# Tutorial on Carrier/Noise & Eb/No Measurements

PARADISE DATACOM

## Introduction

This is the source of much confusion and `Specmanship`. This brief tutorial explains what the terms mean and how to measure and interpret the figures you see. The diagram on the following page attempts to summarise this as clearly as possible visually with the text below adding detail for which there is no room on the diagram. Finally a table provides a quick look up reference to convert values measured on a spectrum analyser to a real Eb/No.

## Derivation of Eb/No from (C+N)/N

It is possible to describe the **Carrier / Noise** ratio within the Rx system at an arbitrary reference point, eg at the input to the demodulator, at the input to the Forward Error Correction decoder (FEC), at the input outer Reed-Solomon FEC decoder, or even at the terrestrial data interface. Many of the terms you come across represent this Rx Carrier / Noise ratio at these different reference points.

It is common to measure **(Carrier + Noise) / Noise** at the demodulator input on a spectrum analyser. Ideally we are after the Carrier / Noise ratio, but as we cannot turn off the atmospheric noise (!) we have to make the measurement with the noise on, and then convert back to simply Carrier / Noise. In the equation below the term (C+N)/N is the (Carrier+Noise) / Noise ratio read from the spectrum analyser.

$$\frac{Carrier}{Noise} = \frac{C}{N} = 10\log_{10}\left(10^{\left(\frac{(C+N)/N}{10}\right)} - 1\right) = \frac{E_s}{N_0}$$

We can express this same Carrier / Noise ratio at the input of the demod in terms of the Energy per *Symbol* / Noise power density, which is written as $\mathbf{E_s / N_o}$. Because `Carrier / Noise` is a ratio of two powers measured in the same bandwidth (the resolution bandwidth of the analyser), this is the same as $E_s / N_o$.

Within the demodulator, each *Symbol* is converted back into the *Transmitted Bits*. For BPSK each Symbol represents only 1 Transmitted Bit, for QPSK or OQPSK each Symbol represents 2 Transmitted Bits, and for 8PSK each Symbol represents 3 Transmitted Bits. The expression $\mathbf{E_t / N_o}$ (or $\mathbf{E_{bt} / N_o}$) represents the Carrier to Noise ratio referenced to this *Transmitted* Bit rate. For QPSK and 8PSK the *Transmitted* Bit rate is higher than the *Symbol* rate, and so $E_t / N_o$ (ie the Carrier / Noise referenced to Transmitted Bit rate) is lower than the $E_s / N_o$ (the Carrier / Noise referenced to the Symbol rate), as this same power is referenced to a higher bit rate. Allowing for the change in bit rate in the demod gives:

$$\frac{E_t}{N_0} = \frac{E_s}{N_0} - 10\log_{10}(No\ of\ Bits\ /\ Symbol)$$

Next in the demod chain comes the FEC Decoder. The receive Carrier / Noise referenced to this point is referred to as $\mathbf{E_{Dec} / N_o}$. In the FEC Decoder the bit rate is reduced as the data is decoded to provide corrected data at a lower rate. Because the data rate reduces through the FEC Decoder $E_{Dec} / N_o$ is higher than $E_t / N_o$ as the same power is referenced to a lower bit rate. Allowing for the change in bit rate in the FEC decoder gives:

$$\frac{E_{Dec}}{N_0} = \frac{E_t}{N_0} - 10\log_{10}(FEC\ Code\ Rate)$$

Following the FEC Decoder (the `inner FEC`, ie Viterbi, Sequential or TCM) comes the Reed-Solomon `Outer FEC` Decoder (if active). This `Outer FEC` RS Decoder operates similarly to the `Inner FEC` Decoder, reducing the bit rate as it corrects errors and finally generating the Composite Information Bit Rate at its output. The Carrier / Noise expressed at this Composite Information Bit Rate is referred to as the **$E_b$ / $N_o$** (or **$E_{bi}$ / $N_o$** or **$E_i$ / $N_o$** with `I` and `b`referring to *Information* and *Bit* respectively). Again because the data rate reduces through the RS Decoder $E_b$ / $N_o$ is higher than $E_{Dec}$ / $N_o$ as the same power is referenced to a lower bit rate. Allowing for the change in bit rate in the RS Decoder gives:

$$\frac{E_b}{N_0} = \frac{E_{Dec}}{N_0} - 10\log_{10}(RS\ Code\ Rate = (\frac{k}{n}))$$

It is this **$E_b$ / $N_o$** that is typically displayed on the front panel of Modem equipment.

Finally, after the RS Decoder comes Deframing and baseband processing such as Drop/Insert. IBS/SMS or IDR deframing does decrease the bandwidth, but the bandwidth reducion effects of deframing are ignored as INTELSAT chose to define the mandatory modem performance in terms of $E_b$ / $N_o$, specifying that this relates to the *Composite* Information Bit Rate (ie it includes framing). Baseband processing has no effect on bandwidth and is totally ignored.

**Practical Implications of Displayed Eb/No**

Practically this means the following:

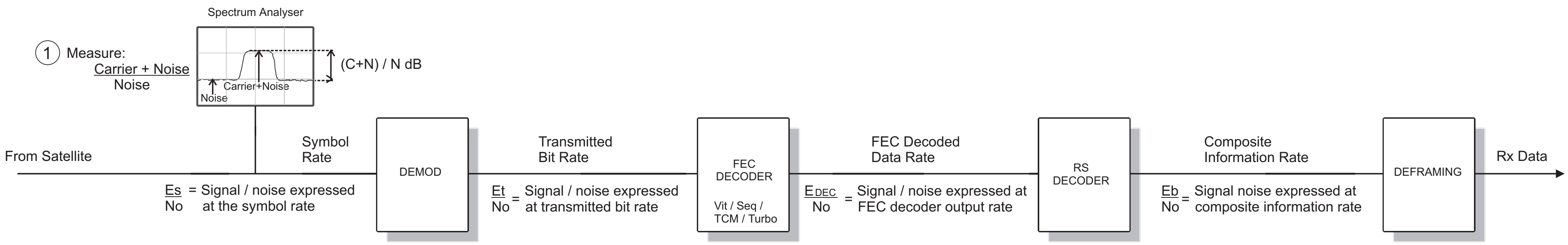
**For a constant Tx Power, you will get the same displayed $E_b$ / $N_o$ whatever the settings of Modulation Inner FEC, and Outer (RS) FEC**, as the $E_b$ / $N_o$ display compensates for these parameters. The bandwidth will vary and so will the Carrier / Noise at the input of the demod, but the displayed $E_b$ / $N_o$ will be steady as it compensates for these parameter changes.

**When comparing BER performance for different Modulation / FEC schemes against $E_b$ / $N_o$**, the plotted $E_b$ / $N_o$ already accounts for the changes in Carrier / Noise caused by the different Modulation and FEC settings. **Any BER difference you see with different schemes is a real difference, you don't have to further compensate for the changes in Carrier / Noise this change induces**.

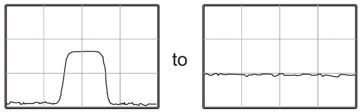
**You do however have to be aware of Bandwidth considerations**. You might find a Modulation / FEC scheme which gives an exceptional BER for a given $E_b$ / $N_o$, but it will of little practical value if the bandwidth it requires occupies a whole transponder.

When you look at BER performance figures **at a *specified* $E_b$ / $N_o$ you get the same BER performance whether the data is framed or not**. Adding framing to a Closed Network link would increase the bandwidth, decrease the Carrier / Noise, decrease the displayed $E_b$ / $N_o$ (as it does not compensate for framing) and therefore degrade the BER. However when brought back to the same $E_b$ / $N_o$ you do of course recover the same BER. Framing therefore does not degrade the BER for a stated $E_b$ / $N_o$, but it does require more power (and bandwidth) to achieve the same $E_b$ / $N_o$.

# Eb/No Explanatory Diagram




(2) Calculate ratio of $\frac{\text{Carrier power}}{\text{Noise power}}$




$$\left(\frac{C}{N}\right) \text{dB} = 10 \text{ Log}_{10} \left(10^{\left(\frac{C+N/N}{10}\right)} - 1\right)$$

(3) Determine $\frac{Es}{No}$

$$\frac{Es}{No} = \frac{\text{Energy per symbol}}{\text{Noise power density}} = \frac{C}{N}$$

Each symbol represents several transmitted bits depending on the modulation:

BPSK = 1 Transmitted bit / symbol
QPSK / OQPSK = 2 Transmitted bit / symbol
8PSK = 3 Transmitted bit / symbol

(4)

$$\frac{Es}{No} - 10 \text{ Log (\#Bits / Symbol)} \rightarrow \frac{Et}{No}$$

| | | |
|---|---|---|
| BPSK | → | - 0 dB |
| QPSK / OQPSK | → | - 3 dB |
| 8PSK | → | - 4.77 dB |

FEC decoder reduces data rate & corrects errors. Data rate reduced by FEC code rate:

1 / 2 Rate = 1 Decoded bit / 2 transmitted bits
3 / 4 Rate = 3 Decoded bits/ 4 transmitted bits
7 / 8 Rate = 7 Decoded bits / 8 transmitted bits
2 / 3 Rate = 2 Decoded bits / 3 transmitted bits

(5)

$$\frac{Et}{No} - 10 \text{ Log (FEC Code Rate)} \rightarrow \frac{E_{DEC}}{No}$$

| | | |
|---|---|---|
| 1 / 2 Rate | → | +3 dB |
| 3 / 4 Rate | → | +1.25 dB |
| 7 / 8 Rate | → | +0.58 dB |
| 2 / 3 Rate | → | +1.76 dB |

RS decoder reduces data rate & corrects errors. Data rate reduced by RS code rate:

Code rate $\frac{k}{n}$ expressed as (n,k,t)
(126,112,7) = 112 Information bits / 126 Decoded
(219,201,9) = 201 Information bits / 219 Decoded
(225,205,10) = 205 Information bits / 225 Decoded

(6)

$$\frac{E_{DEC}}{No} - 10 \text{ Log (RS Code Rate)} \rightarrow \mathbf{\frac{Eb}{No}}$$

| | | |
|---|---|---|
| (126,112,7) | → | +0.51 dB |
| (219,201,9) | → | +0.37 dB |
| (225,205,10) | → | +0.40 dB |

Bandwidth change due to framing is ignored in INTELSAT definition of Eb/No as Eb refers to the composite data rate.

**Summary**

(1) Measure (C+N)/N → (2) $10 \text{ Log}_{10} \left(10^{\left(\frac{C+N/N}{10}\right)} - 1\right)$ → (3) $\left(\frac{C}{N} = \frac{Es}{No}\right)$ (4) $- 10 \text{ Log (\#Bits / Symbol)} \rightarrow \frac{Et}{No}$

(5) $\frac{Et}{No} - 10 \text{ Log (FEC Code Rate)} \rightarrow \frac{E_{DEC}}{No}$

(6) $\frac{E_{DEC}}{No} - 10 \text{ Log} \left(\frac{k}{n}\right) \rightarrow \mathbf{\frac{Eb}{No}}$

**Example**

6.5dB → (5.40dB) - 3dB (QPSK) → 2.40dB

2.40dB + 1.25dB (3 / 4 Rate) → 3.65dB

3.65dB + 0.51dB (126,112,7) → **4.16dB**

## Tables to Convert (C+N)/N to Eb/No

| $\frac{C+N}{N}$ | $\frac{C}{N}=\frac{E_S}{N_0}$ | $\frac{E_t}{N_0}$ | $\frac{E_{Dec}}{N_0}$ | $\frac{E_b}{N_0}$ |
|---|---|---|---|---|
| 4.6 | 2.75 | | | |
| 4.8 | 3.05 | | | |
| 5.0 | 3.35 | | | |
| 5.2 | 3.64 | | | |
| 5.4 | 3.92 | | | |
| 5.6 | 4.20 | | | |
| 5.8 | 4.47 | | | |
| 6.0 | 4.74 | | | |
| 6.2 | 5.01 | | | |
| 6.4 | 5.27 | | | |
| 6.6 | 5.53 | | | |
| 6.8 | 5.78 | | | |
| 7.0 | 6.03 | | | |
| 7.2 | 6.28 | | | |
| 7.4 | 6.53 | | | |
| 7.6 | 6.77 | | | |
| 7.8 | 7.01 | | | |
| 8.0 | 7.25 | | | |
| 8.2 | 7.49 | | | |
| 8.4 | 7.72 | | | |
| 8.6 | 7.95 | **Adjust for** | **Adjust for** | **Adjust for** |
| 8.8 | 8.19 | **modulation:** | **FEC Rate:** | **RS Codec:** |
| 9.0 | 8.42 | BPSK: -0dB | ½ Rate: +3.0dB | (n,k,t)= |
| 9.2 | 8.64 | | | |
| 9.4 | 8.87 | QPSK: -3.0dB | ¾ Rate: +1.25dB | (126,112,7): +0.51dB |
| 9.6 | 9.10 | | | |
| 9.8 | 9.32 | OQPSK: -3.0dB | ⅞ Rate: +0.58dB | (219,201,9): +0.37dB |
| 10.0 | 9.54 | | | |
| 10.5 | 10.09 | 8PSK: -4.77dB | ⅔ Rate: +1.76dB | (225,205,10): +0.40dB |
| 11.0 | 10.64 | | | |
| 11.5 | 11.18 | | | |
| 12.0 | 11.72 | | | |
| 12.5 | 12.25 | | | |
| 13.0 | 12.78 | | | |
| 13.5 | 13.30 | | | |
| 14.0 | 13.82 | | | |
| 14.5 | 14.34 | | | |
| 15.0 | 14.86 | | | |
| 15.5 | 15.38 | | | |
| 16.0 | 15.89 | | | |
| 16.5 | 16.40 | | | |
| 17.0 | 16.91 | | | |
| 17.5 | 17.42 | | | |
| 18.0 | 17.93 | | | |
| 18.5 | 18.44 | | | |
| 19.0 | 18.94 | | | |
| 19.5 | 19.45 | | | |
| >20 | =(C+N) / N (error <0 .04dB) | | | |
| **Example:** 8SPK with ⅔ Rate TCM FEC, & RS=(219,201,9) | | | | |
| (C+N)/N= 8.4dB | C/N= 7.72dB | 8PSK so subtract 4.77dB | 2/3 Rate FEC, so add 1.76dB | RS Code (219,201,9) so add 0.37dB |
| **Measure** | **Look up** | **Calculate Eb/No = C/N - 2.64dB** | | |