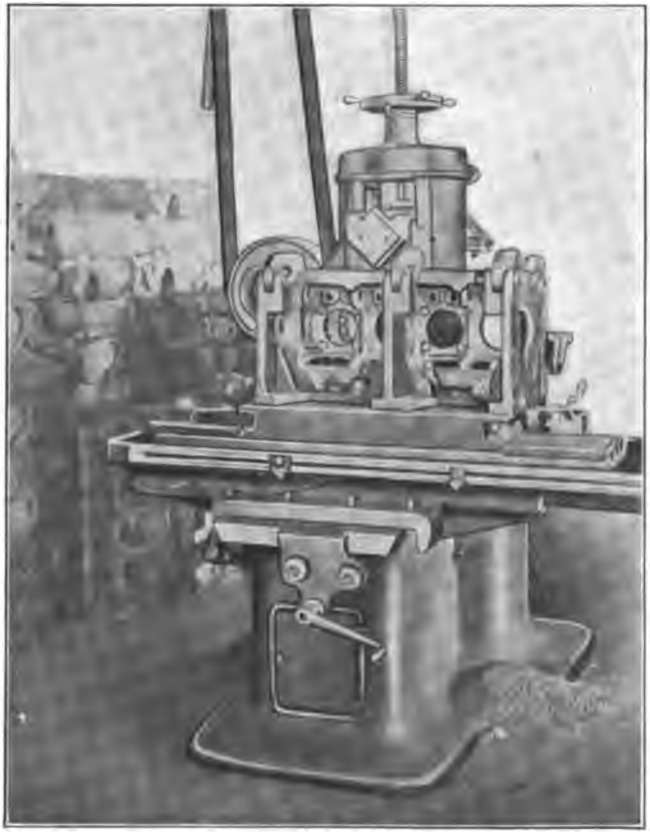

Fig. 20. Milling Engine Frames

& Sharpe, Cincinnati, and Leland & Faulconer machines are fitted with heavy jigs, and large inserted tooth cutters are used on this work. Fig. 20 illustrates the L. & F. machine milling the top face of the engine frame where the cylinders bolt on. This machine is very satisfactory for manufacturing, as the low table permits rapid handling of work, and its heavy construction, large bearing surfaces and all geared feeds and speeds provide for heavy and rapid cutting.

Fig. 21 shows the method of boring the seats for the cylinders in the engine frame. This operation follows that shown in Fig. 20. The cutter heads have a floating drive and are centered by the ground pilots entering inserted bushings in the jig bosses. The whole jig slides forward, and back against a stop to facilitate inserting and removing the work.

Fig. 22 shows the lower half of the crank-case (shown in Fig. 19 with crank-shaft in place) clamped in the jig for drilling 24 holes for

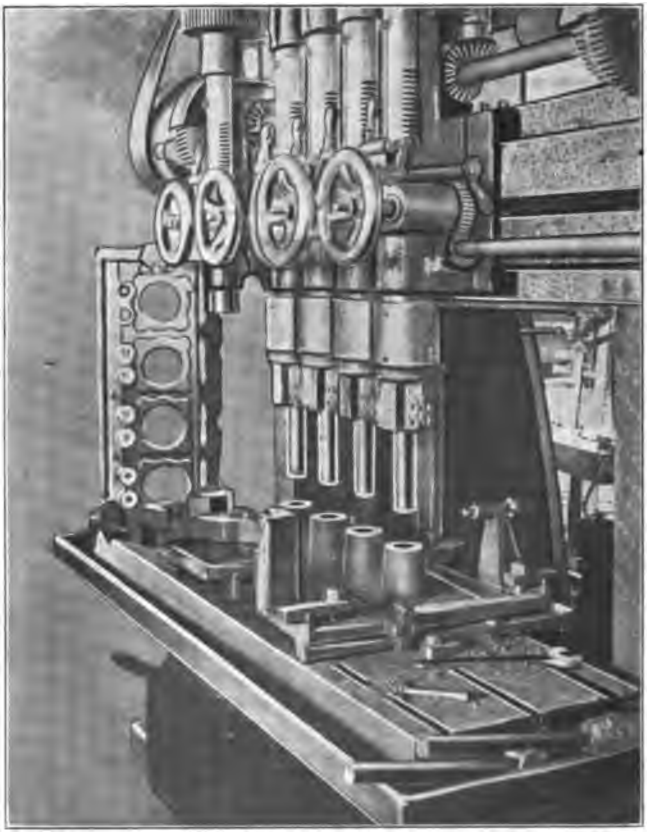

Fig. 21. Machine for Boring Frames

studs and cap-screws. The 24-spindle Baush machine drills these holes in about two minutes, including inserting and removing the work. A similar style of jig is provided for the upper half of the crank-case, which has 18 holes to be drilled in the lower face.

Fig. 23 shows the jig provided for boring the cam-shaft bearing seats in the upper half of the crank-case. These seats are indicated by the letter *A*, and are a very close fit for the five bronze bearings which carry the cam-shaft. The work locates over the two large bosses in the center of the jig, and rests on hardened and ground plugs inserted in the base. The swing clamps shown bear directly over the plugs. The boring tool, which is driven by a face-plate fixture, is seen projecting through one of the guides. The B. & S. plug gage seen on the lathe carriage, allows only 0.002-inch variation in the size of the holes. A similar type of jig (not shown) is used for boring the main bearing seats in the lower half of the crank-case, and

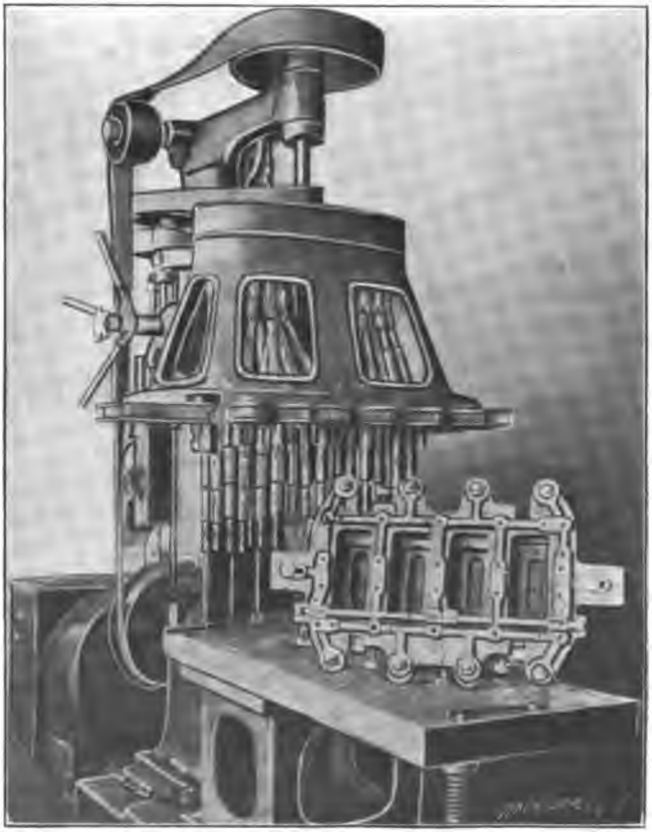

Fig. 22. Twenty-four-spindle Machine for Drilling the Frames

an adjustable hand reamer with a very long pilot is used for finishing them. The variation in size allowed on the bearing bushings is only 0.0015 inch and only 0.001 inch on the shaft bearings.

Cam-shaft

Fig. 24 shows both the cam-shaft drilling and reaming jigs on the same machine table, for convenience. The drill jig (seen in front) is of steel with hardened bushings with an adjustable stop-screw in the end. This jig gives the correct position of the holes for the eight cams and the drive geams. As the holes are to be reamed in pairs and each pair is 90 degrees from the others, the reaming jig is designed with a view to extreme accuracy. In operation the first hole reamed

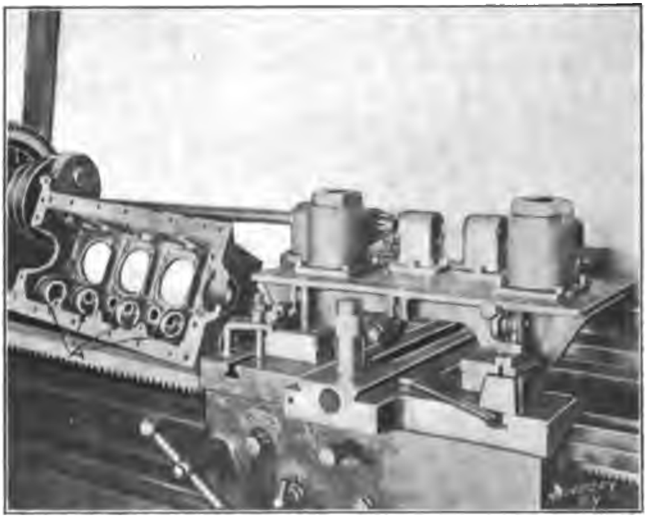

Fig 23. Fixture for Boring Cam-shaft Bearings in Engine Frame

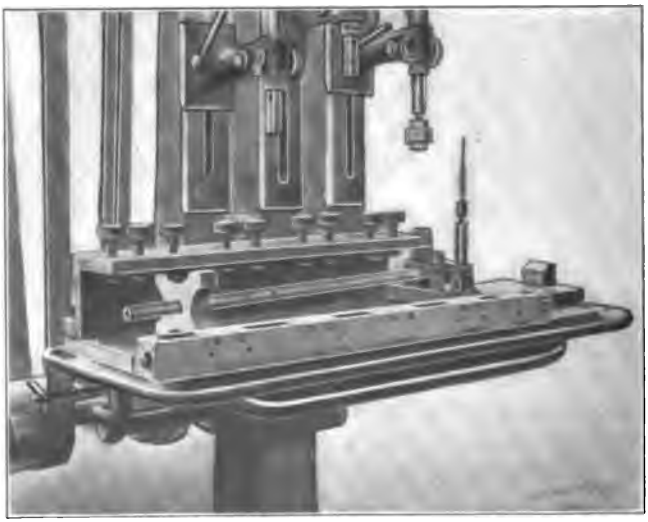

Fig. 24. Fixtures for Drilling and Reaming Cam-shafts

is the one by which the drive gear (Fig. 15, page 18) is pinned on. The taper reamer is guided by the bushing in the clamping fixture at the right, and the collars are so adjusted as to ream the hole to the required size. The shaft is then slipped through the square, hardened and-ground steel block seen at the left in the illustration, and a master pin is inserted. The block is then slipped along in the frame of the jig and clamped by the screws seen on top of the fixture as the various holes come under the reamer. The projecting block seen at the extreme right end of the jig, forms a rest for the cam-shaft as it is passed along. As the taper holes in the cam shaft, cams and cam-gears, must bear the correct relation to each other, a set of master pins is provided for testing the depth of the reaming. These are hardened and ground tool-steel pins having two fine lines 0.020 inch apart around them at the point where they project through the hole in either the shaft, the cam or the gear. As a variation of 0.001 inch

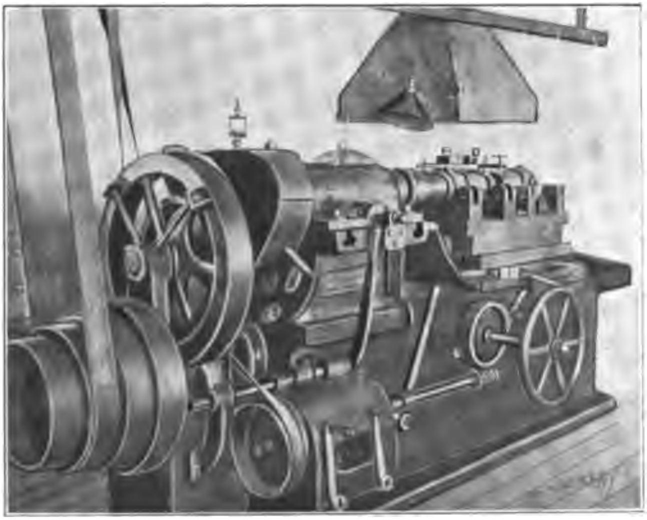

Fig. 25. Boring Cylinders

in the diameter of a standard taper pin hole permits the pin to enter 0.040 inch deeper into the hole, the accuracy of this work can be realized when it is known that no hand reaming is required in assembling the cam-shaft. The cams are drilled and reamed in similar jigs, which, in all cases, locate the cams by the eccentric portions. The inlet cams are alike and interchangeable, as are also the exhaust cams. The cams are of selected steel, hardened and finished by grinding on the working surfaces in correct relation to the pin holes.

Cylinders

Fig. 25 illustrates the method of boring the cylinders in a double spindle Beaman & Smith machine, with a turn-table fixture whereby two cylinders may be changed while two others are being bored. As the cylinder castings are very uniform in size, the boring leaves the walls very uniform in thickness. After being bored and reamed, the cylinders pass to the testing bench where water pressure of 700 to

800 pounds per square inch is applied to test them for leakage. Those passing the test are taken to the screw machine department and put on an expanding arbor in a large Potter & Johnston machine for facing and tapping the top and turning the portion of the cylinder which enters into the crank-case of the motor. The machine and tools for these operations are seen in Fig. 26. The turret tools in the foreground are those used in roughing out and boring the upper end of the cylinder for the cylinder head nipple. The heavy overhanging turret tool finishes the flange on the cylinder for the copper water jacket. The rear cross slide carries the tools for roughing this flange and also the flanges through which the studs pass for fastening the cylinders to the engine frame, while the forward cross-slide tools fin-

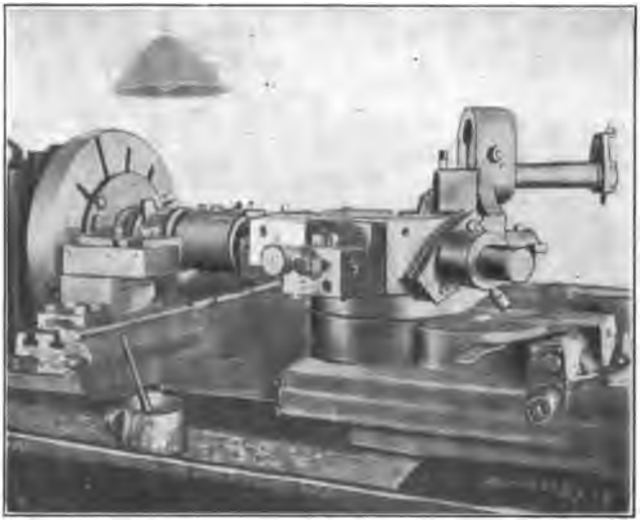

Fig. 26. Turning Cylinders

ish the stud flanges and a portion of the cylinder where it enters the bored seat in the engine frame.

The cylinders are finished by grinding in Brown & Sharpe and Heald machines. A heavy angle-plate fixture, bored and faced to a very close fit on the cylinder diameter, is fitted to the table of the machine as shown in Fig. 27. The cylinder is clamped to this fixture exactly as it is held later in the assembled motor. Cooling water is supplied to the outside of the cylinder, and the air tube seen at the extreme right conveys the particles of metal and emery to a suction fan at the rear of the machine. The "Go" plug gage seen on the machine table, is 4 inches in diameter and the "Not Go" gage is 4.002 inches in diameter.

Pistons and Rings

The second operation of roughing off the pistons in a Gridley automatic turret lathe is shown in Fig. 28. The first operation is not shown, as it consists only in chucking and roughing off the outer diameter of the head end for about an inch to permit the steadying roll passing over the end. The upper roll has but a slight travel, as it

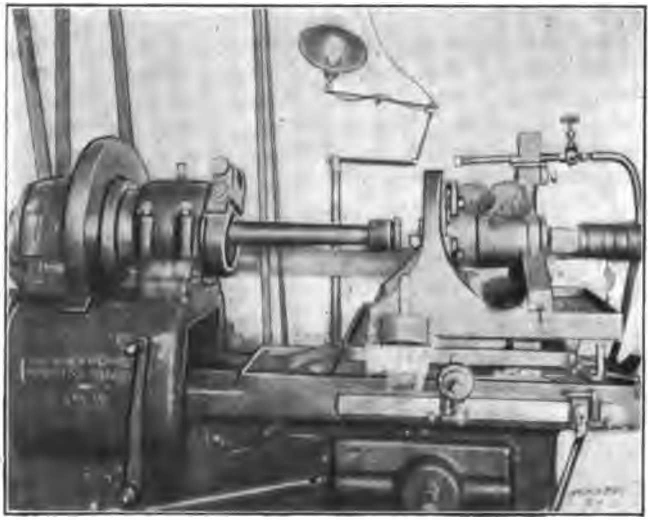

Fig. 27. Grinding Cylinders

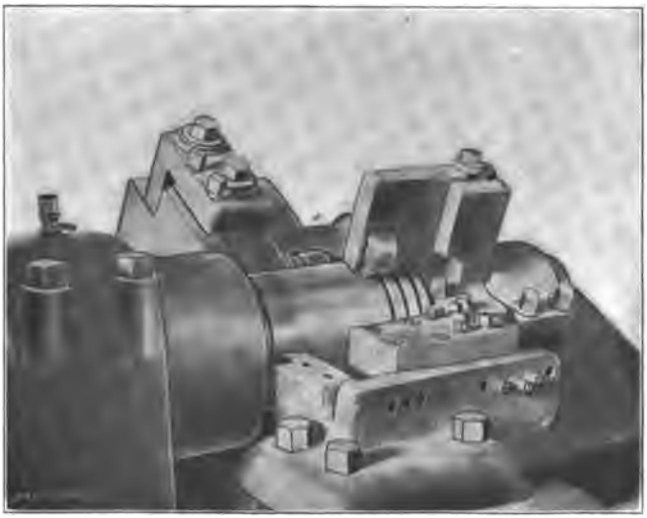

Fig. 28. Turning Pistons

forms a part of the end facing tool. The heavy turning tool is carried in the rear tool holder, which also carries another roller; this roller supports the piston against the side thrust on it, caused in cutting the ring grooves. The view shows the very heavy character of the tools, and the provisions for adjustment. The piston is held by an internal draw-in fixture, thus permitting the turning tool to travel its entire length. The finish is by grinding in heavy Brown & Sharpe and Norton machines, as illustrated in Fig. 29. The greatest variation in size permitted is 0.002 inch. A finishing cut is taken from the open end of the piston in a special reaming fixture just before grinding, which prevents any possible distortion of the piston due to changes in the metal after the open end has been machined. The piston pin hole is bored in box jigs and 0.001 inch is left for hand reaming previous to assembling the piston and connecting-rod. A final light finishing cut is taken from the piston ring grooves after the piston is ground.

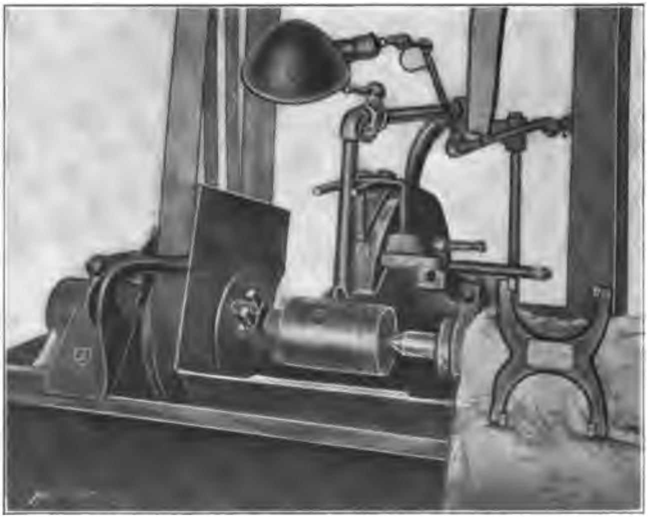

Fig. 29. Grinding Pistons

The piston rings are of a special close-grained iron mixture and are turned and bored on Gridley machines, and finished by grinding. The ring joint is the standard 45-degree angle joint, which has always given good results in practice.

Connecting-rods

The connecting-rods are drop forgings of H-section, having a pressed-in bronze bushing bearing for the piston-pin, and a hinged cap carrying babbitt-lined bronze half-bushing bearings for the crank-pins. While the machining of the rods requires a set of very complete and accurate jigs and tools, limited space prevents their illustration. Two of the fixtures for testing the alignment of the assembled rods, however, are shown in Figs. 30 and 31. Fig. 30 shows the method of locating the piston-pin bushing central with the crank-pin bearing, which is held in the hinged end of the rod by large brass dowels. A plug is placed between the half bearings. and the adjusting screw tightened down sufficiently to hold them tightly in place. The piston-pin bushing having been pressed in approximately central and hand reamed, is then slipped on the ground arbor which is pressed into the casting and positively held by a large hexagon nut. The knurled nut *A* is then screwed on the outer end of the arbor, thus holding the piston-pin bushing against a ground shoulder on the fixed arbor. The micrometer screw is then brought up until it touches the edge of the crank-pin bearing, a reading taken, and the screw backed away. The nut *A* is then loosened, the connecting-rod slipped off, turned over and replaced on the arbor and another reading of the micrometer screw is taken. The difference in the two readings thus indicates the amount the two bearings are out of line with each other. For

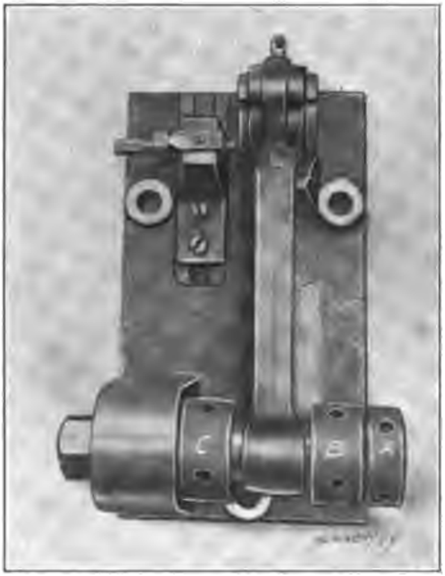

Fig. 30. Fixture for Testing the Relative Lateral Positions of Connecting-rod Bearings

overcoming this variation, the two knurled nuts *B* and *C* are provided. Nut *B* is internally threaded to fit a threaded portion of nut *A*, and in use screws up against the face of the connecting-rod forging for pressing it farther onto the bronze bushing. Nut *C* which is internally threaded to fit a portion of the fixed arbor, operates to move the rod forging in the opposite direction. When the rod is thus centralized, a dowel of brass tubing is put in, which prevents disalignment and also conveys oil to the piston-pin bearing.

For testing the parallelism (both vertical and horizontal) of the rod bearings, the fixture shown in Fig. 31 is provided. In operation, two ground arbors which are tight-fits in the rod bearings, are inserted, and the rod laid in the fixture as shown. A pair of flat springs *A* press the smaller arbor against the inserted hardened and ground plugs opposite them. A similar pair of plugs are seen at the other end of the fixture; between these and the arbor is inserted the taper strip seen in the foreground. The taper is such that the cross lines which are about ⅛ inch apart each give a reading to 0.001 inch. The two flat strips attached to the lower end of the fixture are so placed for convenience in reading any variation in the position of the taper strip. As all four horizontal surfaces on which the ends of both the

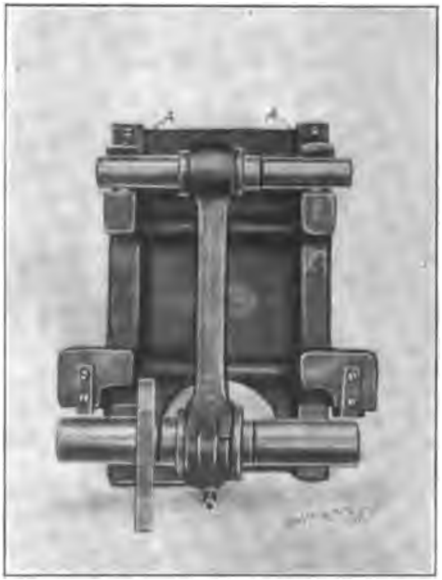

Fig. 31. Fixture for Testing the Parallelism of Connecting-rod Bearings

arbors lie are ground to the same plane, any "wind" in the connecting-rod is seen by the failure of all four points to touch at the same time.

Bevel Gear Templet Milling Machine

A pair of bevel gears are used to drive the short vertical commutator shaft from the cam shaft of the motor, and as the relative positions of the commutator to the cam-shaft and main shaft of the motor must be accurately maintained, the necessity of correctly cut and carefully mounted gears is apparent. For producing these gears, a specially designed machine is employed, which is shown in Fig. 32. The machine is one of the templet type, whose templet or form (seen on the arm at the top of the machine) is primarily developed by rolling contact with a rack. This produces a magnified tooth form which is mathematically correct, and even if it contained any errors these would be reduced in the actual work in the same proportion which the gear tooth bears to the form. Hence, very accurate bevel gears may be cut on this type of machine, and a brief general description of its main features may be of especial interest.

The machine consists of two principal parts: the work spindle and its driving and indexing mechanism, and the cutters with their driving mechanism. The cutters are driven by round belts, at a high speed, and are mounted on geared spindles which are carried in two vertical slides, which, in operation, have a reciprocating motion on

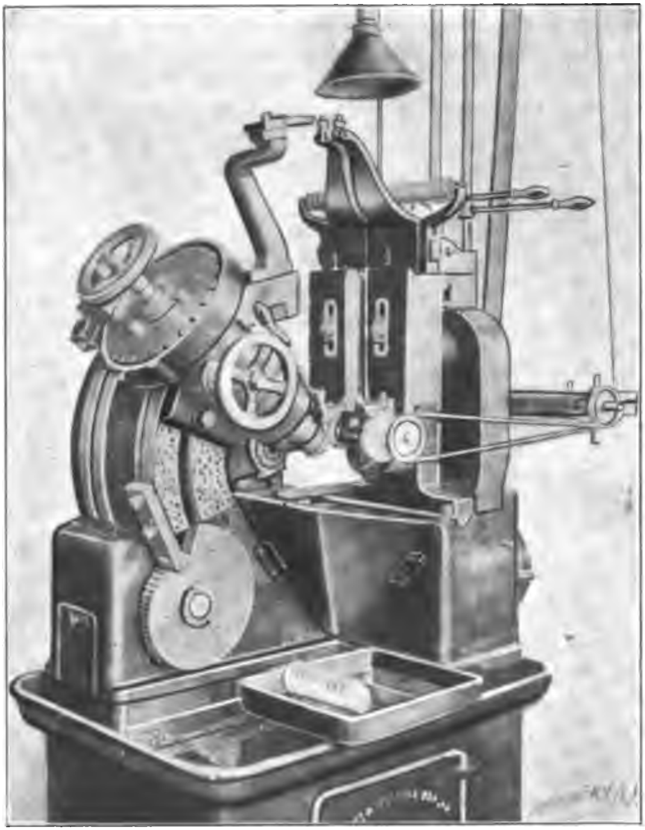

Fig. 32. Bevel-gear Milling Machine of the Templet Type

lines divergent from the cone center of the gear to be cut. The cutting edges of the cutters are thus always traveling along lines which become the clearance lines of the gear tooth. The gear blank is roughed out on a special gashing machine as the templet milling machine is not intended for roughing.

The work spindle is carried in the head, which has a working range of 75 degrees between the horizontal and vertical planes. This head is locked to the movable graduated quadrant, which is pivoted at a point coincident with the center of the gear. The work spindle has an end movement of several inches, for convenience in changing the gear blank, and has a draw-in arbor attached to the hand-wheel seen above the index plate, for locking the gear blank in position. The index plate is seen at the top of the work spindle. The index trip is set at the desired position on the rear slot of the stationary quadrant. In operation the large cam under the work spindle raises the pivoted quadrant to which the work spindle is locked, and gradually feeds the work forward between the two cutters, which are gradually forced to change their position by the action of the large tooth form entering between the two rolls on the cutter slide arms.

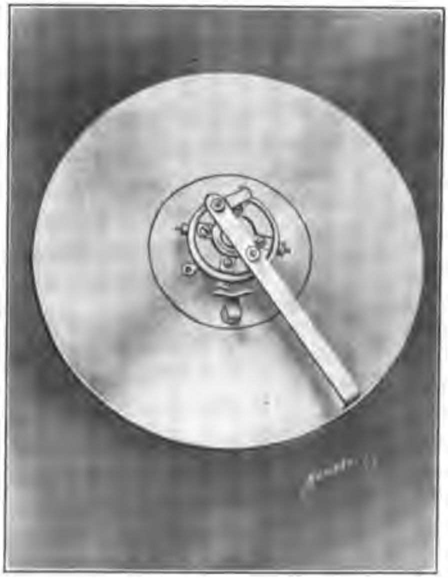

Fig. 33. Fixture for Testing the Accuracy of Commutator Contact Points

The indexing is, of course, automatic, and occurs at the position of the cam shown in the engraving. This cam has, as shown, an edge consisting of a series of small steps, rather than a gradual curve, and is so geared to the cutter spindle mechanism that the work is fed into the cutters at the ends of the stroke of the cutter slides, rather than during a cut. The index mechanism shows careful thought in its design, in that the index pin enters the slots in the index plate in such a manner as to have no sliding contact on the master edge of the slot. An automatic trip stops the machine when the gear is finished. This machine is one of a series which was built by this company (then the Leland & Faulconer Manufacturing Company) in 1898-1899, for producing either soft or hardened and ground bevel gears, the machine being designed to produce finished soft gears, or semi-finished gears for hardening.

Commutator Testing

Fig. 33 shows a fixture employed for testing the accuracy of the spacing of the contact points of the commutator. This fixture consists of a central portion carrying the commutator shaft, and of an outer graduated steel disk movable on the central part of the fixture. In operation, a commutator is slipped on over the stationary shaft and the bearings adjusted. The commutator brush is then placed on the shaft and locked in place, leaving the commutator body free to be revolved. A battery and coil which are a part of the fixture, indicate the electrical contact by the buzzing of the coil. The pointer is then put in place and clamped, and the commutator turned until a contact is indicated. The large outer disk (about 18 inches in dia-

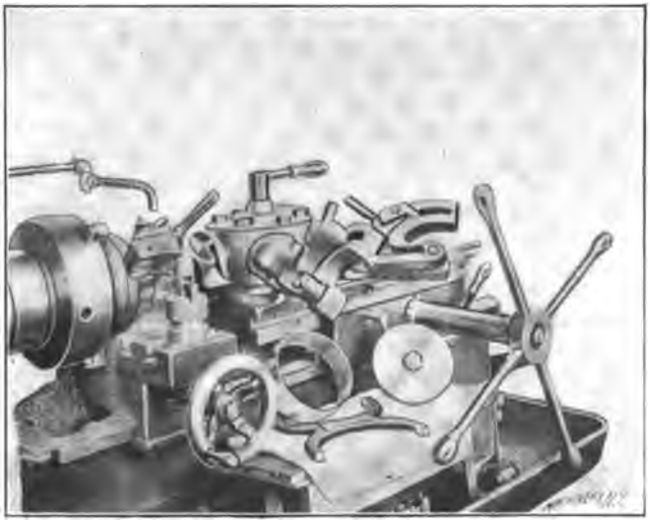

Fig. 34. Turning a Spherical Ring

eter) is then turned around under the pointer until one of the 90 degree graduations are directly under the pointer. The commutator and pointer are then turned to bring the other contacts to the brush, and their variation read on the large disk, which is graduated in degrees at four equi-distant points around its edge. The requirement is that the commutator contacts be spaced 90 degrees apart, and the variation allowed is only one-half a degree, as the relation of the firing to the piston and valve movements must be very exact.

Fig. 34 illustrates a nice piece of screw machine work in the brass shop. The ring seen leaning against the machine is of bronze. The diameters of these rings range from 6.497 inches to 6.500 inches and the bore from 5.878 inches to 5.880 inches. The outside is spherical in shape, and the ring forms a part of the rear universal joint housing which swivels on the rear axle driving shaft casing, and also slides

in to compensate for the rear spring action. Slight variations in size and a fine finish are necessary to make this point oil tight. A casting is seen in the machine, and a roughing cut is being taken from the outside. It has already been rough bored, enough metal being left for a fine finishing cut to be taken after the outside is finished. The castings have heavy flanges for inside chucking, so that little trouble is experienced by their springing after being parted. The illustration shows the construction of the spherical turning tools, and two of the gages used.

CHAPTER III

SYSTEM FOR THE RAPID ASSEMBLY OF MOTOR CARS*

From a mere corner in the machine shop in the days when the automobile was built in lots of but two or three at a time, the assembling room has grown to such an extent that, in many factories where the output is large, it occupies an entire floor of the main building, and has come to be considered as one of the three or four most important departments of a modern motor car factory. A corresponding increase in responsibility has attended the growth in size and importance of the assembling room, and to-day, unless well managed and equipped with the most up-to-date devices for the convenient and rapid handling of parts, it can easily "eat up" the profits on a whole year's output of low or medium-priced cars. Without requiring the services of an excessive number of men, it must take care of the parts from the machine shop and the parts-assembling room as they are turned out, and not allow a great number of finished pieces to accumulate at any time in the stock room. The work of assembling must also be done thoroughly, so that, when tested, the complete car need not be sent back for overhauling and readjustment of parts. In short, the assembling room must work in harmony with each of the other departments in doing its share toward producing a car of maximum quality at minimum cost of production—and that share is by no means small. But not alone are the best systems and business management, proper interior arrangement and most up-to-date devices necessary, but the highest class of skilled mechanics must be employed as well. A motor and transmission may be composed of the best of materials and have bestowed upon them the most skilled workmanship available, but unless they are placed together in the completed car with each shaft lined up, each bearing scraped and fitted and each gear in position to mesh properly, all this expensive material and labor may count for naught. The assembling room cannot, to any great extent, compensate for poor machining, but it *can* absolutely ruin the best products of the machine shop.

That the leading automobile manufacturers have been brought to a realization of the importance of the use of the best systems, equipment and labor in their assembling rooms is particularly well exemplified in the factory of the Chalmers-Detroit Motor Car Company at Detroit, Mich. Probably the most convincing proof of this statement will be found in the fact that, for the 3,000 complete cars turned out by this company last year, not more than 30 men were employed at any one time on the assembling room floor. More remarkable than

* MACHINERY, October, 1909.

this, however, is the high record established for a day's work. In ten hours, the 30 men in this department assembled 35 complete cars! Of course this does not include the assembling of the small parts of the motor, transmission and rear axle, as these are taken care of in other departments, but when it is remembered that the chassis assembly *does* include the installation of all these parts in the frame, the adjustment of each to its new position, the attaching of all springs, wheels, running-boards, foot-rests, steering gear, and the wiring and piping of the motor, it will be realized that the system and equipment employed in this department must be perfect in every respect, in order to turn out this amount of completed work.

The headquarters of the assembling department may be said to lie in the finished stock room, which occupies a large section of the floor of the main factory on which the assembling room proper is located. To this finished stock room come all finished parts such as nuts, bolts, screws, front axles, springs, and wheels, and the previously assembled motors, transmissions, steering gears, and rear axles. These are all classified and placed by themselves, the smaller parts being kept in bins which extend in long rows down one end of the room. Lists pasted in conspicuous places along these bins show the exact number of each size and kind of bolts, nuts and other pieces required for the various models of cars made here, and hand trucks having bodies divided into compartments are drawn down past the bins and filled with the necessary number of small parts for two cars. In the larger divisions of the truck box or body are placed the axles, steering gear, running boards, foot rests, and other bulky parts of the car. Each truck is filled with a sufficient number of the proper parts for the complete assembly of two cars and is then rolled into the assembling room, adjoining the stock room, and placed between two pressed steel frames which form the foundations, as it were, of the two chassis to be assembled. Having received the required number of parts of the proper kind, three men now devote their entire time to assembling the two chassis—and it is here that the advantages of "team work" are exhibited. Having become accustomed to this method of assembling, each man knows just what he is to do, and always has the other chassis at hand to which he can turn his attention when he is liable to interfere with the work of his two companions. It is highly specialized work, each team of three men devoting their whole time and energy to the installation and adjustment of the various parts of two cars until they are ready for the road test. As the three men finish the first two chassis, another truck is brought in containing parts for two additional cars, and the team then devotes its attention to cars three and four. The motors are not included in the quota of parts comprising the truck load, but are carried in separately by differential hoists which travel on overhead tracks and pass in two lines down the sides of the assembling room in front of the two rows of chassis. When the frame is ready for the installation of its motor, the latter is lowered in place. This system renders each car independent of the stock room after the truck load of parts has been received,

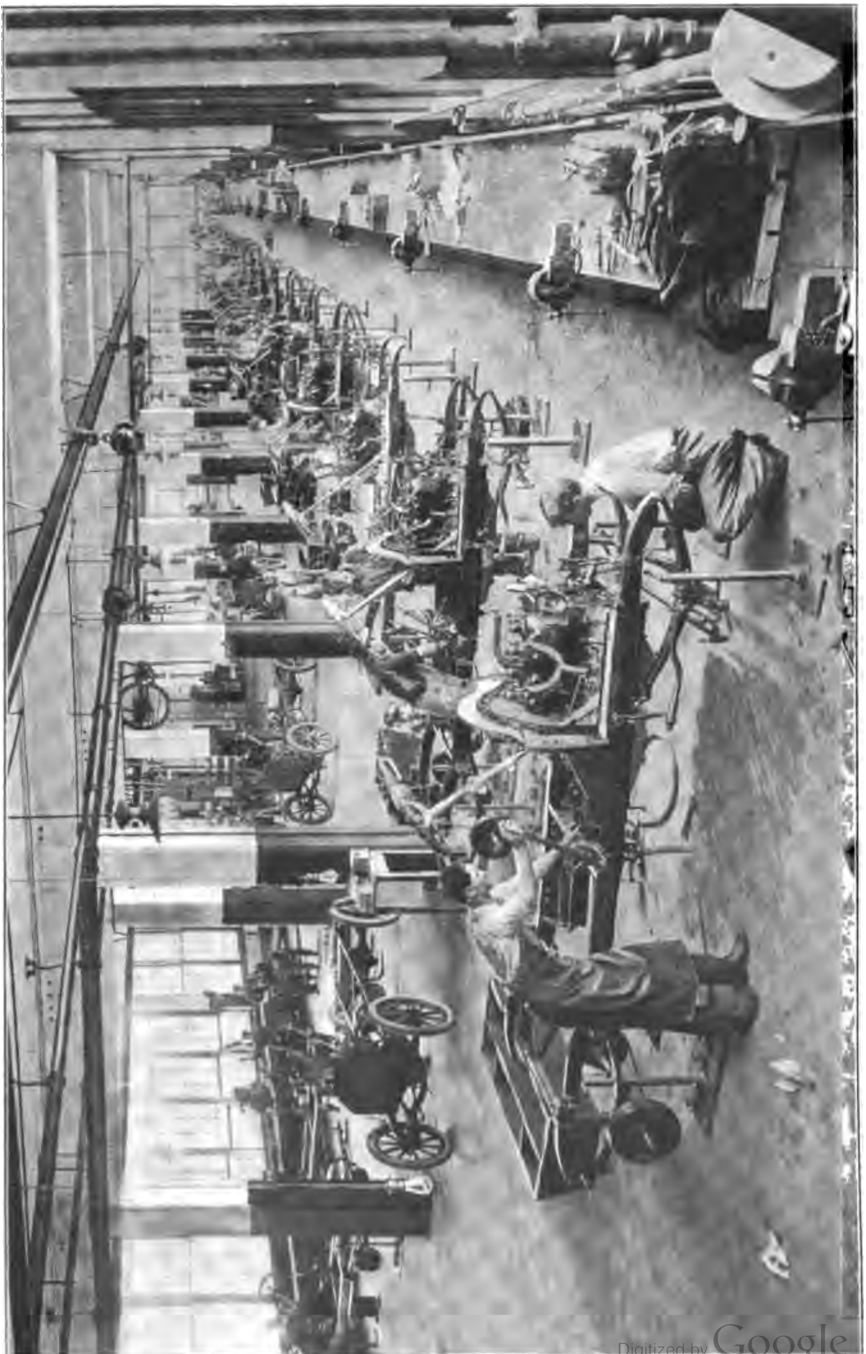

Fig. 35. View of the Assembling Room, showing Arrangement of Overhead Track and Differential Hoists, the Trucks, each of which holds the Parts for Two Cars and the Adjustable Frame Supports

and the work bench, vise and kit of tools near every chassis reduce to a minimum the number of steps necessary to be taken by each workman.

The arrangement of the rests for holding the frames rigidly in place is very ingenious and entirely does away with the use of saw-horses or other movable and bulky supports. There are four of these supports for each frame, as shown in Fig. 35, and when not in use, one or all may be let down into the floor. Each of these supports consists merely of a vertical iron rod, bent at right angles at its upper end and forged into the shape of a hook. A corner of the frame rests on this horizontal portion of the rod, while the hooked-shaped ends of the two opposite supports prevent lateral motion in either direction.

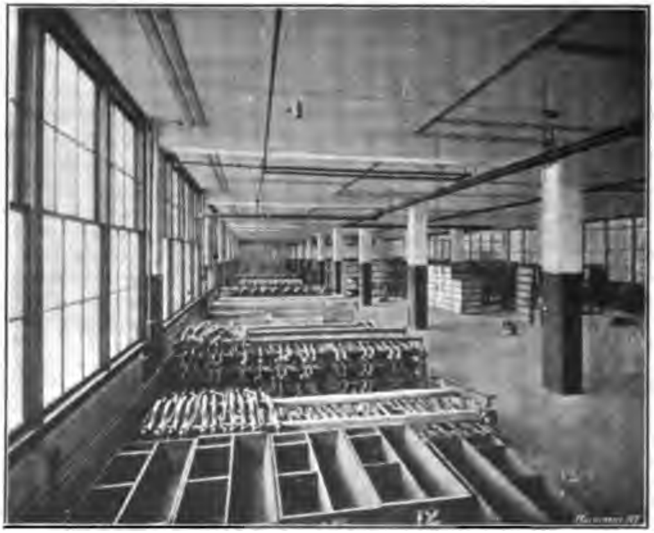

Fig. 36. View of Stock Room, showing Trucks in which Parts are taken to the Assembling Room

Each rod is supported by a pin passing through it at the proper distance from the end, which rests across the top of the base-plate which is bolted to the floor and through which the end of the rod passes. By giving a partial turn to the rod, the pin is allowed to pass through a slot in the base-plate, and the whole support is thus dropped until its top is flush with the floor. In order that the supports may accommodate themselves to various lengths of frames, the rear pair of every set of four base-plates is made with four sets of holes, in any of which the rods may be placed. The sets of supports are placed at such intervals along the floor that sufficient space between the frames is allowed to enable two teams of men to work on adjoining cars without interference. While it may seem a small matter, the facility with which these supports may be put in place, adjusted or removed from the floor helps to make possible, in no uncertain degree, the record for the rapid assembly of cars of which this factory can boast.

Although not a part of the assembling room proper, the department in which the pressed-steel frames of channel-section are prepared for the chassis, has an important part in facilitating quick assembling. When the frames arrive at the factory, forty or fifty holes must be drilled for the various parts which are to be attached, such as the gear shift, brake levers and their supports, the motor, transmission, running boards, fenders, lamp brackets, springs, and the like. Most of these, with the exception of the motor and transmission, are riveted in place before the frames reach the assembling room. These operations are performed in the frame riveting room, which contains several unique and ingenious arrangements that, so far as efficiency is concerned, bring this department on a par with the assembling

Fig. 37. Room in which the Frames are drilled and riveted by Pneumatic Tools

room. The frame is first placed on a set of supports similar to those used in the assembling room, except that a tension rod and turnbuckle connect both pair of rods for the purpose of holding the frame more rigidly in place. A single track over this set of supports carries a differential hoist, from which is suspended a large jig (see Fig. 37) containing a guide hole corresponding to every hole necessary to be drilled in the sub-frame, which carries the motor and transmission. This jig is clamped securely in place and the holes drilled by means of pneumatic drills connected to flexible piping. When all the holes are drilled in this manner, the frame is removed to another set of supports a few feet distant, where it is held rigidly in place in the same manner as that before described. Above this second set of supports is an oval track of the same length and width as the frame. From the traveler on this track is suspended a cable terminating in a single pulley through which passes a chain. On one end of this chain is a heavy, penumatic riveter, which is counterbalanced by an iron weight attached to the other end of the chain. This enables the tool to be placed at any height desired without unnecessary exertion. A small forge (not shown in the illustration) in one corner of this room heats the rivets before they are driven into the frame. By means of the oval track and pulley, any vertical or horizontal plane bounded by the frame may be reached with the riveter, and four or five men in this department are usually able to keep the assembling room supplied with the required number of frames. After being finished in this department, however, the frames in all cases are taken directly to the finished stock room, from which they are drawn out to the assembling room as needed. This stock room, in facts, acts as a sort of clearing house for the whole factory, and no part ever reaches the complete car until it has been inspected, checked and entered in the stock room records.

The keynote of this system is specialization. Every man knows what he has to do—and he does it. There is no overlapping of departments. It is scarcely ever necessary for the men in the assembling room to step into the stock room, and the men in the stock room are supposed to keep the men in the assembling department supplied with the necessary parts for the cars that have been ordered to be finished that day. Each team in the assembling room follows its two cars through until they are ready for the road test, and it is then easy to place the responsibility for any defect where it belongs. When this system is supplemented with such labor and space saving devices as are used in the assembling and frame riveting rooms, and when, at the head of it all is able, efficient and experienced management, one can begin to understand the conditions which allow the immense increase in production and the reduction in cost of the American-made motor car of to-day.

CHAPTER IV

TREATMENT OF GEARS FOR AUTOMOBILES*

There is probably no part of an automobile that is subjected to more use or greater abuse than the transmission. Carrying as it does practically all of the power developed by the motor, and, receiving at the hands of a careless driver the strains imparted by a suddenly

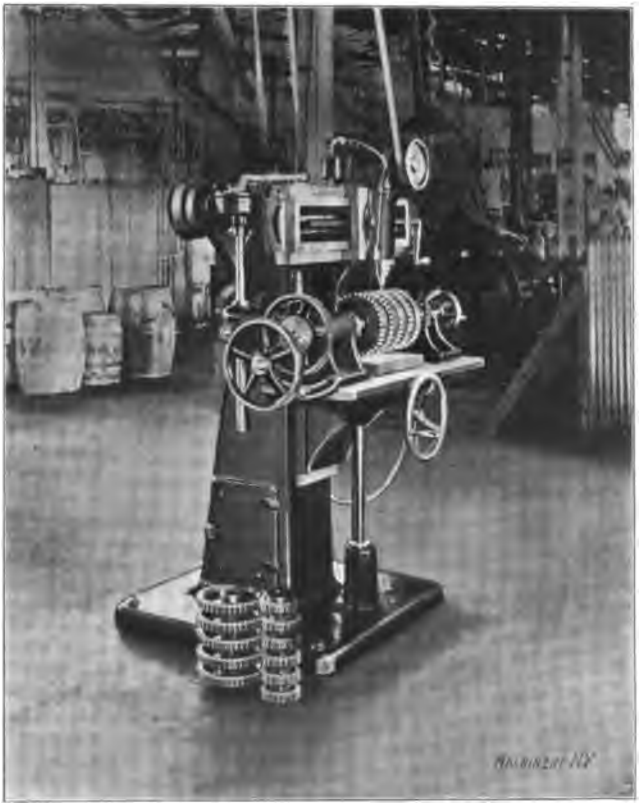

Fig. 38. Chamfering the Teeth of Spur Gears in the Winton Factory

applied load or a too rapid shifting of the speeds, it is small wonder that the gears of the transmission must be made of the highest grade of materials, and that the care and workmanship bestowed upon each must be of the best. The ordinary automobile transmission consists of a series of different sizes of spur gears mounted on two parallel

* MACHINERY, October, 1909.

shafts with means for sliding the gears on one shaft into mesh with those on the other, as desired. In this manner various speed ratios are transmitted from the motor to the main driving shaft, although on the majority of automobiles the high speed drives the car direct, without the interposition of any of the gears of the transmission.

As a saving in weight is an important factor to be considered in the design of a transmission, the gears must be made as small as possible and yet be sufficiently strong to carry suddenly-applied loads with no attendant danger of breaking. Owing to the methods by which the speeds are changed, and the clashing and "bruising" which take place when the gears are shifted, the transmission must also be made of a material which is hard as well as tough. Different kinds of steel have been used and each has been treated by various methods in an effort to discover the perfect gear material, but although this

Fig. 39. Gear Case-hardening Room in the Premier Factory

is yet to be found, the transmission of a modern, well-made automobile, when intelligently handled, will last nearly as long as the car itself. Of the various kinds of carbon steel which have been employed for transmission gears, nickel, chrome-nickel and silico-manganese seem to have more adherents among the leading builders than any other materials. In most factories the gears are case-hardened after being cut, and in this manner the combination of toughness with the desired hard surface is obtained. Gears which have been treated in this way have been taken out of cars after having been run many thousands of miles, and in some instances, the original tool marks on the faces of the teeth were still visible.

Methods employed for cutting gears in automobile factories do not differ in any essential features from those used in any well-equipped machine shop or manufacturing concern. Most of the automobile makers purchase their transmission gear blanks outside and cut and finish them in the factory. Many of these blanks of special steel are imported from France, but a few of the leading factories have laboratories of their own in which experiments on high-quality materials for transmission purposes are continually in progress. Six or seven spur gear blanks of the same size are generally placed on the mandrel of the cutter at once. A continous cut extending throughout the width of all these blanks is then taken for each tooth, and in this manner six or seven gears are finished at once and are made absolutely uniform.

After the teeth have been cut, the gears are taken to the heat treating room to be case-hardened. In the Middle West, and a few other sections, many of the case-hardening ovens are heated by natural gas obtained from near-by wells. In the Maxwell factory, at Newcastle, Indiana, a special machine has been installed for the manufacture of gas from "distillate"—a hydro-carbon obtained from the oil refineries. This machine is set up in the power house connected with the factory, and the gas is stored in a tank located in the same building. It is conducted from here to the heat-treating ovens in which it is used for case-hardening, tempering and annealing. Still another method for obtaining heat for the ovens is in use at the Ford factory, in Detroit. Petroleum, or crude oil, is vaporized and forced by air pressure into a series of special burners located under the ovens. By regulating the amount of air or vapor or both, the ovens can be kept at a uniform temperature, or the amount of heat generated may be varied at will between almost any limits. The temperatures of the ovens are indicated by an electric pyrometer connected with each, and pieces to be case-hardened are kept at a heat of 1,600 degrees F. for a length of time which depends on the depth below the surface to which it is desired to carry the treatment.

In several factories the final operation bestowed upon the gear, before assembly in the transmission or the motor, is the sand blast which serves to scour off any roughness or stains which may have been left on the surface during the cutting or the heat treatment. In the National factory, at Indianapolis, this operation is conducted in a small building separated from the remainder of the shop. The sand is kept in a bin in one corner and is sucked up by a centrifugal blower and forced by the air pressure through a pipe which terminates in a nozzle. The sand, being forced out at high velocity by the air pressure, may be directed at all parts of the pieces to be cleaned. This is one of the most efficient methods of polishing and finishing a gear and does not injure the hard metal surface in any way.

As silence of operation of all moving parts is one of the principal requisites for a motor car of to-day, it is necessary that the teeth of all gears shall be made to mesh perfectly and smoothly with all of those on the other gears with which they come in contact. In order to obtain silence of operation, the gears are run with each other for some time and each tooth is worn to a more perfect fit. The first few weeks of operation by the customer would wear the gears in properly, but, in order to produce a perfect car, this is done before it leaves the factory. Most of this "running in" of the gears can be accomplished by the thorough road test to which the whole car is subjected before leaving the shop, but many of the leading factories supplement this with additional methods for obtaining the required wear on the transmission. A special frame is used in the Marmon factory, in Indianapolis (see Fig. 40), in which the transmission, driving shaft, differential, and rear axle and wheels are set up. An idler and a driving pulley, with a belt shifter, are attached to the front end of the transmission shaft and connected by belt to a countershaft driven from the main line shafting. When the power is applied and the different speeds of the transmission are thrown into mesh by the shifting lever, every gear of the whole car, with the exception of those

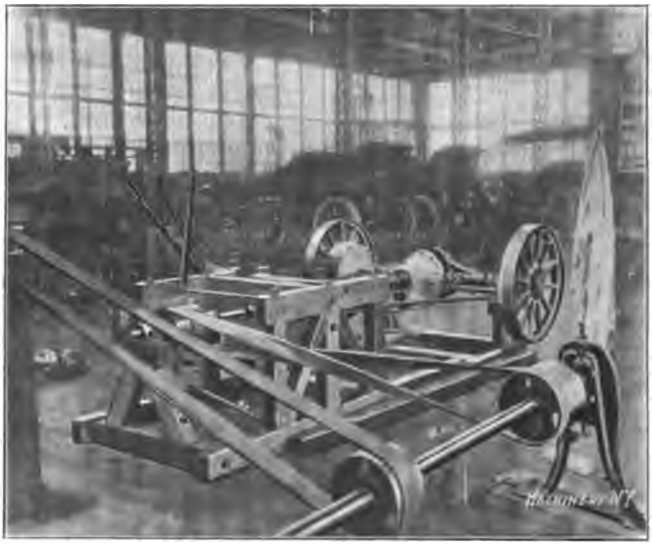

Fig. 40. Running in the Transmission and Differential Gears in the Marmon Factory

used on the motor, will be set in motion. The gears of the engine are worn in when it is operated under belt power before installation in the chassis. Somewhat the same method is pursued in the Packard factory, in Detroit, the only difference between the two being that here, instead of allowing the wheels to run free, a brake is attached to the end of the driving shaft by means of which a variable load may be applied to the gears in mesh. A section of the testing room is devoted to this purpose, and as the transmission and rear axle are assembled, they are brought in, placed on special frames provided for the purpose and connected by belts to the overhead shafting. As the gears of the transmission and differential are run in, the loads are increased until all are worn perfectly smooth.

Before their final installation in the motor and transmission, all of the spur gears for the Winton cars, made in Cleveland, are set up in a special case and run in under belt power. The bearings in these special cases are set at the proper distances apart to accommodate the various gears of a train, thus wearing in the gears so that all of those for similar parts are absolutely interchangeable. The case is made oil tight and a mixture of finely powdered emery and lubricating oil is fed through an opening in the top so that this grinding material will come in contact with all the teeth of the gears in mesh in the train. This grinding is continued until each tooth has been worn perfectly smooth and to an accurate fit with the teeth of the other gears with which it comes in mesh. For the gears used in the front of the motor to drive the cam, pump and magneto shafts—gears which always occupy the same relative position in regard to each other—a tooth of each is marked when in the grinding case with the corresponding teeth of the others with which it meshes. This is done so

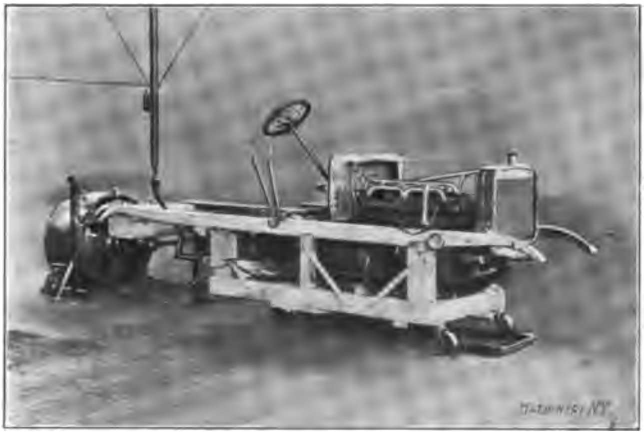

Fig. 41. Preliminary Run of Engine and Transmission to wear in the Parts

that each gear of the train may set up in the motor in the same corresponding position as that occupied while being worn to a perfect fit with the others in the case. It is evident that every tooth cannot be of *exactly* the same size and shape, and if each tooth is allowed to mesh with those with which it came in contact while being ground, more perfect rolling contact will take place and less friction and noise will result. The marks made on the gears are also useful for timing the magneto and valve cam shafts when an occasion arises necessitating the removal of any of these parts from the motor. Of course, it is impossible to carry this practice to the transmission, for most of the gears on one shaft revolve independently of those on the other, and it is very seldom that the same teeth of two gears will come into mesh on succeeding occasions. This practice, however, may be applied to the bevel gears of the driving shaft and rear axle and the pinions of the differential. As a further means of wearing the gears of the transmission to a perfect fit, the motor, transmission and driving shaft are installed in the chassis as shown in Fig. 41, and the motor is run while the various speeds of the transmission are thrown into mesh in order to wear in every gear thoroughly. During this run an electric dynamometer, by means of which a variable load may be applied, is connected to the end of the driving shaft.

An ingenious device for testing the accuracy of gears is used in the factory of the Grabowsky Power Wagon Co., of Detroit. This consists of a standard having three pins or bearings set in it on which the gears of the transmission are placed as shown in Fig. 42, thus forming a replica of the planetary transmission as used in the car. The middle upright bearing is stationary while each of the other two is movable in a horizontal direction and is connected to a micrometer at either end of the base of the instrument. A master gear is set on one of these bearings, while the pinions to be tested are placed on the other two. When the two movable bearings have been so

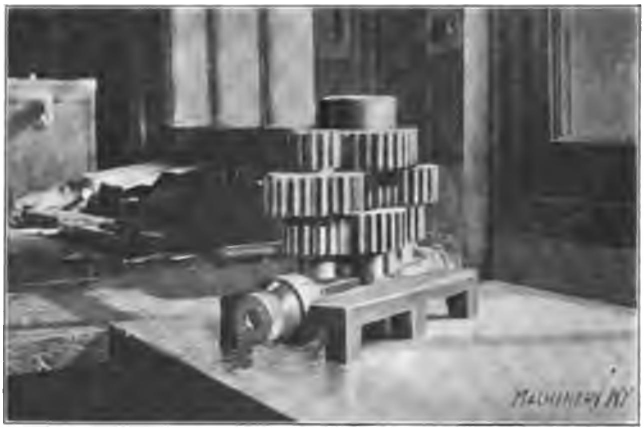

Fig. 42. Device for Testing the Accuracy of Gears

adjusted that all of the gears mesh perfectly, the readings of the two micrometers may be observed and the amount, in thousandths of an inch, by which the gears are "off" may thus be determined accurately. Certain limits of variation are necessarily allowed, but if any gear is below one or above the other, it is thrown out. Inasmuch as the distance between the centers of the gears must be constant in the transmission case, this instrument is useful in determining just what gears are acceptable without the necessity of installing them in the case.

Many of the gears used in the forward end of the motor for driving the cam, pump and magneto shafts are made of manganese-bronze. The Premier car, however, made in Indianapolis, employs a laminated gear for the magneto shaft, built up of alternate layers of bronze and fiber. These layers are pinned firmly together and the gear is then cut by the usual methods. This makes an exceedingly quiet-running gear, as the layers of fiber or rawhide cushion the impact of the teeth as they meet, and the whirring or grinding sound familiar in many all-metal gears is practically eliminated. It has been found by means of a series of exhaustive tests conducted in this factory that the silent running of this gear is brought about by a slight rounding or "bulging" of the face of the rawhide sections caused by the absorption of the lubricating oil in the pores of the fiber and the pressure against its sides. This, as mentioned above, effectually cushions the impact of the teeth, but if this bulge becomes too great, the teeth will not mesh properly, there will be a tendency to "jam" and more friction will be set up than would be the case were an all-metal gear used. Of course the wider these fiber sections are, the greater will be the bulge to each, and it has been found as a result of these experiments that laminated gears composed of layers of rawhide about 1/8 of an inch thick, alternating with bronze disks of the same dimensions, give the best service for this purpose. When sections of this thickness are used, a sufficient bulge is formed to cushion the impact satisfactorily, and yet this is not great enough to change the shape of the teeth materially. These experiments are still in progress at the factory in question in order the more accurately to determine other facts and figures concerning the best form of laminated gears, and this is only one of the many instances which give evidence to the fact that the American motor car manufacturer is now fully awake to the importance of paying attention to the most minute details of design.

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CONSTRUCTION AND MANUFACTURE OF AUTOMOBILES

By RALPH E. FLANDERS

CONTENTS

49-55 Lafayette Street, New York City

CHAPTER I

DESIGN AND CONSTRUCTION OF A HIGH-GRADE MOTOR CAR*

The following description of a 40 H. P. automobile, built by the Stevens-Duryea Company, of Chicopee Falls, Mass., may, except for certain important details which will be specifically mentioned, be taken as typical of the design of high-grade cars in general. In Fig. 1 is shown a side view of the "Model Y," 40 horsepower, six-cylinder machine, with 36-inch wheels and 142-inch wheel-base. An automobile may be divided into two parts—the body and the "chassis." The former is the product of the carriage-maker's art, the latter of the mechanic's

Fig. 1. Stevens-Duryea "Big Six" Motor Car, 1910 Model

and engineer's. The chassis of this machine is shown in Figs. 2 and 3, to which reference will now be made.

The mechanism and body of the car are supported by a frame whose side members, of chrome-nickel steel, are shown at *A*. These are connected by four cross pieces, and are supported on the front and rear axles by the spring connections shown. The cross pieces are also pressed from chrome-nickel steel, and are hydraulically riveted to the side frames. A platform spring suspension is used at the rear, hung on connecting shackles designed to overcome the side roll met

* Machinery, October, 1909.

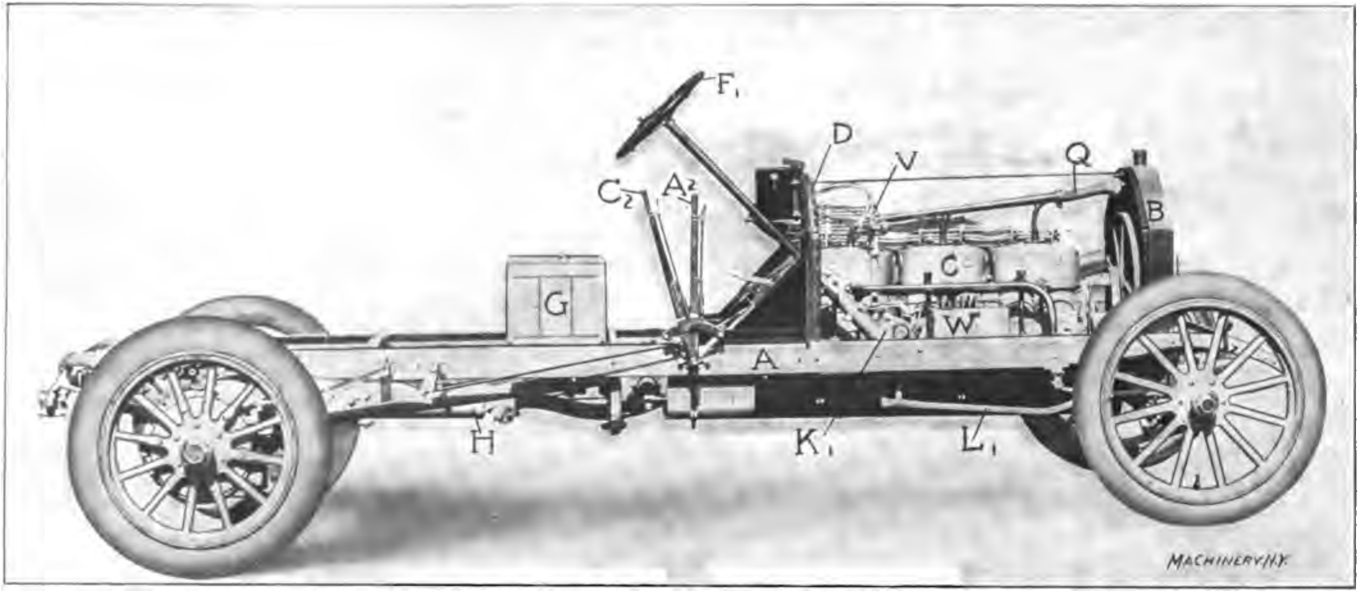

Fig. 2. Side View of Car with Body Removed, showing Chassis

when rounding curves in large and fast cars. The springs are made from steel selected after careful tests of both American and imported materials. The cost of the brand selected was far in excess of that of the nearest competitor, but it gave an endurance under repeated shock and reversal of stress not met with in any other make.

On the chassis frame are mounted, first the radiator *B*, next the engine C, then the dash-board *D* with its steering and controlling mechanism, the clutch and speed change mechanisms at *E* and *F* respectively, the gasoline tank *G*, the muffler *H* for the exhaust, the propeller shaft *J* for transmitting the power to the rear axle, and the rear axle

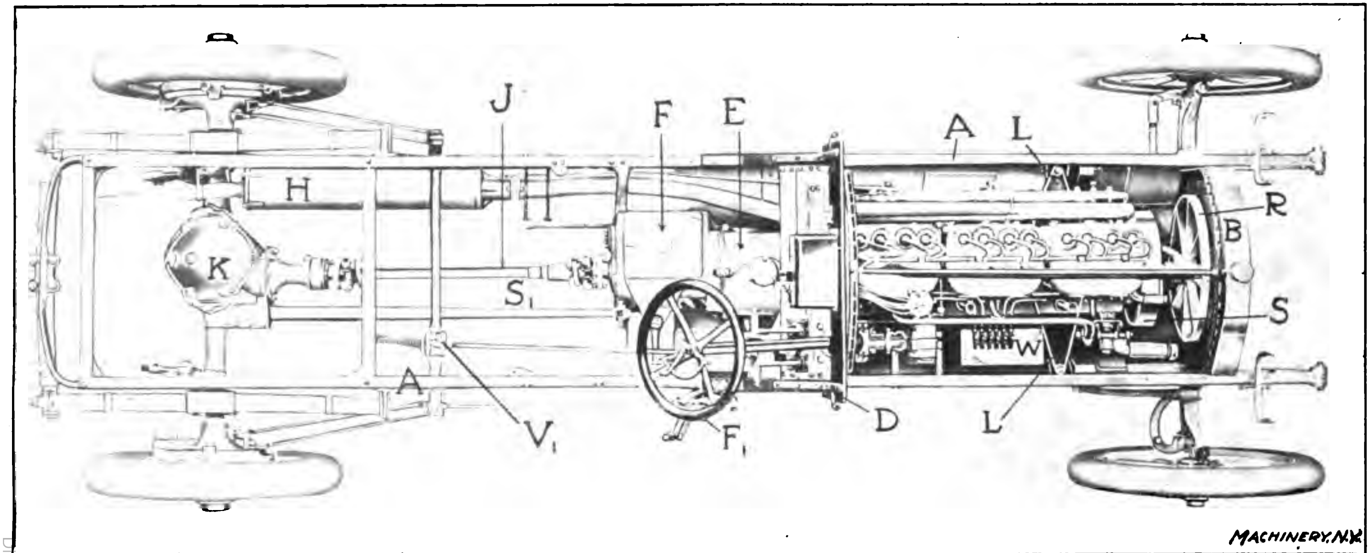

Fig. 3. Top View of Chassis, showing Arrangement of the Mechanism

with its differential gearing at *K*.

The engine is shown more clearly in Figs. 4 and 5, which show the "unit power plant" form of construction, one of the important original features of the design. This peculiarity consists in mounting the engine,, clutch, and transmission casings as a single rigid member, supported by a three-point bearing on the flexible frame. Supports *L* bear on the two side frames, while pivot *M* is riveted to one of the cross pieces. This allows the whole of the contained mechanism to run without distortion or bending, even on roads which rack the frame severely, and thus results in less friction and lighter structural parts, giving a high

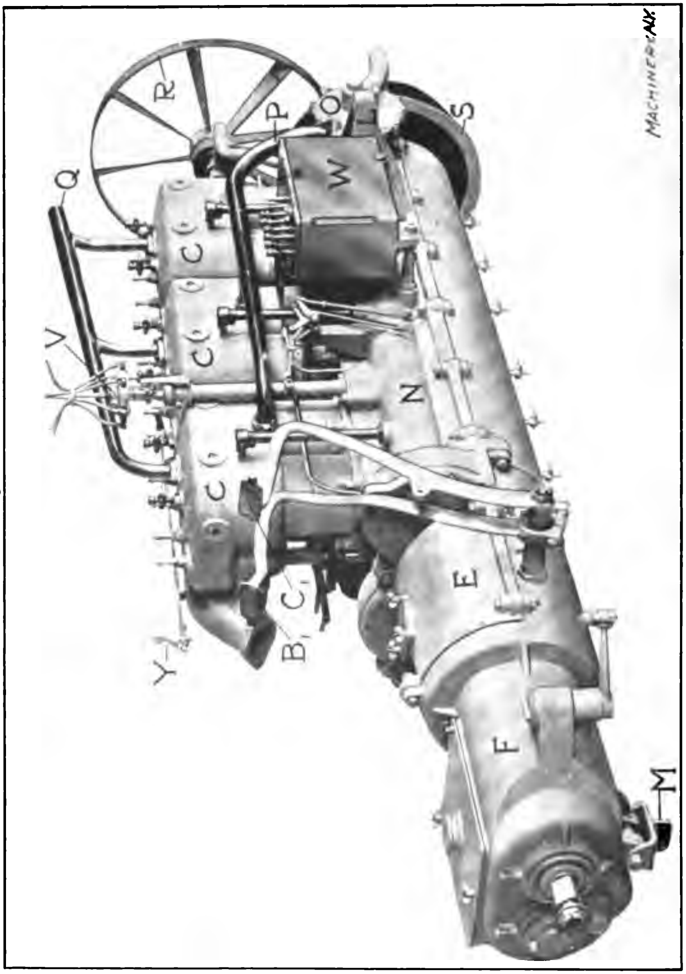

Fig. 4. The Unit Power Plant, comprising Engine, Transmission, etc.; View taken from the Timer and Lubricator Side

available horsepower per hundredweight of load. It also permits the power plant to be assembled as a whole and to be bolted in place without fitting. This construction, which is the distinctive point in the design of this motor, has been successfully followed by the builders for the last five years, and it is one of the things which serve to give an attractive mechanical appearance to the whole mechanism. Only one double set of universal joints is required, that connecting the propeller shaft with the transmission gearing at one end, and the differential gearing at the other.

The cylinders are grouped in three two-cylinder castings *C*, bolted to the crank case *N*. As is common with internal combustion engines in ordinary practice, they are water jacketed, there being a continuous

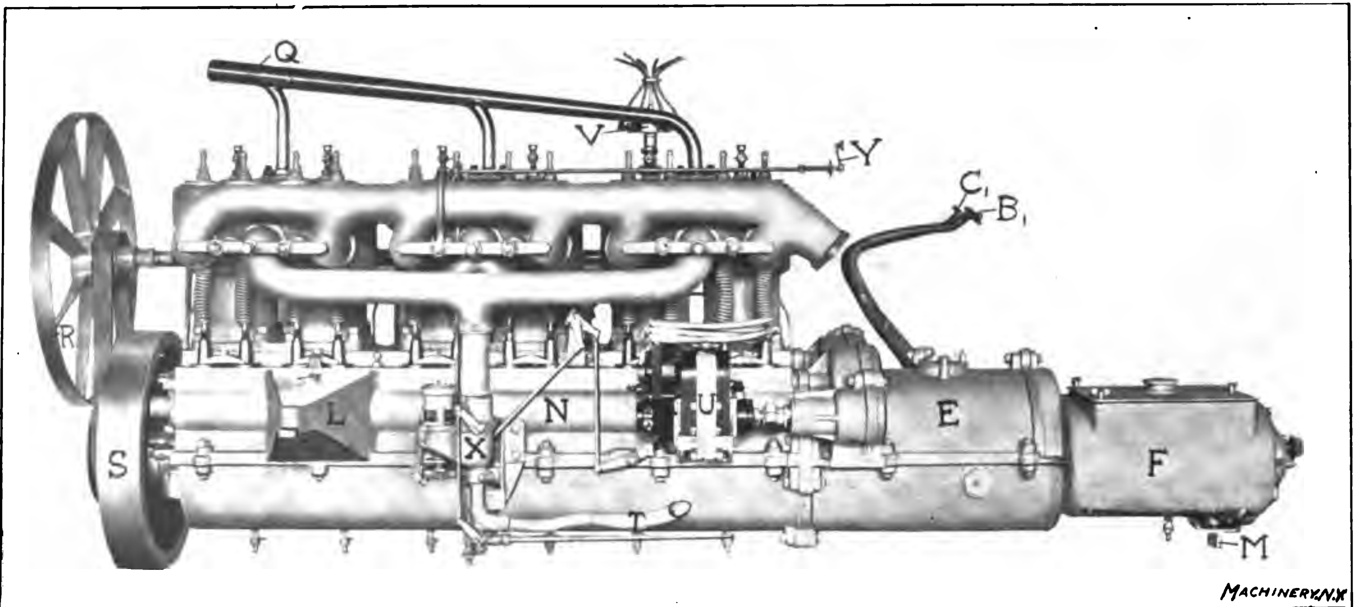

Fig. 5. The Left or Valve Motion Side of Power Plant, showing Carburetor, Magneto, Arrangement of Piping, etc.

circulation from radiator *B* through centrifugal pump *O* and pipe *P* to the water jackets, thence back again through the return pipe *Q* to the top of radiator *B*. Here the heated water is cooled by passing through sheet metal channels, having a large radiating surface exposed to the draft of wind produced by the passage of the machine through the air. This draft is increased by an aluminum fan *R* belted to the pulley on the outside of flywheel *S*. An automatic tightening arrangement is provided for the belt.

It should be mentioned that the placing of the fly-wheel at the forward end of the crank-shaft, as here shown, is unusual, the common construction being to locate it between the crank-shaft and the clutch. It tends, in particular, to bring more of the weight onto the front wheels, off from the heavily loaded rear wheels of the machine, and permits the reducing of the clearance over the roadbed in the center of the chassis, where there is the greatest danger of striking on high water-bars, railroad crossings, etc. It will be readily seen that more clearance is required at the center of the machine than at the axles, when crossing a hump in the road.

Lubrication, Ignition, etc.

Two shafts mounted in the crank casing, one on each side, above and parallel to the crank-shaft, are driven from it by enclosed gearing.

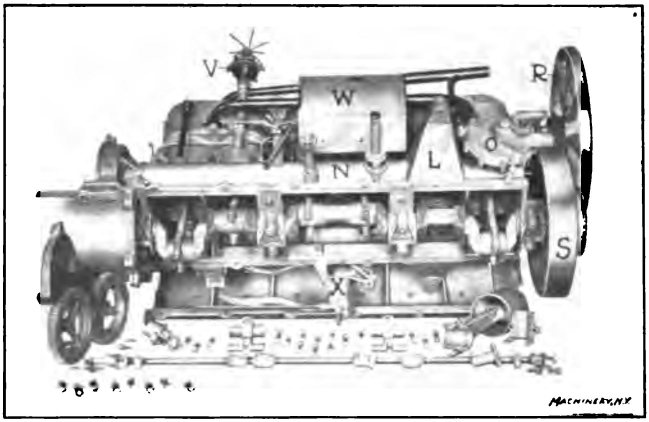

Fig. 6. View of Engine from Beneath, showing Removal of Piston, Cam- and Lay-shafts, etc., without Dismantling

The one at the side shown in Fig. 5 is the cam-shaft and is provided with twelve sets of cams for operating the six inlet and six exhaust valves, whose stems and closing springs are plainly shown in the engraving. The driving gear of this cam-shaft is also connected with a pinion on the armature shaft of the magneto, whose function will be described later. The shaft on that side of the machine shown in Fig. 4, is known as the lay-shaft. Its office is the driving of the timer *V*, which controls the ignition, the driving of the forced lubrication mechanism at *W*, and of the water jacket circulation pump *O*.

The lubricator gives a forced oil supply with sight feed, and is always in operation when the engine is in motion. The six-throw crank-shaft is mounted in four bearings in the crank case, with two cranks between each pair of bearings. The boxes at these points are connected with the lubricator *W*. The lower half of the crank case forms a reservoir for the oil escaping from the main bearings. The connecting-rod splashes into this and thus supplies the pistons, connecting-rod bearings, etc., with the necessary lubrication.

The ignition in each cylinder is effected by either of two systems, the one by storage or dry battery and induction coil, and the other by means of a magneto *U* connected by gearing with the crank-shaft. The battery and spark coil is used in starting, while the magneto is used for regular running. The spark coils and switches are located on the dashboard. A lever on the steering wheel, as will be described, is connected with the commutator or timer *V*, which distributes the current to the six cylinders in such a way as to enable the operator to advance or retard the spark at will.

The Carburetor and Fuel Supply

An important and rather delicate piece of apparatus essential to the operation of the gasoline engine, is the carburetor, shown at *X* in

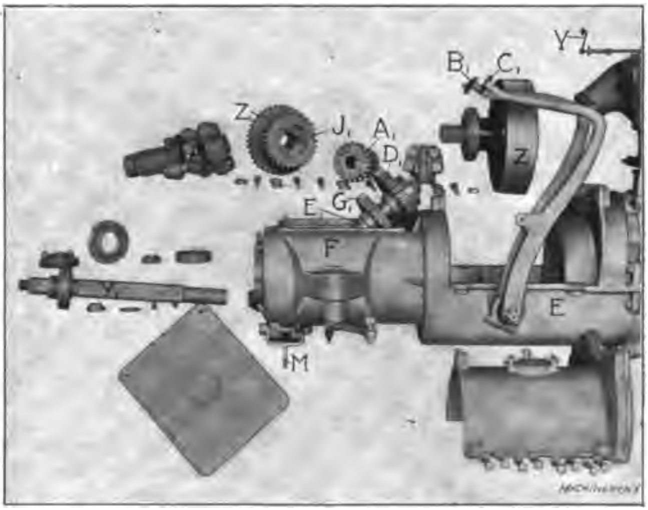

Fig. 7. Clutch and Transmission Gear Members Dismantled to show Construction

Fig. 5. This receives a supply of gasoline through a feed pipe from the tank *G* (see Fig. 2), a supply of air through *T* heated by the exhaust gas for vaporizing the gasoline, and a supply of fresh air to furnish the oxygen for the charge. The gasoline is received in a float chamber, where the level of the liquid is maintained by a suitable float and valve. An automatic valve provides for a constant proportion of oxygen and fuel at widely-varying speeds. The carburetor is provided with a throttle which controls the needle valve connection in the feed pipe, together with the butterfly valve in the suction to the cylinders, thus providing the driver with means for varying the amount of charge furnished the machine; this controls the speed without shifting the gears in the transmission case. The automatic air valve is controlled from the seat by a handle *Y* on the dash-board, which permits the obtaining of a proper mixture for the starting. A button at the front of the radiator, where the machine is cranked for

starting, also provides means for flooding the carburetor with fuel for a send-off. The throttle is controlled from a lever on the steering wheel, concentric with the spark control lever, or from an "accelerator pedal" on the foot-board.

The gasoline supply tank *G* is located under the front seat. It contains a partition near the bottom which saves about three gallons out of its twenty gallons' capacity, for use in emergency. By the manipulation of cut-off valves passing through the left side frame of the chassis, it is possible to use this reserve supply after the tank has been otherwise exhausted. This provision is a great comfort to the motorist at critical times.

The Clutch and the Transmission Gearing

In casing *E* is mounted the clutch *Z* (Fig. 7) connecting the engine with the transmission to the driving wheels. This is of the multiple

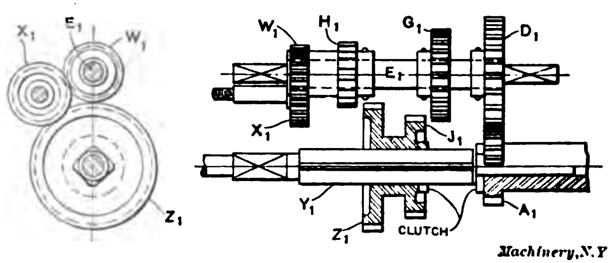

Fig. 8. Sketch showing Arrangement of Gears in Transmission Case

disk type, with alternate disks keyed to the driving and driven members. The driving disks have a wired asbestos facing which makes a superior friction surface, and gives a high resistance to heat as well. This construction obviates, and in fact makes impossible, the use of oil in the clutch. The friction surfaces are held in engagement by a spring, and are released by a pedal B_1, which projects through the foot board at the driver's side of the machine. The spring is so proportioned as to give a smooth, easy engagement, entirely out of the control of the driver, who thus finds it impossible to start the machine with a sudden shock. The second foot lever, C_1, is connected with the rear wheel brakes, as will be described. The driven member of the clutch is connected with the driving shaft in the transmission case or speed box *F*. Contained within it is a mechanism which, by the aid of the sliding gears, clutches, etc., permits of the obtaining of three forward and one reverse speed.

The operation of this gearing will be understood from the sketch shown in Fig. 8. Gear A_1 receives its movement from the clutch. It meshes with gear D_1 keyed to the secondary shaft E_1, which is thus in motion whenever the engine is running and the clutch is engaged. This shaft carries also gears G_1, H_1, and W_1, the latter of which drives, in turn, the idler X_1. Squared shaft Y_1 is directly connected by means of propeller shaft J (Fig. 3) and the universal joints with the rear axle. On Y_1 is mounted the double sliding gear Z_1. Clutch teeth are provided in the faces of the gears A_1 and J_1.

In the position shown in Fig. 8, the transmission is in the neutral position, so that the motion from the clutch is not transmitted to the axle. The right-hand end of shaft Y_1 lies loosely in the revolving gear A_1. When the sliding gear is thrown to the extreme right, the clutch faces of A_1 and J_1 are engaged, so that shaft Y_1 is driven directly, and at the highest speed, from the clutch. By shifting it a step to

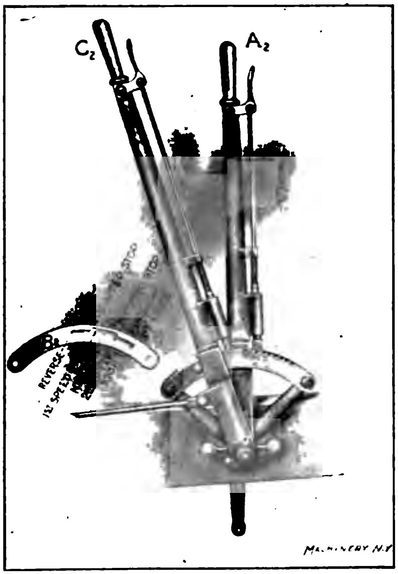

Fig. 9. The Speed Gear Control and Emergency Brake Levers

the left, J_1 is thrown into mesh with G_1, thus giving a lower rate of speed through the back gear shaft E_1. A still further movement to the left, past the neutral point shown in the engraving, brings Z_1 into engagement with H_1, giving the lowest forward speed. A final movement to the left engages Z_1 with idler X_1, thus reversing the drive.

The shifting of gears Z_1 and J_1 is effected by a forked lever connected with lever A_2 (Fig. 9) at the side of the machine, which thus controls the speed changes. This lever is provided with a latch connected with a pin in the slot of the quadrant B_2, operating in a manner easily understood from the engraving. It will be seen that it is possible to move between the reverse and the lowest speed, or between the second and the high speed, without touching the latch, and it is possible to make all the movements rapidly and precisely by the sense of touch without looking at the quadrant at all.

The Differential Drive

Propeller shaft *J* leads from the transmission case *F* to differential case *K* on the rear axle. The bevel gear M_1 (Fig. 11) is connected with the two rear wheels by a differential mechanism, whose function it is to give an equal tractive force to each of the two wheels, but at the same time to permit either of them to run ahead or lag behind the other as may be required in rounding curves, riding over obstructions, etc. The principle of this mechanical movement will be understood by referring to Fig. 10.

Referring first to the sketch at the left, N_1 is the pinion on the propeller shaft and M_1 is the driven bevel gear, concentric with the axle.

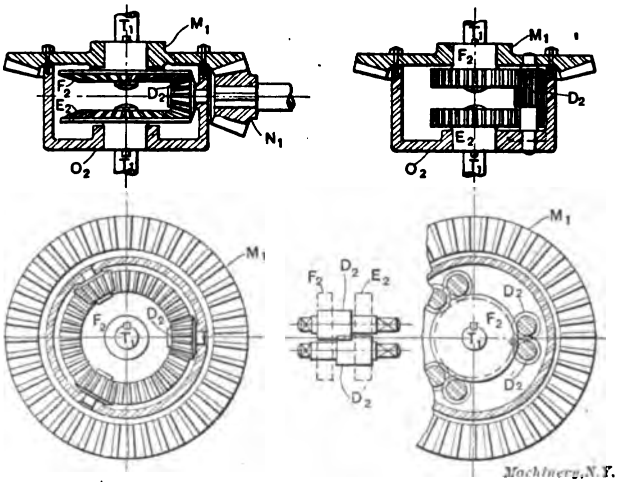

Fig. 10. Sketch showing Principle of the Bevel and Spur Gear Types of Differential Gearing

This gear and shell O_2 to which it is bolted, revolve freely on the hubs of E_2 and F_2. Within the shell are mounted radial pivots on which revolve, loosely, bevel pinions D_2. These engage with bevel gears E_2 and F_2, connected respectively with the right- and left-hand axle shafts T_1. It will be seen that under ordinary conditions the rotating of gear M_1 carries gears E_2 and F_2 along with it, by the pull exerted on them by the bevel pinions D_2, which are stationary; thus the two rear wheels are driven at the same rate of speed. Suppose now that the right-hand wheel be held from turning, so that gear E_2 is stationary, then the rotation of bevel gear M_1 will roll pinion D_2 about on E_2 with a compound action, which will give F_2 twice the rate of speed it had before. In the same way, F_2 can be held from revolving, in which case E_2 will have twice its normal speed, or either of them may be slowed down, in which case the other is speeded up correspondingly. The driving force on both wheels, however, is always the same.

An alternative form of this device is shown at the right of Fig. 10, in which each of the bevel gears D_2 is replaced by a pair of spur pinions D_2 and D'_2, meshing with each other and with spur gears E_2 and F_2 as shown. A little study will show that the action of this device is identical with that shown in the sketch at the left of the figure, the only change being the employment of spur gearing in place of bevel gearing. The differential used on the Stevens-Duryea machine is of the second or spur gear type.

The Full Floating Type Rear Axle

The differential gearing is contained in the casing O_1, which forms the central member of the axle. Tubular extensions to both sides

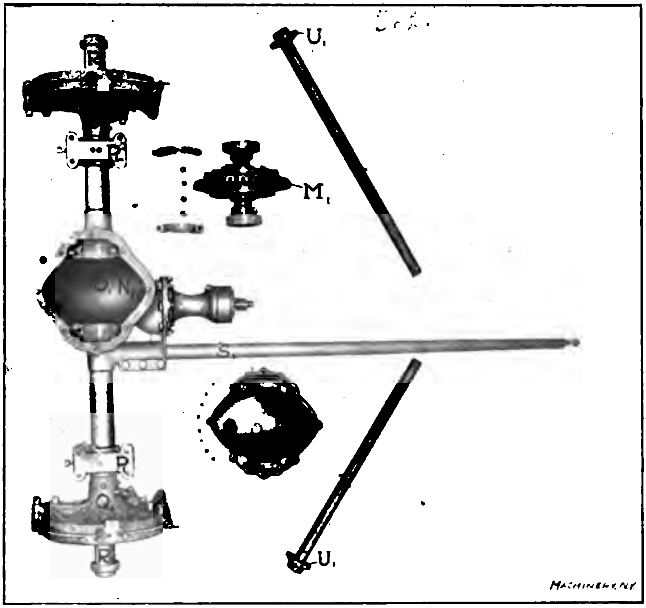

Fig. 11. The Full Floating Type Rear Axle, Differential Gearing, etc.

carry the spring supports P_1 on which the weight of the car rests. The brake flanges Q_1 and the wheel bearings at R_1, all of which are solid with each other, are non-rotating. The rear axle, however, is permitted to rock in spring supports P_1. The torque rod or tube S_1, which is fast in case O_1, extends toward the center of the chassis, where it is hung in a spring suspension as seen in Fig. 3 permitting a limited vibration up or down, with a constant force urging it toward a central position. This construction furnishes the resistance against the climbing of pinion N_1 on bevel gear M_1. In case of sudden starting or stopping, a limited amount of climbing either way is permitted, the torque rod being raised or lowered against the spring pressure to correspond. This greatly decreases the danger of gear breakage.

The construction just described belongs to what is known as the full floating type axle. The wheels are mounted on ball bearings on stationary journals R_1. Shafts T_1 are provided with squared driving ends engaging sockets in the differential gearing in casing O_1 at one end, and similar sockets cut in driving dogs U_1 at the other end. These latter members have driving slots engaging dove-tails in the hubs of the wheels, to which the power is thus transmitted. The squared ends of shafts T_1 are rounded to permit a slight rocking movement in their sockets in the differential gearing and driving dogs U_1. This permits the springing of the rear axle under the load without cramping the driving mechanism.

To allow for the springing of this axle under the load, the two sections of tubing on either side, between members O_1 and Q_1 are held in bored seats which point downward at an angle of ½ degree from the horizontal on each side. Thus the rear axle wheels point in toward each other at the bottom at an angle of ½ degree from the vertical, giving a much better appearance than would be the case if they should by some mischance point the other way. It would take a load in excess of any which would ever be applied to spring the axle and bring the wheels into the vertical plane. It is stated that when the wheels are exactly vertical, they have the appearance of being sprung out at the bottom, into the position occasionally seen in a vehicle of the "one-horse-shay" type.

The Brakes

The brake mechanism of the automobile is of the utmost importance, as is realized by anyone who has had anything to do with these machines whether as driver, passenger or pedestrian. It is usual to provide two complete sets of braking machanism, one for regular use and the other for emergency. That for regular use is controlled by the foot lever C_1 (see Fig. 4), which is connected with a reach rod leading to double cranks on a transverse rock-shaft at V_1 (Fig. 3). One section of this rock-shaft is connected with the brake at the right side of the machine, and the other at the left. An equalizing lever between the two insures an even pressure on each of these two brakes, even though one be much more worn than the other. The brake is of the band type, applied to the outside of a brake rim fast to the hub of the wheel. The emergency brake is operated by lever C_2 (Fig. 2). This, by means of a second rock-shaft concentric with V_1, controls internal expanding ring brakes in the hubs of the wheels.

The Control of the Machine

The steering gear will be best understood from Figs. 2, 3 and 12. The wheel F_1 is mounted on a tubular shaft which carries at its lower end a worm engaging the segment of a worm-wheel G_2 in casing K_1. To the hub of this segment is connected a bell crank H_2 which, through the operation of the steering rod L_1 (see Fig. 2) and suitable connecting cranks and links, turns the front wheels to the right or left as may be required. Spring cushions are provided at the ends of steering rod L_1 so that sudden shocks and twists of the wheels are not transmitted to the worm-gearing and the steering wheel, even when traveling at a high rate of speed. As most mechanics doubtless know, the center line of the pivots about which the wheels are swiveled meets the road at about the point where the tire touches it. This makes it possible to turn the wheels easily when standing still, and decreases the danger of accident while running, as well.

As previously stated, the throttle control and the timing of the spark are effected from levers placed at the hub of the steering wheel.

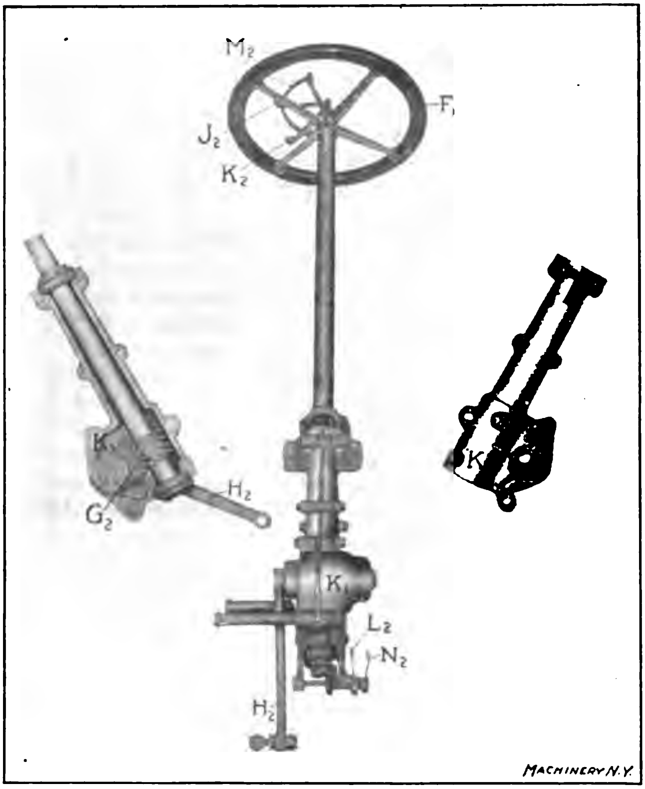

Fig. 12. The Steering Post, with its Throttle and Sparking Connections

Lever K_2 controls the throttle. This is mounted on a tube passing through the steering wheel tube and connected at its lower end by bevel gear segments with a bell crank L_2, which is, in turn, connected by suitable rods and levers with the carburetor. Inside of the throttle lever tube is still another fixed tube on which is mounted the segment M_2, which is thus held stationary. This is provided with notches for locating lever K_2, and lever J_2 as well, which latter controls the timing of the spark. This is mounted on a rod which passes through the center of the system of tubes and is connected by bevel segments with lever N_2 leading to the commutator or timer V.

It may be well to recapitulate as to the functions of the levers, etc., used in the control of the machine. At the front of the radiator is the crank by which the motor is turned, for starting. By the side of it is a button connected with the carburetor, for flooding the latter at starting to obtain a rich mixture on the first stroke. On the dashboard is mounted a lever Y, for setting the automatic air valve to supply the proper amount of oxygen for starting. Beside it is a switch for throwing the ignition spark from the battery to the magneto when the machine is changed from the starting to the running condition, and *vice versa*. On the dashboard are also mounted the spark coils. Through the foot board project the two pedals B_1 and C_1 controlling the clutch and the operating brake respectively, as described. Hand lever C_2 and A_2 control the emergency brake and the speed changes respectively.

Two small pedals are also provided on the foot board. One of these is connected with the throttle in such a way that this may be controlled by the foot instead of by the hand if required. It is called the accelerator. By its use, when the hand throttle lever has been set to a certain point, the valve may be opened clear out to the maximum, as desired, by the foot, thus giving immediate control under varying conditions of traffic. The other pedal operates a valve which cuts out the muffler. This is occasionally done to make the exhaust audible, for finding out how the engines are working, and also for removing the back pressure, and thus giving every ounce of power possible on critical occasions.

These levers, pedals, etc., with the main and supplementary gasoline supply valves previously mentioned, give the driver complete control of a powerful, swift machine, if he has the knowledge, experience and nerve to use them properly.

General Considerations in Automobile Design

A glance at the illustrations will serve to show that the chassis of the modern high-power automobile is a rather complicated, highly specialized, and carefully designed piece of mechanism. It is within the memory of the child in kindergarten when this was not the case, and the writer has painful memories of his duties as consulting physician to one of the best of the machines in existence six years ago. At that time, the mechanism of the automobile did not have the homogeneous, appropriate structure that the successful machines of the present day possess. It had a gasoline engine, an epicyclic speed change mechanism, a jack-in-the-box differential gear, and chains leading to the rear wheels of a "horseless carriage." Over the mechanism thus described wandered a maze of levers, braces, pipes, wires, etc., supported at intervals at any part of the mechanism which happened to be in convenient reach. That, however, was before the automobile "found itself." The present development has been the result of the experience of many men with break-downs and failures, as well as of an enormous amount of theoretical work in the matter of testing of materials and analysis of conditions. These theoretical and practical results have been combined on the drawing board, and the resulting machine has the appearance of having been *designed* rather than simply *built*.

The guiding principles in the design of the automobile relate to strength, power, lightness, durability, accessibility, and economy in operation. The matter of economy in construction and materials is about the last thing to be thought of, instead of the first, as with many other classes of machinery. The severe and often reckless usage received by one of these machines demands special treatment in the design and construction which should not ordinarily be necessary.

As an illustration of what has been said in this respect, attention may be called to the method of connecting the driving members of this machine, from the engine through to the wheels. In no place throughout the length of the chassis are keys used for this work. Reliance is everywhere placed on square joints or dovetailed flanges. The crank-shaft is connected with the driving member of the clutch by a square taper socket. The driving member of the clutch is connected by a square socket with the driving shaft of the transmission gearing. The sliding gears of this mechanism are mounted on square shafts, and the same squared drive is used for the universal joints, propeller shafts, pinion shafts, etc., through the intermediate pinions in the differential gearing at M_1 in Fig. 11, and through driving shafts T_1, to the driving dogs on the wheel hubs. These latter, as well as the side plates of the differential gearing, drive or are driven by the engagement of dovetailed teeth. The possibility of the shearing of keys, always present in machine parts subject to shock, is thus avoided. The makers believe themselves to be the only firm employing a complete drive of this kind.

In the matter of accessibility, a study of Figs. 6 and 7 will be found interesting. By removing the lower crank chamber casing and turning the crank-shaft to the proper position, the piston and piston rod may be removed without further trouble, and without removing cylinders or cylinder heads. The same is true of the cam- and lay-shafts. The covers provided for the clutch and transmission casings give evidence of care in providing easy means for inspection and removal of all parts likely to need attention. With a well-designed machine the man on his back under the motor car is a mere figment of the imagination.

CHAPTER II

AUTOMOBILE MANUFACTURING METHODS*

The subserviency of manufacturing considerations to considerations of strength, durability, accessibility, etc., mentioned in the preceding chapter, results in the design of parts which require special and interesting provisions for their economical production. Only a few of the operations particularly noticed in the Stevens-Duryea factory will be described here. They will serve, however, to give an idea of the general practice in such work, and will illustrate the ingenuity required for the solution of some of the problems.

Operations in the Machining of Cylinders

In Fig. 13 is shown a Beaman & Smith combined horizontal and vertical milling machine engaged in surfacing the base, exhaust and

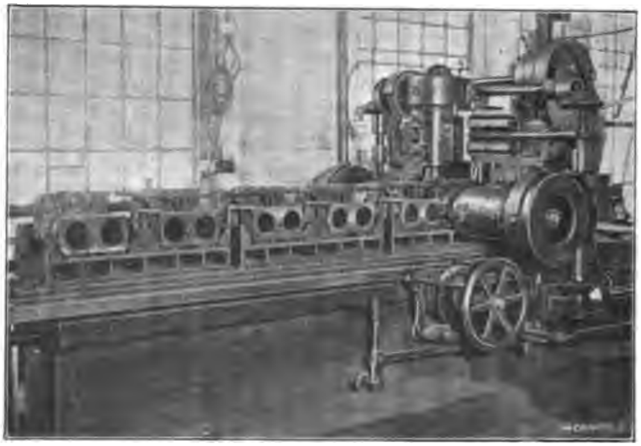

Fig. 13. Gang Milling Operation. Surfacing Cylinder Sides and Ends

inlet flanges, and the spark plug bosses of a series of cylinder castings. The work is mounted in gangs according to the most approved methods. The picture is chiefly interesting in that it shows that the builders take advantage of wholesale manufacturing methods even in the building of a $4,000 machine. Of course, an extensive use of jigs and fixtures, besides reducing the cost of manufacture, results in a greater uniformity in the product, and thus gives the advantage of an easy renewal of worn or damaged parts.

Fig. 14 shows a Beaman & Smith boring machine with fixtures mounted on the rotating table for holding four double cylinder castings. This table can be rotated and adjusted across the bed of the machine.

* MACHINERY, October, 1909.

On each side of the table, double boring heads may be fed in along the bed, one carrying roughing and the other finishing cutters, the feeds and speeds of the two heads being independent. A set of two castings being in place on the roughing end, the head is fed into them and one hole in each casting is roughed out. The work-table is

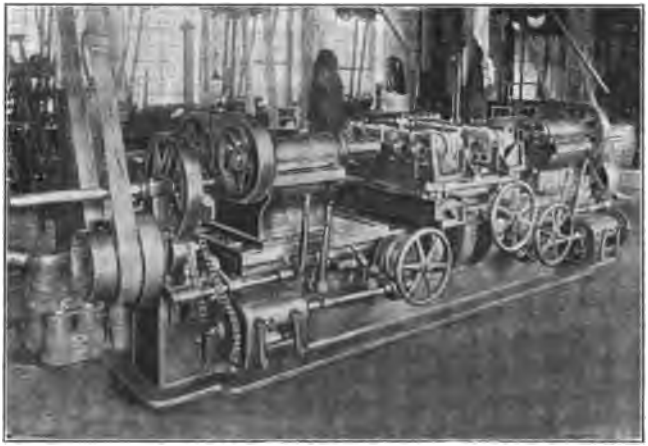

Fig. 14. Four-cylinder Boring Machine with Revolving Table

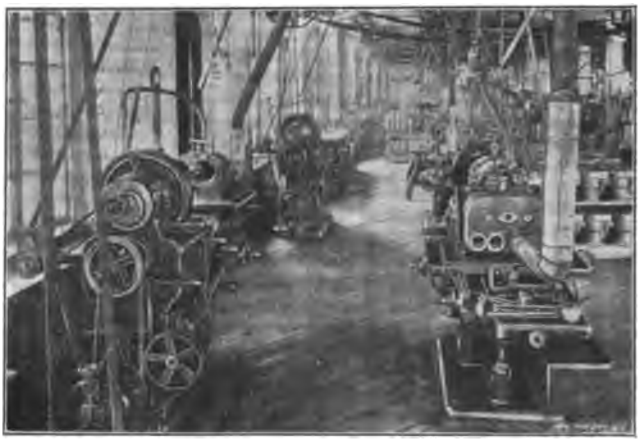

Fig. 15. Grinding the Cylinders. Note Connections for Exhausting the Dust and the Use of the Water Jacket for Cooling

then shifted, by means of the hand-wheel, against suitable stops, and the other bore of each cylinder is roughed. The table is then indexed to bring these castings to the finishing side, where the same operation is repeated, the boring being here carried to size for grinding. This rotating of the table, in turn, brings a new set of the cylinders up to be rough-bored. The process is continuous, the work being removed from the finishing side and new cylinders clamped in, while the rough boring is being completed.

For setting out the cutters in the boring bars, the construction shown in Fig. 16, at the left, is used. It will be seen that a taper-headed screw is used for forcing the blades out simultaneously. The cutters *B* bottom on this taper-headed screw *C*; fillister head screws *D* serve to keep the blades forced down to their bearing on *C*, and so draw them firmly against the side of the slot. By this means two or more blades may be set out simultaneously for regrinding to exact size. A similar arrangement (see view at the right of Fig. 16) is used for cutters in the middle of long boring bars, except that the taper point of a screw tapped into the bar from the side, is used in place of the corresponding taper-headed screw in the first case.

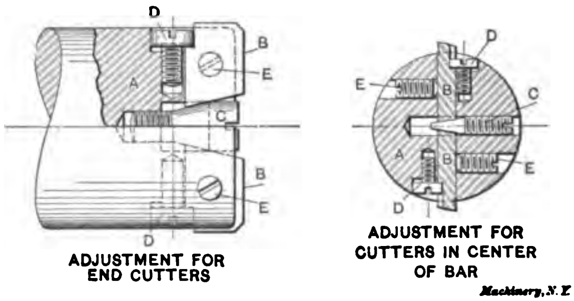

Fig. 16. Adjustment used for Boring-bar Blades

The bore of these cylinders is finished in Heald internal grinding machines especially built for this work. These are of the type in which the work remains stationary while the axis of the spindle is revolved about the center line of the bore and parallel with it, on such a diameter as to bring the outer periphery of the wheel in contact with the inner surface of the bore. The grinding spindle is fed out so as to rotate in a larger circle as the diameter of the bore is increased. An interesting feature shown in Fig. 15 is the provision of a flexible suction tube for drawing out the dust of the grinding through the inlet and exhaust ports, and also the provision made for water cooling. The water is not applied directly to the wheel, as in an ordinary external grinder, but is forced instead through the regular water jacket of the cylinder casting. This reproduces, in a measure, the conditions met with in actual use, and so tends toward accurate work.

Machines and Fixtures for Grinding and Lapping

There are other operations of interest in the grinding department besides that of finishing the bore of the cylinders. Extensive use is made of the Pratt & Whitney face grinding machine for finishing flat surfaces; in fact, it has largely displaced the vertical milling machine for this work, on parts in which the surface to be finished is clear of projections or obstructions to the sweep of the wheel. The faces of the various casings, covers, inlet and exhaust pipes, etc., are finished on this machine. In the past most of these parts have been made from castings on which 3-16-inch of stock had been left, in accordance with the usual practice of milling. The castings come true enough to shape, however, to permit of this finish being reduced to 1-16 of an inch, or

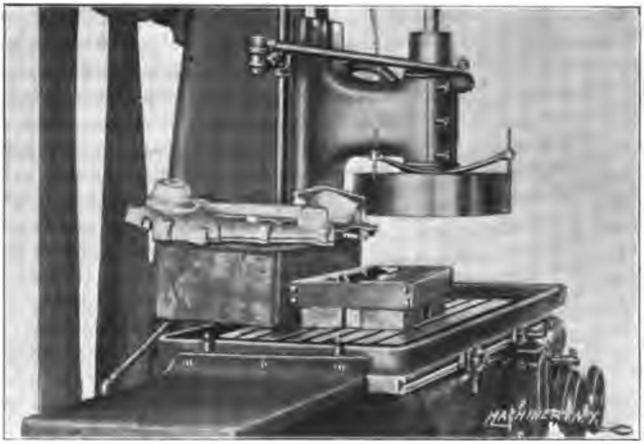

Fig. 17. The Acme of Simplicity in Fixture Making. Face Grinding the Steering Gear Casing

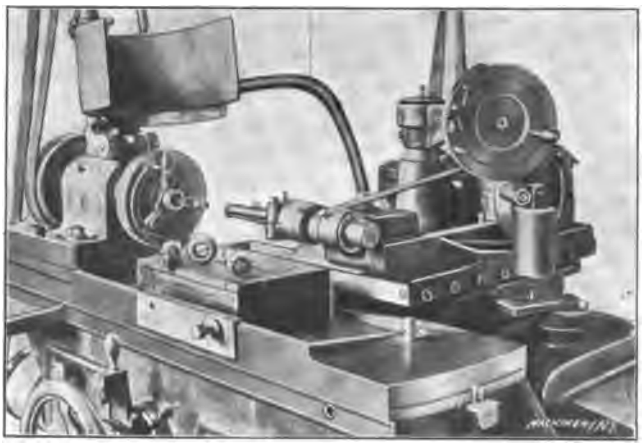

Fig. 18. Grinding the Bore of the Cams Concentric with the Cylindrical Surface

thereabout, when finished by grinding, thus materially reducing the time required. Even when removing 3-16-inch of stock the grinding machine has proved its superiority to the milling machine in the matters of cost, finish and accuracy. The foreman of the grinding department discovered that a little experimenting and investigating along the line of the grading of wheels made a tremendous difference in their durability and effectiveness in removing metal. For aluminum work a vitrified carborundum wheel of about No. 24 grain and grade H hardness is used, a soda compound being employed for cooling.

The cover side of the steering gear casing is one of the parts surfaced on the face grinder. An exceedingly simple fixture is used for holding it. This fixture, as may be seen in Fig. 17, is nothing more or less than a mass of lead melted and poured around a sample casting as a form. The work is set into the bed, thus prepared to receive it, and is supported on the table by its own weight, no fastening being necessary. The castings come uniform enough so that they fit well in this device, except at certain points around the gates and sprues, where it is found necessary to relieve the form slightly to allow for these variations. It may be mentioned that the other or main member of the steering gear casing has a boss projecting above the finished

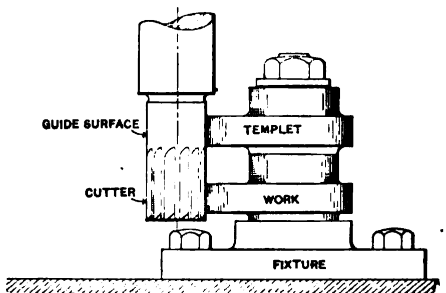

Fig. 19. The Simplest and Stiffest Arrangement for Cam Cutting

surface of the joint, making it necessary to mill that surface. The joint is thus formed of one ground and one milled surface.

In Fig. 18 is shown the operation of grinding the holes in the cams. It is quite important that the cylindrical portion of the cam shall be exactly concentric with the cam-shaft to prevent shock or jar during the period when the valves are supposed to be closed. To make sure that this surface is concentric, the cam is located by it in the grinding fixture as shown. After the fixture has been mounted on the faceplate of the machine, the gripping surfaces of the two jaws at the right are ground out by the internal grinding attachment, to the radius of the cydindrical dwell of the cam. The cam is clamped against the surface thus prepared, by the lever, which forces a wedge across and down upon the cam, holding it firmly into the corner in both directions.

It will be seen that this car does not employ the integral cam-shaft. By giving careful attention to the locating of the cams on the shaft and by being careful to obtain a strong drive fit between them, the difficulties of loosening and dislocation, which the integral construction is expected to cure, have been avoided. It is thus permitted to cut the cams in a way which gives the best chance for producing accurate shapes and smooth finish. The obvious scheme shown in the sketch, Fig. 19, is followed, the operation being performed on a profiling machine. The connection between the forming cam and the work is so close that the difficulties of springing and chattering, met with in the construction of the more elaborate machines required for integral cam-shafts, are avoided.

Another faceplate fixture for internal grinding is shown in Fig. 20, where it is employed for grinding the hole in the hardened nickel steel sockets used for the universal joints (see Fig. 7, Chapter I). The socket is held in the same way as when in use, by a nut screwed onto its threaded shank. It is also located in the same way, a pin in the fixture engaging a slot in the flange as shown. A limit of 0.0005

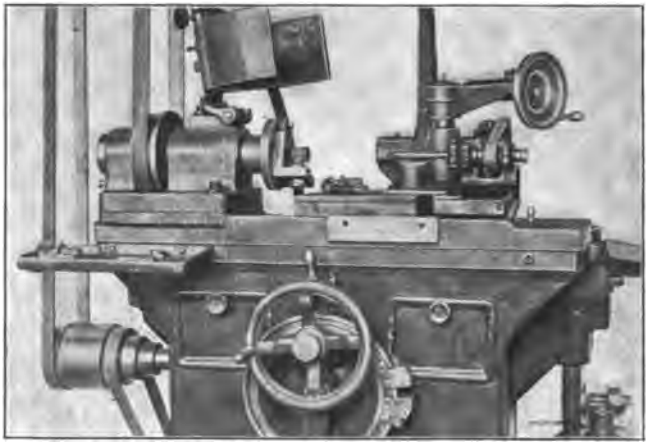

Fig. 20. Grinding the Holes in the Universal Joint Pivots

inch only is permitted in this operation, and an allowance of about 0.003 inch for the depth of the hole is the maximum, just enough being permitted for proper lubrication by the grease supply provided. This fixture is kept in place on the machine practically throughout the season. If at any time it is necessary to remove it, however, it can again be trued up by clamping a model socket in place, inserting a plug in the ground hole, and truing up the plug. These studs are held in the same way in the screw machine for roughing out the hole preparatory to grinding. The form of internal grinding spindle used should be noted. One of them is shown detached in Fig. 18, lying on the table of the machine. These spindles and their bearings are self-contained, interchangeable and adapted to work in holes of various sizes. The clutch drive provided rotates the spindle without side pressure on the bearings.

Machining the Members of the Squared Drive

As previously mentioned, the use of keys is eliminated in the drive of the Stevens-Duryea machine, their place being taken by square sockets throughout. A tapered square drive is used to connect the crank-shaft with the driving member of the clutch. The method of machining this is shown in Fig. 21. It has been found advisable to keep the milling machine set up for this work, continuously, owing to the difficulty of making a good taper square fit. When the machine has once been set, it is kept so throughout the season. An ordinary dividing head is used, as shown, tipped up to the angle of the taper. To the faceplate of this dividing head is clamped the fly-wheel flange of the crank-shaft. The outer end of the crank-shaft is supported in a suitable steady-rest as shown. For shorter lengths of crank, filling pieces are employed, having flanges bolted to the faceplate at one end, and to the work at the other. The use of filling pieces permits machining of the full line of crank-shafts without disturbing the adjustments.

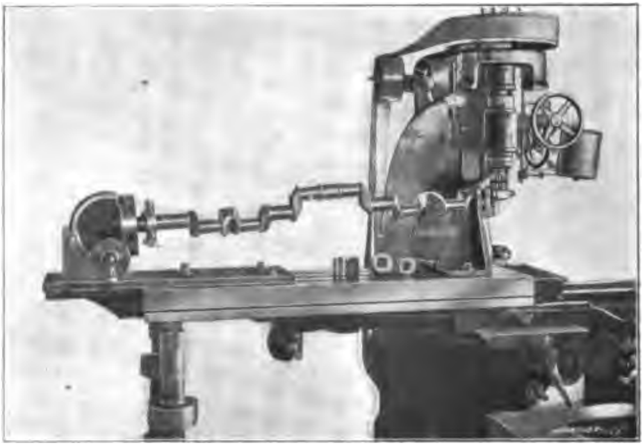

Fig. 21. A Vertical Milling Machine set up for Milling the Tapered Square Drive on the Crank-shaft

The automatic cross-feed is employed in feeding the work past the end mill in the vertical milling attachment. The table has to be so far overhung that an out-board support is provided as shown, which permits this cross-feed. This consists of a sliding guide, supported by two standards, reaching to the floor and provided with jack screw adjustments for careful leveling.

The squared holes of the drive are finished on a La Pointe broaching machine in the usual manner. The further machine shown in Fig. 23 is engaged in finishing taper square holes in the clutch driving flange, this being the member into which the taper squared end of the crank-shaft shown in Fig. 21 fits. The hole is first reamed out to a taper a little larger than the distance across the flat of the finished hole. The work is then mounted on a broaching machine on the fixture shown in place. As may be seen, the broach cuts one corner of the square hole, and one-half way up each of the two adjacent sides, into the relief formed by the taper hole. A dog is fastened to the hub of the work, and the latter is mounted on a taper plug fitting the hole, with the tail of the dog located by a pin in the faceplate of the fixture, the latter being mounted on the faceplate of the machine at an angle as shown, to agree with the angle of the corner of the tapered sides.

One pass of the broach finishes one corner of the tapered hole. The broach is then returned to the starting position, the work is drawn off the taper plug, the dog indexed to the second pin on the faceplate, the work is put in position and the second corner broached. This operation is repeated until the four corners have been machined, and the square hole finished, the work being centered on the taper plug of the fixture throughout the whole operation. A taper square gage is shown lying on top of the broach in the engraving. This is used for testing the fit of the holes and the accuracy of the work, and a most accurate fit is made on this by no means easy operation. In the machine in the foreground, another operation is being done—that of

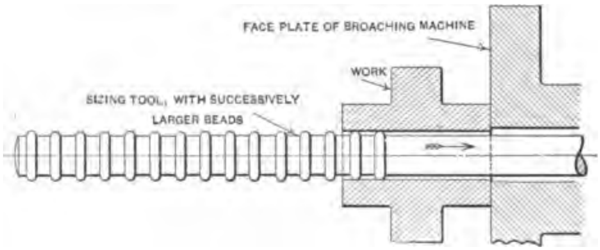

Fig. 22. Method of Sizing Phosphor-bronze in the Broaching Machine by Compression

broaching the driving slots in the driving clutch members for the multiple disks.

Sizing Round Holes in the Broaching Machine

Another unusual operation for which the broaching machine is here used, is that of sizing holes in hard phosphor-bronze bushings. This material, as any mechanic who has had any experience with it knows, is as hard on a finishing reamer as anything well can be. It is tough, elastic and slippery, and the less there is to ream the more difficult becomes the operation. Instead of reaming such holes, the tools shown in Fig. 22 are used in this shop. It will at once be seen that the operation is that of compressing the metal in the sides of the hole, until it has been enlarged to the finished size. The tool is drawn through the work. Each of the rounded rings or beads is a little larger than its predecessor, thus gradually compressing the metal the desired amount. The finished hole springs to a size smaller by some few thousandths than the diameter of the largest ring on the tool, so that the size of the latter has to be determined by experiment. This allowance varies slightly also, as may be imagined, with the thickness of the wall of metal being pressed. In such a part as that shown in Fig. 22, for instance, after drawing through the sizing tool in the broaching machine, it will be found that the hole will be somewhat larger in the large diameter of the work than in the hubs. It has been found that this difference in size can be practically avoided by passing the sizing tool through the work three or four times. Few pieces of this kind are found, however. The operation is a rapid one as compared with reaming.

An Adaptable Lapping Machine

The machine shown in Fig. 24 was built mainly in the factory, use being made, however, of the adjustable columns of a Taylor & Fenn

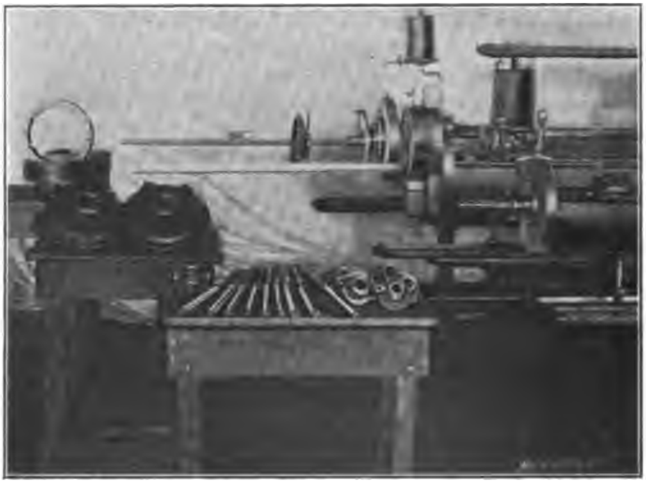

Fig. 23. A Set of Interesting Broaching Operations

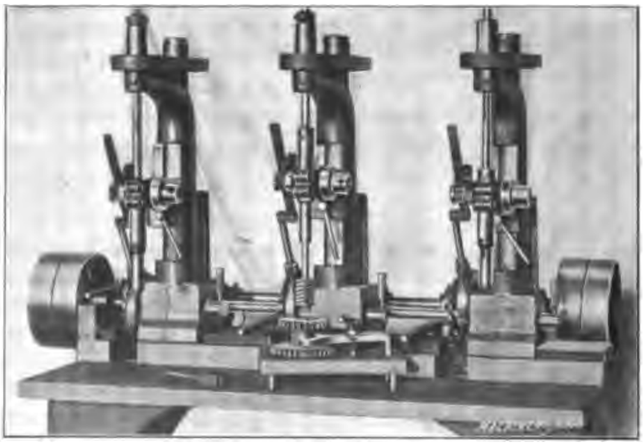

Fig. 24. Machine for Circular and Square Lapping Operations

sensitive drill press. This special machine is intended for lapping out the square holes of the drive, but is provided also with a rotary movement in addition to the vertical movement thus necessary, so as to provide for cylindrical lapping as well. The driving pulley at the right gives the reciprocating motion, while the pulley at the left rotates the spindles through the medium of the regular geared speed drive. The sprocket wheels shown, driven from the right, are loose on the driving shaft, and carry eccentrics whose rods are extended to form racks engaging, through a suitable clutch connection, the pinion shafts by which the spindle quills are fed up and down. It is thus possible to give a rotating and reciprocating movement to the spindles, either together or separately.

Separating Piston Rings

Another milling operation is shown in Fig. 25. It is a common practice to make piston rings on an automatic machine specially rigged up for the purpose, separating the rings from the finished casting by means of a series of parting or cutting off tools, each of

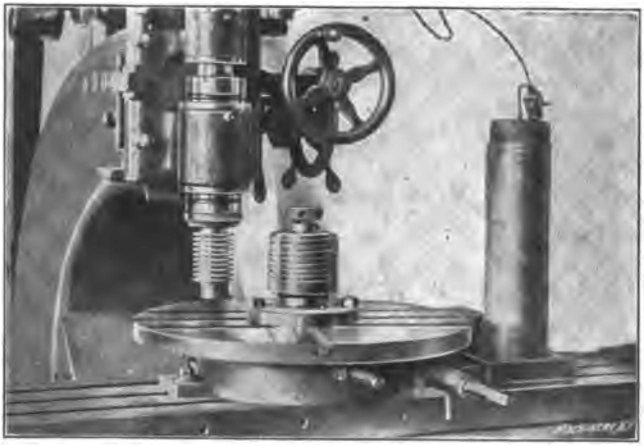

Fig. 25. Cutting out Piston Rings in the Vertical Milling Machine

which is set a little in advance of the other so that the rings will cut off in regular succession. The parting tool, however, especially when used in severing cast iron work like this, having an eccentric bore, leaves a considerable burr. In the method of severing the rings shown here, the eccentric cylinder is first finished complete on the turret machine. Then it is mounted on an internal expansion chuck on the faceplate of the cylindrical attachment of the Becker vertical milling machine, as shown. This chuck is provided with clearance grooves for the gang of saws shown in the engraving. These are sunk into the cylinder, and then the work is rapidly revolved, cutting out the eight rings at once. The saws are permanently mounted on their arbor, with separating collars ground to the proper thickness.

Examples of Fixtures Used for Drill-press Operations

The drilling department seems unusually small, when compared with the size of the whole plant, and gives the appearance of being worked at high pressure. The large output required is evidently maintained by the universal use of highly developed jigs for all manufacturing operations. Multiple spindle drill presses are used to almost the entire exclusion of the single spindle type.

Fig. 26 is interesting as showing the development of the jig for a comparatively simple operation—that of drilling the cotter pin hole in a headed cylindrical stud. In the first apparatus employed (not shown) the stud was pushed into a hole up to its head, and held there by a lever, one piece being done at a time. This rigging had two faults. One piece at a time is held, and trouble with chips and burrs was experienced, as might be imagined. An improvement on this device is shown in the two jigs at the right, where a base with a set of V's is provided in which several of the pins may be placed, their heads being pressed up against the end of the V-block by

Fig. 26. Interesting Drill Jigs for a Simple Operation

springs. The cover being clamped down on the work, the parts are thus held for the drilling operation. This, however, was not quite easy enough to clean to suit the ideas of the tool designer, so the fixture shown at the left was used for the next tool of this kind that had to be made. Here hinged sides are used instead of springs as in the previous case. These sides fold up and press the heads of the work against the edges of the V-block. When they are turned down and the cover of the V-block is raised, the top surface of the V-block is all clear, so that the presence of chips shows inexcusable carelessness on the part of the operator. When the sides are folded up against the work and the cover is brought down, the latter, by means of wedge surfaces, presses the sides in, holding the heads of the work firmly in place and clamping them down on the V-block at the same time.

The jig shown at work in Fig. 27 is used for drilling and reaming the connecting-rod holes. It is of the "four-legged table" variety, with suitable clamps and hook bolts for taking the strain of the cut without permitting noticeable deflection and consequent inaccuracy in the work. A feature of the construction which is, perhaps, old enough, but probably new to many, is the provision made for both drilling and reaming with a fixed bushing, thus avoiding the use of slip bushings of different diameters. For drilling, the jig is used as shown in the engraving, with the work clamped beneath the plate and the jig bush-

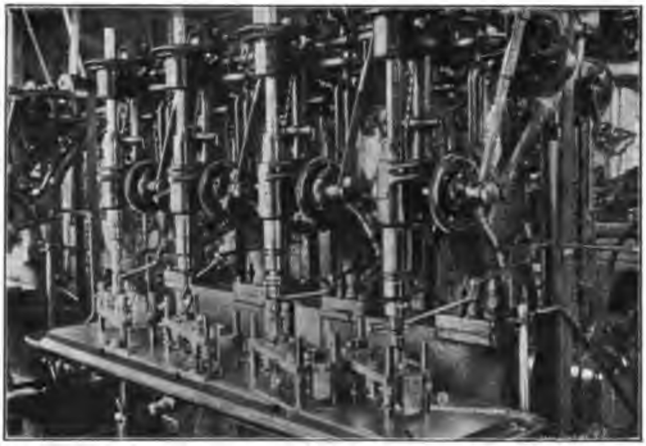

Fig. 27. Gang Drill used in Drilling and Reaming Connecting-rod Ends

Fig. 28. An Unusual Array of Automatic Chucking Machines; Thirty-one are used in this Department

ings above, guiding the drills. For reaming, the jig is reversed and a reamer is used having a pilot, which passes through the work into the jig bushing (now on the under side of the plate) by which it is guided.

Fig. 28 shows what is by long odds the largest aggregation of automatic chucking machines the writer has ever seen. There are thirty-one of the Potter & Johnston type. Practically every turned part not made in the screw machine from the bar is produced on these machines. That old standby, the engine lathe, appears to be about the rarest machine tool in the shop.

Fig. 29 shows a section of the engine assembling room. It will be noted that machine tools are few and far between, the only ones in

Fig. 29. The Engine Assembling Department

sight being a drill press, speed lathe, and two or three grinding stands for sharpening tools. This shows that the manufacturing operations have been performed with great exactness. The question of assembly is simply one of bolting and screwing the separate parts together. The engines here shown are of the four- and six-cylinder type. The overhead trolley lines should be noted.

CHAPTER III

MANUFACTURING AUTOMOBILE EQUALIZING GEARS*

The present chapter deals with operations which do not present any especially unusual or spectacular features, yet they have a value derived from the fact that they are closely related to the operations which produce the bulk of the product of the machine shops of the country; for that reason they should attract the attention of mechanics interested in accurate and economical work. The operations for making a complete, compact machine unit—a differential or equalizing gear for automobile use, is described from beginning to end. The completeness of the job gives it a suggestive value that would not be

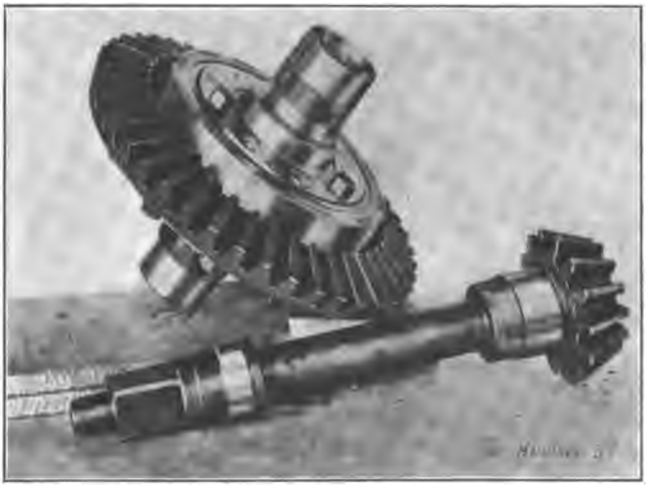

Fig. 30. The Equalizing Gear Complete, with Bevel Gear and Pinion

offered by a series of miscellaneous operations, however interesting. The value of this description, however, does not depend on its completeness alone, as many of the specific shop operations give evidence of a high degree of manufacturing ability.

Description of the Equalizing Gear

Figs. 30, 31 and 32 show assembled, dismantled and detail views, respectively, of an equalizing or differential gear, designed by Mr. A. A. Fuller, of the Providence Engineering Works, Providence, R. I. The determining feature of this design is the necessity for getting a maximum of strength and effectiveness in a minimum of space—coupled,

* MACHINERY, December, 1909.

of course, with reasonable cost of manufacture. This problem was attacked by scientific analysis. It was possible, without great difficulty, to obtain reasonable strength in the casing which contains the equalizing gearing. The crucial point was in the design of the equalizing gears themselves. In determining the proportions of the gears, curves were drawn showing the strength of the teeth for lay-outs of varying pitch and number of teeth, arranged to be contained within a casing of a given diameter. The strength and bearing area of the pivots, and the strength of the pinions as limited by the thickness of the shell between the bottom of the tooth and the bore, had also to be reckoned with. The tooth shapes were not confined to standard forms, but various pressure angles and heights of addendum were

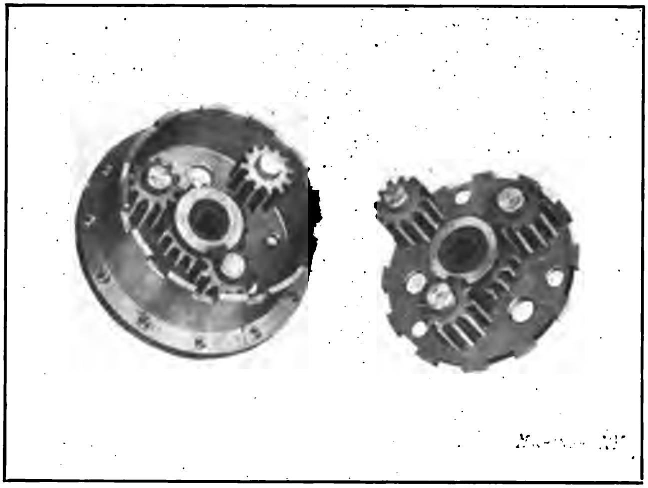

Fig. 31. A Small Size of Equalizing Gear Dismantled to show Construction

investigated. By comparing the curves for various possible designs, a certain pitch, number of teeth and shape of tooth for the various gears were found for each diameter of casing, so proportioned that if any of the dimensions were changed, the mechanism became weaker instead of stronger. These proportions, worked into a design satisfactory in other particulars, have been adopted as standard, and the makers feel confident that it is impossible to enclose in the same space gears of greater strength than they are offering in the design illustrated herewith. As this confidence is based on mathematical calculations and has been further tested by many months of experience, it seems reasonable that they should hold to it.

Referring particularly to Fig. 32, the mechanism is contained within case *B* and covers *A* and *A'*. It revolves in the rear axle gear casing on ball bearings, mounted at the ends of casings *A* and *A'*, and the driving bevel gear is carried on the periphery of case *B*, to which it is clamped by hexagon-head screws *H*. The pivots *E* are riveted into the flanges of covers A and A', three in one side and three in the other. These pivots carry pinions F and F' meshing with gears C and C'; the latter run in bronze bushings D and D' forced into the two covers, and are provided with broached square holes by which the floating wheel shafts are driven. As will be seen in Fig. 31 in connection with Fig. 32, gear C meshes with pinion F', which also meshes with pinion F, the latter in turn engaging gear C'. Thus, when gear C is turned, gear C' is revolved in the opposite direction, and *vice versa*, thus forming a spur gear differential mechanism.

Attention may be called to some of the features which make for strength in this design. It will be seen, for instance, that the gears have teeth of special shape and of very coarse pitch and few numbers of teeth. The pinions have eight teeth and the gears sixteen each. In

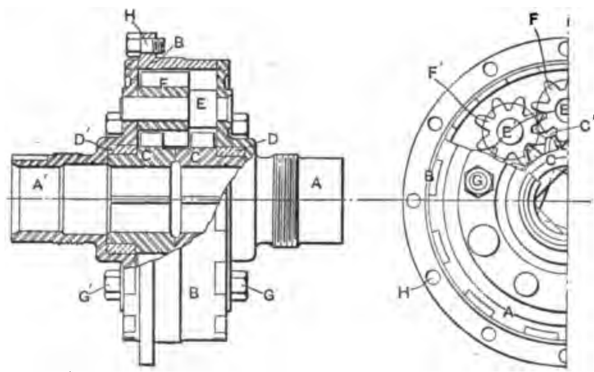

Fig. 32. Details of Construction of the 7-inch Equalizing Gear

designing the mechanism by analysis, as described, it was found that this construction was necessary for strength. Older designs of this kind, more commonly met with, in which the pinions are smaller in proportion to the gears, have repeatedly proved their weakness by breakage.

Mention should also be made of the solid way in which the parts composing the casing are fastened together. The casing B is provided with tongues locking into the grooves cut in covers A, so that the strain of transmission is taken on these interlocking members and is not taken by the bolts, dowel pins or similar parts. So far as this torsional strain is concerned, the casing is as strong as if it were made of solid metal—an impossible construction, of course. Through bolts and nuts G and G' clamp the whole casing firmly together.

The proper meshing of the bevel gears can be controlled by shifting the whole casing axially in its bearings. Nuts are mounted, for this purpose, one on the threaded diameter of A and the other at the same point on A'. By loosening one and tightening the other the teeth of the gears can be brought more closely into contact, or *vice versa*.

The provisions for oiling should be noted. The casing on the rear axle is provided with a bath of oil in which the bevel gears run. Three holes cut in the exterior of *B* (not shown in Fig. 31, but visible in the detail views of the operations in Fig. 33, and at the right of Fig. 34, where these holes are being drilled) admit oil from this bath into the interior spur gears. Pivots *E* and pinions *F* are grooved, as are also gears *C* and *C′* permitting a flow of oil through the whole structure, kept in constant motion through the revolving of the parts.

In describing the manufacture of this device we will take up each part in turn. The manufacture of the bevel gears will not be described

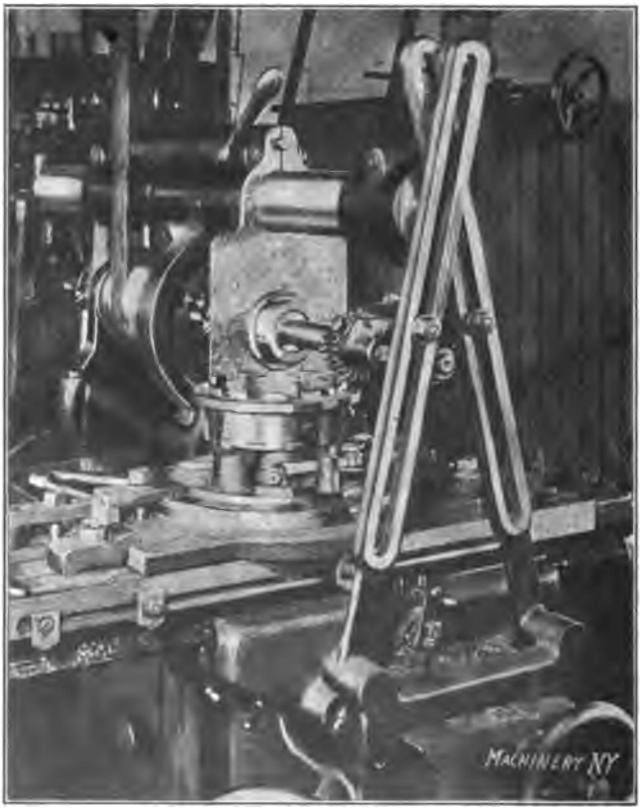

Fig. 33. Milling the Drive Tongues in the Gear Case—Second Operation

in detail, as their design is determined by the maker of the car in which the device is to be installed. The first part to be considered will be the gear case, shown at *B* in Fig. 32.

Operations in the Manufacture of the Gear Case

The case is made from a malleable iron casting on which the first operation, naturally, is that of snagging to remove fins, gates, etc. The second operation is performed in the Jones & Lamson flat turret lathe, of which large use is made in this shop. The casting is placed in the chuck of the machine with the flange outward. In this operation the hole is finished to size, the flange is turned, and the projecting end is faced. The regular equipment is used for this purpose, the only special tools being gages for the inside diameter of the hole and the outside diameter of the flange.

In the third operation, performed in the same machine, the part is grasped by the finished flange in special soft chuck jaws, which have been turned in place to fit the diameter they are to receive. This gives assurance that the work done in this operation will be true, within reasonable limits, with the cuts previously taken. Regular flat turret lathe equipment is used for this operation as well, suitable gages of

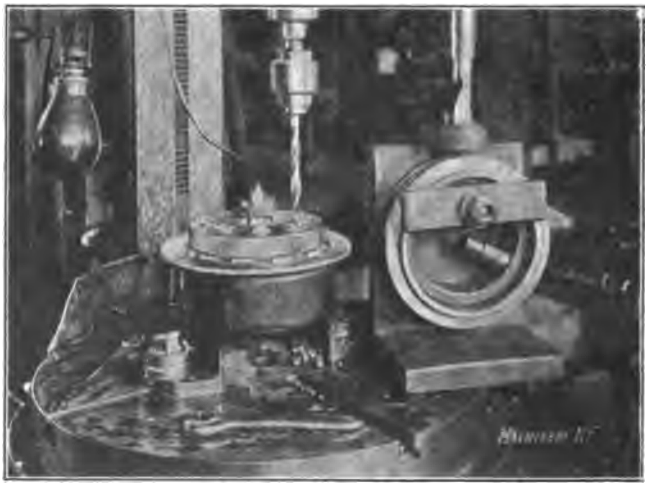

Fig. 34. Drilling the Three Oil Supply Holes in the Case (see Fixture at the Right), and Drilling the Bolt and Pivot Holes in the Cover

simple construction being provided. The next operation, shown at the right of Fig. 34, is drilling the three holes which admit oil to the interior of the case. This jig is of the simplest possible construction, consisting of a knee with a turned seat on which the work is placed, and an overhanging lug carrying a drill bushing. A clamp provides for holding the work, and a plug, entering a suitably located hole in the seat, provides means for indexing the second and third holes drilled, from the one previously completed. The other operation shown in this engraving will be described later on.

The tongues which interlock with the grooves in covers *A* and *A'* (see Fig. 32) have next to be milled. The fixture for doing this is shown in use in Fig. 33. It consists of a base provided with an index plate and a revolving table, by means of which the work may be indexed step by step to cut the various tongues. These are shaped by straddle mills which form the opposite sides of the tongues parallel, so that they fit into corresponding grooves milled into the covers by a straight-sided cutter. In the operation illustrated, tongues have been cut on one side of the casing, which is located in its seat in the fixture by the interlocking of these tongues with grooves provided to receive them as shown. This assures alignment of the cuts on each edge of the case. In the first operation the uncut edge of the work is simply set down onto

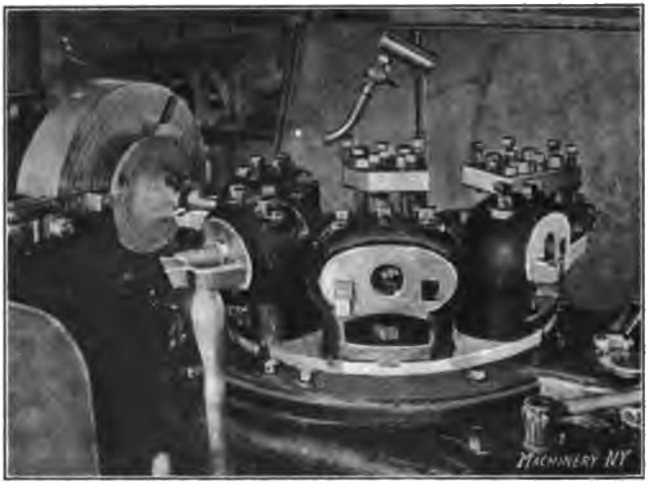

Fig. 35. The First Turret Lathe Operation in Finishing the Gear Case Covers

Fig. 36. Second Operation on the Flat Turret Lathe using Special Jaws

this seat. It is held down by three clamps, provided with noses which enter the three holes drilled to admit oil to the interior of the mechanism.

It is interesting to see the expertness with which the operator cuts out these tongues. The automatic feed is set at the highest point practicable when cutting the full depth. As this would be less than the maximum possible when the cutter is entering the work, he begins with a hand-feed at a considerably higher rate, throwing in the automatic feed when the cutter gets down to work. Although the machine is of modern construction, the workman feeds in all the belt can handle. The gear casing is now complete except for certain operations performed on it in assembling, as described later.

Operations on the Gear Case Cover

The gear case covers are made from machine steel drop forgings. After the snagging, the first operation is the simple one of putting a 1½-inch hole through the center of the forgings. This is a drill press operation and is merely done to remove stock, it being, of

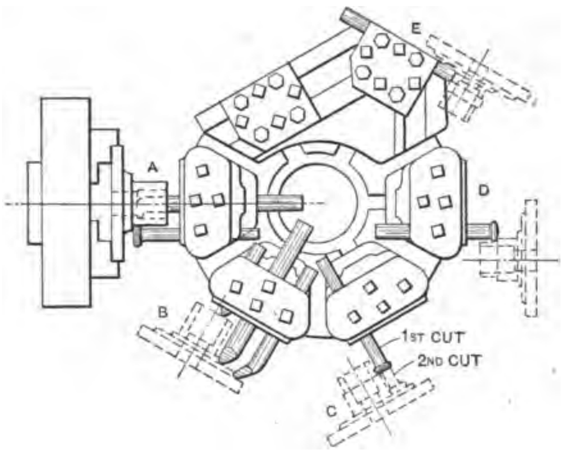

Fig. 37. Layout of Tools on the Flat Turret Lathe for the Operation shown in Fig. 35

course, impracticable to form the hole in the forging. It is next clamped by the rim with the hub projecting, in the chuck of the flat turret lathe. This first turret lathe operation is shown in Figs. 35 and 37, the latter diagram indicating the arrangement of the tools.

The first cut is shown at *A*. An outside turning and boring tool, acting in conjunction, rough turns the hub and rough bores the hole. At the next station, *B*, three tools simultaneously face the end of the hub and the two surfaces of the flange. Two cuts are taken with these, one for roughing and one for finishing. A third cut is taken with the same tools fed axially against the work to form the two grooves in the face of the flange, as most plainly shown in Fig. 32. At the third station *C*, another turning tool removes the stock on two diameters of the hub, two cuts being taken. At *D* a finishing cut is taken over the smaller diameter, while at *E* a form tool shapes that portion of the hub extending from the threaded diameter to the flange. This operation is completed in about 18 minutes.

In the second operation (see Figs. 36 and 38), the completed end of the piece is grasped in soft jaws turned to fit the surface they grasp, assuring true running of the surfaces made in the two operations. The tool at *A* bores out the large diameter of the hole, which is for clearance only. The reamer at *B* finishes the small diameter to size. The tool at *C* faces the flange, taking two cuts, one to rough out stock and the second to bring it to size. A flat-nosed tool at *D* finishes the flange. The tool at *E* roughs out the counterbore, while that at *F* finishes it. This latter tool is fed directly in, boring the diameter of the counterbore to size until the bottom is reached, when the sliding head is fed outward, so that the same tool faces the bottom of

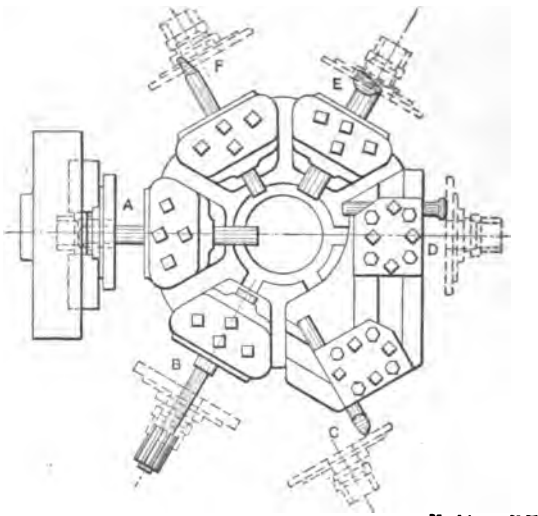

Fig. 38. Layout of Tools in the Operation shown in Fig. 36

the counterbore. The finishing is thus done by turning cuts instead of forming cuts, giving a higher degree of accuracy. Work of this kind shows the flat turret lathe to very good advantage. In the layout of tools shown in Figs. 37 and 38, there were probably no special tools of any kind required, with the exception of the form tool *E*, the rest being stock turning tools of the kind which form the regular equipment of the machine. It may have been necessary in some cases to give the tool a knock of the hammer on the blacksmith's anvil to bend it in one direction or the other, but nothing more would be needed. The cross sliding head and the multiple stops come into play in such operations as those at *B* and *C* in Fig. 37, and *F* in Fig. 38, giving each separate tool a wide range of usefulness, especially when it is so made that it can be used for both turning and facing jobs.

Of course there are all sorts of opinions about such matters, but in the question of hand *versus* automatic machines, this company believes that the conditions favor the use of the hand turret lathe in its work. The simplicity of the tooling is an important factor on contract work. The management can never be sure of the long continuance of any job, so that anything approaching costliness or elaboration is prohibited. Furthermore, it is reasonably certain that one hand machine will turn out more work than one automatic, particularly when, as in this shop, there is an inducement, such as the premium system, for the workman to get the very most out of his machine. He is constantly changing his feeds and speeds as the varying diameters, depth of the cut and condition of the tool require. He is thus able to take heavier cuts without injuring his cutting edges than would be possible without constant personal supervision.

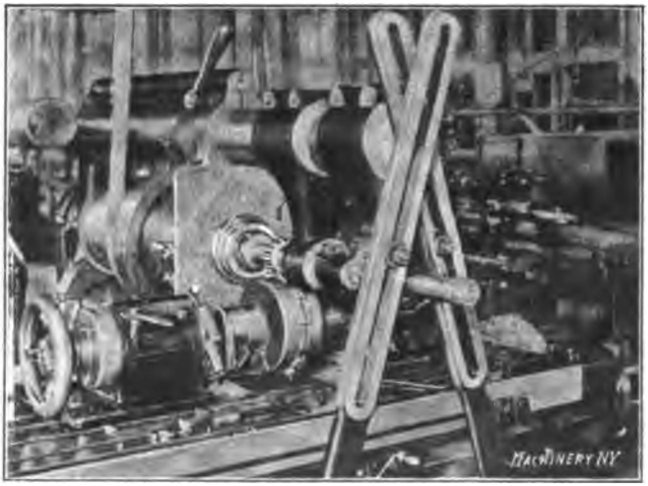

Fig. 39. Milling the Driving Slots in a Pair of Gear Case Covers

Probably three or four changes are made in each operation to one that would be made on an automatic machine. As another advantage, this greater production of the machine means a much less capital outlay per dollar of output.

It certainly does keep the operator busy to get the most out of one of these lathes. There is no possibility of his running more than one machine, on this particular work at least. Cuts are taken very rapidly and changes of feed and speed follow each other in constant succession. There is a line of demarkation at the point where the intensity of production on the part of the hand machine and the lower capital charge on machines, buildings, stock, etc., balance the higher output per man and the consequent lessened labor cost for the automatic machines. In accordance with their judgment, some shop managers will draw the line at one point and some at another. It is fortunate for the builders of both types that all men do not come to the same conclusion when reasoning from the same premises.

In Fig. 39 the milling machine is shown rigged up to cut the driving slots in a pair of the gear case covers. The two are mounted together face to face on a special iron arbor, having a driving tail cast integrally with it in place of the usual separate dog. A formed cutter is used which shapes the bottom of the slot to the true radius of the inside diameter of the casing *B* (see Figs. 32 and 33) assuring a tight fit. This operation and that shown in Fig. 33 have to be done to close limits with good indexing plates, only 0.001 inch variation being allowed on the thickness of the slot and the tongue. This means that in order to make a good fit the dividing must be very accurate. In the cases the writer has seen assembled, these parts drove together with a very little gentle urging from a lead hammer. Not much of

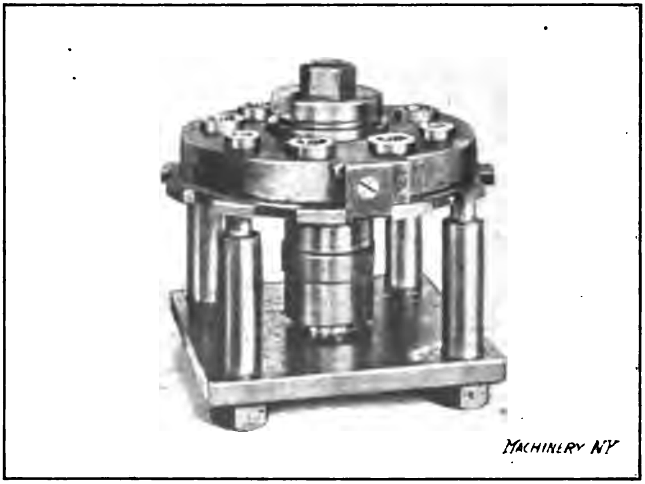

Fig. 40. Jig for Drilling the Bolt and Pivot Holes in the Gear Case Covers. Another Jig for the Same Operation is shown at the Left of Fig. 34

anything else seemed to be required. In Fig. 40 is shown a jig for drilling the bolt and pivot holes in the gear covers. It is of simple construction, the cover being supported on four legs and located by a central spindle over which it is dropped and by which it is clamped, an open side collar and nut being used as shown. The bushing plate set over the work is located to bring the holes in right relation with the slots, by a tongue entering the latter. In the next operation the covers are mounted on a special faceplate, as shown in Fig. 41. This faceplate is surfaced true in place and is provided with an expansion mandrel centered integrally with it. The gear case is slipped on over this mandrel and tightened in place by turning on a wedge screw. While thus held the countersink in the outer end of the hub, the seat for the ball bearing, and the threaded diameter are turned. The thread is also cut. This is done by the Rivett-Dock threading tool, shown in operation. These operations of countersinking, turning and threading, altogether, average about eight minutes time for each piece. When the turning was in progress, the writer timed the lathe and found it was making 250 revolutions per minute, which gives about 150 surface feet per minute for the cutting speed.

A fixture and mill of obvious construction are used for cutting the keyway by which the inner race of the ball-bearing is made fast to the hub.

Equalizing Pinions, Studs and Gears

Studs *E*, Fig. 32, are made on the Gridley automatic turret lathe with the regular tools and equipment, the job being, of course, one of the everyday variety for this machine. Oil grooves are milled, and then the burrs are removed by hand. The equalizing pinions are drilled, reamed and turned on the flat turret lathe. The ends are squared accurately to length in the engine lathe.

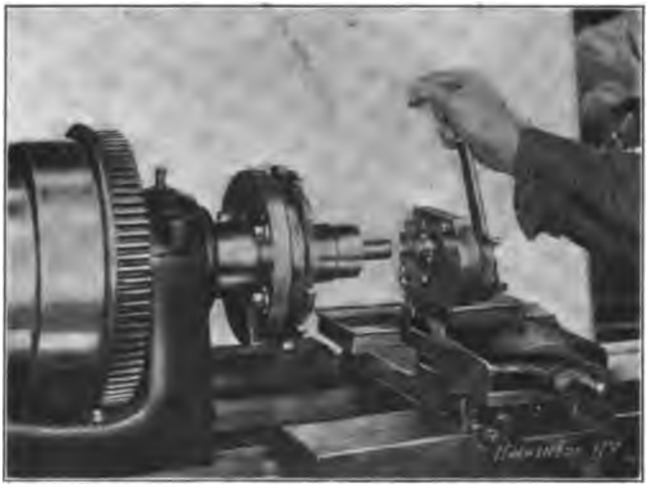

Fig. 41. Threading the Gear Case Covers with a Rivett-Dock Threading Tool

The equalizing gears are cut off to length from the bar stock (all gears and pinions are made of chrome-nickel steel) and are bored, reamed, faced and filleted at the large end in the Jones & Lamson machine. The hole is reamed accurately to size so as to furnish a guide for the broach in forming a square hole. This is done on the La Pointe machine at a single pass of the broach, which is a long one, having some 24 inches or thereabouts of cutting length. The outside surfaces of the gear are then rough turned on a square expansion chuck somewhat similar to that shown in Fig. 41 for the gear case cover, except, of course, that it is mounted on a square surface instead of a round one. In the next operation it is finish turned all over.

The spur gears and pinions are cut in a triple head indexing device which is one of the standard attachments on the Brown & Sharpe milling machine. Three cutters operate on three gangs of work simultaneously. By giving special shapes to the gears and by being very careful, both in centering the cutters and setting them to the proper depth, first-class results have been obtained—better than are needed in fact, since normally these gears are stationary or nearly so, being in operation only when rounding corners, in the case of a deflated tire on one side, or the slipping of a wheel in the mud. After removing the burrs by file and reamer, the gears and pinions are hardened by the regular process recommended by the makers of the steel (the Carpenter Steel Co.), with such modifications as the blacksmith of the shop has found advisable.

The equalizing gear bushings *D* and *D'*, Fig. 32, are cut from a bronze bar in the flat turret lathe, being turned and bored complete to size. A stack of them are placed on the Mitts & Merrill keyseater for cutting the internal oil grooves. The radial oil groove is cut on the

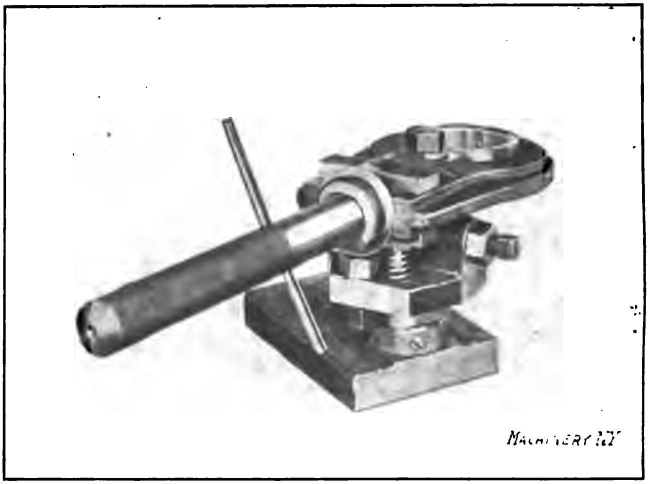

Fig. 42. A Special Fixture for Cutting Oil Grooves in the Equalizing Gear Bushing

interesting tool shown in Fig. 42. This device is a modification of the principle used in attachments for slotting screws with a saw held in the speed lathe. The knurled handle shown controls three motions. By screwing it in or out the bushing is tightened or released in the jaws by which it is held. Tripping it up or down drops the bushing away from or brings it up toward the revolving cutter, while springing it to one side brings the bushing out from under the cutter where it can be removed without interference. A wire finger locates the work with relation to the internal groove previously cut.

Assembling

The operation of assembling the parts to make the complete mechanism includes some operations worthy of notice. In Fig. 43 is a case assembled with its two covers, and dropped into a cast-iron reaming stand, where it is held from revolving by the projecting pin shown, which enters one of the three holes in its periphery. A line reamer is used, giving assurance that the two bearings in each cover will be true with each other. After this line reaming the covers are marked, numbered and burred so that the same parts will be reassembled together.

Studs *E* are next riveted to the covers, three on one side and three on the other, a hand hammer being used for this purpose. The ends of the rivets are cupped to facilitate this operation. The pinions are assembled on the studs, three on each side. The bushings are pressed

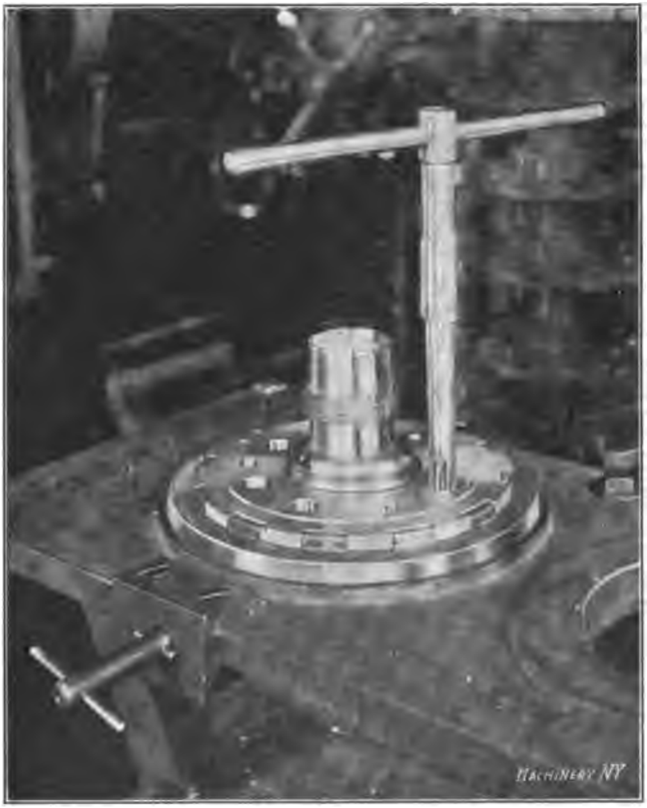

Fig. 43. Line-reaming the Pivot Holes in the Assembled Gear Cases and Covers

into the covers under the arbor press, and burred. The equalizing gears *C* and *C'* are dropped into place and the whole structure is then assembled. A square wrench inserted through the bore into the squared hole in *C*, permits the gears to turn until they are all engaged. Three bolts and nuts *G* and *G'* are now passed through, binding the whole solidly together.

It is of extreme importance in the quiet running of an automobile that the bevel gears run true. For this purpose the bevel gear seat on the outside diameter of the casing is not finish turned until it has been assembled as described. To do this, the mechanism is mounted on the lathe on large centers, bearing on the countersinks in *A* and *A'*. These countersinks, being formed in the same operation with the ball bearing seats and the threads, are true with them. After this turning and facing, a jig fitting on this accurate seat is used for drilling the flange holes through which screws *H* pass to fasten the bevel gear to the casing.

The gear is pressed into place in its seat by a simple contrivance which illustrates the demand for conveniences created by the prem-

Fig. 44. A Convenient Fixture for Assembling the Gears on the Gear Case

ium system. On the bench in front of the workman is a cast-iron seat (Fig. 44) in which the bevel gear is placed face downward. The complete differential mechanism is then placed over the gear in a position to be forced down into it. The workman now reaches up above his head and brings down the hand-wheel, clamping screw and clamp shown, which is suspended by a counterweight so as to move freely up and down and remain stationary in any position. Entering the screw in the nut in the base of the device and turning the hand-wheel, forces the casing down into the gear and thus completes the assembling. The tap bolts are now put in and are wired through holes drilled through their heads, to prevent them from turning. This completes the making of the equalizing gear.

A Good Tapping Record

While the making of the bevel gear has not been described, it will not do to pass over one of the operations met with. This is the operation of tapping the holes by which the gear is held to the flange. These holes are 5-16 inch in diameter and 13-16 inch deep and are

Fig. 45. A Tapping Operation and Operator with a Remarkable Record—75,000 Blind 5-16-inch Holes in Chrome-nickel Steel without breaking a Tap

blind, being tapped to a bottom and not through. The tapping is done in a Cincinnati drill press (Fig. 45), using an Errington friction chuck. Tapping in chrome-nickel steel by power is, it will be agreed, no "fool of a job." One of the difficulties met with is the tendency of the metal to seize the tap and break it when backing out.

The operator shown broke many taps in becoming familiar with his job, but since he has gotten into the swing of it, he has tapped 75,000 of these blind holes in chrome-nickel steel without breaking a tap.

The credit of this record must be divided between the man, the machine, the chuck and the tap, but there is enough to make a respectable showing for all four. The operator's increase of efficiency was obtained with practically no change in the tools or methods, being due simply to the training of his judgment in the feeling of the tap, and in the use of excellent tools. It might be said that a firm of the highest

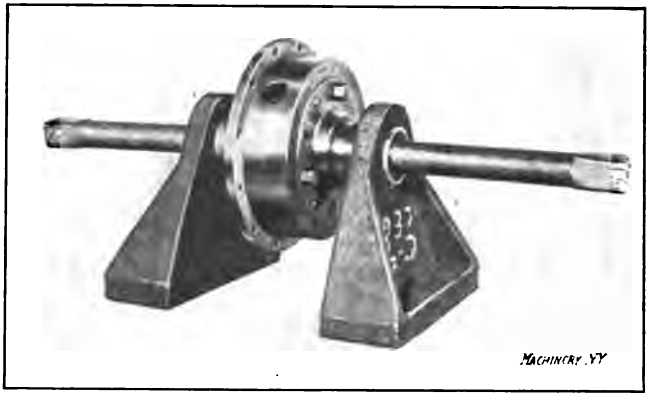

Fig. 46. A Completed Equalizing Gear Set up for Testing to Destruction

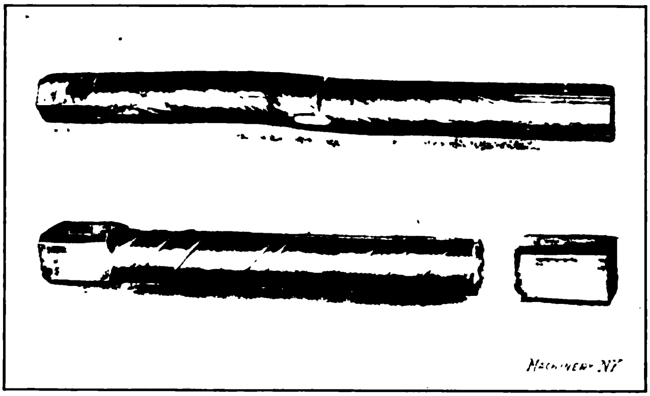

Fig. 47. Condition of Shafts Broken in Tests shown in Fig. 46; the Gears were Uninjured

reputation for accuracy and for skill in manufacturing had asked ten cents a hole for the job. This operator runs two taps in each of the twelve holes in a gear, twenty-four holes in all, in from 15 to 18 minutes.

Tests on the Finished Casings

Of course, the object that was aimed at in designing these equalizing gears for sale to manufacturers of automobiles, was to give them such strength that some other part of the machine would break first. In order to find out whether or no this result had been obtained a number of tests were made in the laboratory of the engineering school of Brown University. In Fig. 46 the casing is shown as mounted in brackets for a torsion test, the power being applied through 1-inch, 3½ per cent nickel-steel shafts, specially treated. These failed at 20,300 inch-pounds, twisting through 800 degrees before rupture. Samples of broken shafts are shown in Fig. 47, and give some idea, in combination with the figures just given, of the excellence of the material used in these shafts. No damage of any kind was found inside the gear casing, the mechanism being unbroken and running as easily and smoothly as before.

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